EXAMPLES IN PHYSICS

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BY

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FOURTH EDITION: REVISED AND ENLARGED

London

MACMILLAN AND CO.

AND NEW YORK

1896

First Edition printed 1888
Second Edition 1893
Reprinted 1894, 1896

PREFACE

THE value of the mental training obtained by solving algebraical problems and geometrical riders has so long been acknowledged that these form an essential part in all mathematical teaching. Although similar practice is quite as necessary in studying physical science, it is by no means equally easy for the student of physics to obtain it, for only the more recent text-books contain any numerical examples, and these are generally insufficient in number and not carefully graduated. It is quite common to find students who have a correct knowledge of the general principles of physics, and can apply it intelligently in making a physical measurement, but who are yet unable to solve an easy problem or to calculate the results of their experimental work.

There can be no doubt that the best way of acquiring the necessary practice is by means of a regular series of quantitative experiments in the laboratory, carried on side by side with the more general work of the lecture-room; but such concurrent work is not always practicable, especially with large classes and in the earlier stages. Just as the student of dynamics has at first to confine his attention to questions of a more or less ideal nature, so in

some departments of experimental physics (for example in electrostatics) the beginner must for a while content himself with somewhat theoretical problems in place of laboratory work.

The examples in the present book (amounting to over one thousand in number) consist for the most part of questions and problems framed for the use of the Junior and Middle Physics Classes at the Aberystwyth College. To these have been added (at the end of each chapter) questions from various College, University, and Scholarship papers of recent years. A list of some of the more important examination papers from which these have been selected will be found on page 53, and the source from which each question is taken is in every case acknowledged. A considerable number of typical examples have been solved, and answers (with occasional hints for solution) will be found at the end of the book. Explanatory paragraphs have been inserted in the hope of giving assistance where experience seemed to show that it was most needed, but I have endeavoured not to trench upon the recognised province of the text-books.

The book has not been written with a view to the requirements of any special examination, but I have made use of portions of the MSS. in teaching classes of students taking the Intermediate Science and Preliminary Scientific Courses of the London University, and believe it will be found suitable for students who are preparing for these examinations.

For assistance in reading proofs and working out answers my best thanks are due to Mr. B. B. Skirrow, B.A. of the Mason Science College, Bir-

mingham; to my assistant, Mr. R. W. Stewart; to Mr. F. W. Shurlock, B.A., of the Carmarthen Training College; and to one of my students, Mr. A. H. Leete.

D. E. JONES.

University College, Aberystwyth,
August 1888.

The present edition has been carefully revised and some sixty pages of additional matter have been added. The examination questions at the ends of the chapters have been replaced by new sets of problems from recent papers. I am indebted to several friends who have kindly helped in the revision, and especially to one of my former students, Mr. W. P. Winter, B.Sc.

D. E. J.

Stafford, *January* 1893.

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EXAMPLES IN PHYSICS

INTRODUCTION

1. Units.—In order to measure any physical quantity, we have first to select as our *unit*, or standard of reference, a quantity of the same kind as that to be measured. The ratio between the quantity and the selected unit is called the *numerical value* or *measure* of the quantity. Suppose that we have to measure a definite length l, and that we adopt as our unit a length L: the numerical value (n) of the length to be measured will be

$$n = \frac{l}{L},$$

where n may be any number, whole or fractional.

2. Fundamental Units.—All physical quantities can be expressed in terms of three fundamental units, the choice of which depends upon the ease and certainty with which the standard quantities so selected can be compared with other quantities of the same kind. We might choose as our fundamental quantities a definite length, a definite force, and a definite interval of time: other units, such as those of mass and work, could be deduced from these. But on account of the difficulty of devising a permanent standard of force, the value of which would not change from place to place, such a choice would not be advisable.

The fundamental units usually adopted are those of *length*, *mass*, and *time*; these three elements can be measured with great accuracy, and standards of length and mass can easily be copied, and compared with the original standards.

THE C.G.S. SYSTEM OF UNITS.

A committee of the British Association has recommended the adoption of the centimetre, the gramme, and the second as the three fundamental units. Other units derived from these are distinguished by the letters "C.G.S." prefixed, these being the initial letters of the three fundamental units.

3. Derived Units.

Velocity.—The C.G.S. unit of velocity is the velocity of a point which moves over one centimetre in a second.

Acceleration.—The C.G.S. unit of acceleration is that of a point whose velocity increases by one unit per second. The numerical value of the acceleration due to gravity (g) is 978·10 at the equator, 980·94 at Paris, 981·17 at Greenwich, and 983·11 at the pole.

Force.—The C.G.S. unit of force is that force which, acting upon a mass of one ~~gramme for~~ a second, generates in it a velocity of one centimetre per second.

Special names are given to some of these units; thus the C.G.S. unit of force is called the *dyne*. Assuming the value of g to be 981 (as we shall do throughout), we see that a dyne is $\frac{1}{981}$ of the weight of a gramme.

Work and Energy.—The C.G.S. unit of work is the work done by a dyne acting through a distance of one centimetre, ~~and~~ is called an *erg*. The

same name is applied to the unit of *energy*, for energy is measured by the amount of work which it represents. Since the weight of a gramme is 981 dynes, the work done in raising one gramme vertically through one centimetre against the action of gravity is 981 ergs.

At the Paris Electrical Congress in 1889 it was proposed that a *practical* unit of work, called a **Joule**, should be adopted, and that this should be equal to ten million ergs (10^7 ergs).

Practical Units and Index Notation.—In any uniform system some of the units must be inconveniently large, while others are so small that the quantities with which we have to deal are represented by very large numbers. Electricians find it convenient to use a system of "practical units," each of which bears to the corresponding C.G.S. unit a ratio which is some multiple or submultiple of 10. Thus the **volt** is equal to 100,000,000 C.G.S. units of potential; the **farad** is $\frac{1}{1,000,000,000}$ of the C.G.S. unit of capacity. The prefixes *mega-* and *micro-* are used to signify "one million" and "one-millionth part" respectively. Thus a *megadyne* is a force of one million dynes; a *microfarad* denotes a capacity of one-millionth of a farad.

When very large or very small numbers have to be expressed, it is convenient to adopt the index system of notation, in which numbers are expressed as the product of two factors, the second of which is a power of 10; and it is usual to choose the factors so that the first contains only one integral digit. Thus the velocity of light, which is 300,400 kilometres per second, is expressed as $3{\cdot}004 \times 10^{10}$ centimetres per second. A megadyne is 10^6 dynes, a farad is 10^{-9} $\left(=\frac{1}{10^9}\right)$, and a microfarad is 10^{-15} C.G.S. unit of capacity.

Power, Activity, or Rate of doing Work.—The C.G.S. unit of power is the power of doing work at the rate of one erg per second. The corre-

sponding practical unit, called the **Watt**, is the power of doing work at the rate of 10^7 ergs per second. A horse-power is equal to 746 watts (see p. 15).

The work done by a watt in one second is 10^7 ergs = 1 joule.

Pressure.—The C.G.S. unit of intensity of pressure is a pressure of one dyne per square centimetre. It would be convenient if the pressure of a *megadyne per square centimetre* (10^6 C.G.S. units) were adopted as the normal atmospheric pressure: this standard would correspond to a barometric height of 75 centimetres, but, as compared with any barometric standard, would have the advantage of being independent of the value of g.

Heat.—The C.G.S. unit of heat is the amount of heat required to raise the temperature of a gramme of water through one degree Centigrade. The dynamical equivalent of one heat-unit in ergs is $4{\cdot}16 \times 10^7$: this quantity is called the mechanical equivalent of heat or "Joule's equivalent," and is usually represented by the letter J.

4. C.G.S. Electrostatic Units.

Quantity.—The unit quantity of electricity is that quantity which, when placed (in air) at a distance of one centimetre from an equal and similar quantity, repels it with a force of one dyne.

Potential.—Unit difference of potential exists between two points when the work done against the electrical forces in moving unit quantity of electricity from the one point to the other is one erg.

Capacity.—A conductor is said to have unit capacity when a charge of one unit of electricity raises its potential from zero to unity.

Magnetic Units.

Strength of Pole.—A magnetic pole is said to have unit strength when it repels an equal and similar pole, placed at a distance of one centimetre from it, with a force of one dyne.

Strength of Field.—A magnetic field is said to have unit intensity (or strength) when a unit magnetic pole placed in it is acted upon by a force of one dyne.

Electro-magnetic Units.

Current. — The unit of current is that current which, when flowing along a wire one centimetre in length bent into the form of a circular arc of one centimetre radius, acts with a force of one dyne upon a unit pole placed at the centre of the circle.

Quantity.—The electro-magnetic unit of quantity is the quantity of electricity which in one second passes any section of a conductor in which unit current is flowing.

Electromotive Force or Difference of Potential.—Unit electromotive force[1] exists between two points when the work done against the electrical forces in moving unit quantity of electricity from the one point to the other is one erg.

Resistance.—A conductor is said to possess unit resistance when unit difference of potential between its ends causes unit current to flow through it.

Capacity.—A conductor has unit capacity when a charge of one unit of electricity raises its potential from zero to unity.

5. Practical Electrical Units. — The following

[1] Usually contracted into "E.M.F."

system of units, based upon the C.G.S. electro-magnetic units, was devised by the British Association committee, and is in general use among practical electricians. It will be noticed that in this system the units of current, electromotive force, and resistance have been chosen so as to be of suitable magnitude for the electrical measurements which most frequently occur.

Current and Quantity.—The practical unit of current is the **ampère**, and is one-tenth (10^{-1}) of the C.G.S. electro-magnetic unit of current. It follows that the **coulomb**, or practical unit of quantity, is also one-tenth of the corresponding C.G.S. unit.

Electromotive Force.—The practical unit of E.M.F. is called the **volt**, and is 10^8 C.G.S. units. This is a little less than the electromotive force of a Daniell cell, the E.M.F. of a standard Daniell of the Post-Office pattern being 1·08 volt.

Capacity.—A conductor is said to have a capacity of one **farad** when it is charged to a potential of one volt by a coulomb of electricity. The farad is 10^{-9} of the C.G.S. electro-magnetic unit of capacity.

Resistance.—A conductor is said to possess a resistance of an **ohm** when a difference of potential of one volt between its ends causes a current of one ampère to flow through it. The ohm is therefore equal to 10^9 C.G.S. units of resistance.

Material standards, intended to represent the ohm as above defined, were issued by the B.A. committee, but it was soon found that their resistance was somewhat too small. For the sake of distinction these standards and the copies of them which have since been made are known as "**B.A. Units.**" The B. A. unit has the same resistance as a column of mercury one square milli-

metre in cross-section and 104·8 centimetres long.

At the International Conference of Electricians held at Paris in 1884, it was agreed that a column of mercury 106 cm. long and 1 sq. mm. in section should be adopted as the practical unit of resistance, and called the "legal ohm." The name chosen was rather unfortunate, inasmuch as the Congress had no power to legalise any unit. So also was the choice of a round number (106) for the length of the mercury column. The best determinations made by Lord Rayleigh and others had already shown that a column 106·2 or 106·3 cm. long would more nearly represent the true ohm. Hence there was for some years a tendency for every man to choose as seemed fit to him between "B.A. units," "legal ohms," and "true" or "Rayleigh ohms."

The Electrical Standards committee of the British Association has recently (1892) made a recommendation which appears to meet with general approval, and will put an end to the uncertainty above referred to. The actual resolutions of the committee are: (1) That the resistance of a specified column of mercury be adopted as the practical unit of resistance. (2) That 14·4521 grammes of mercury in the form of a column of uniform cross-section, 106·3 cm. in height, at 0° C., be the specified column.

It will be observed that the cross-section of the column is not stated, in so many words, to be one square millimetre. Indeed, the specification looks somewhat uncouth; but it has to be remembered that in practice the cross-section of such a column is determined by weighing. Thus the above resolutions may be taken as defining the practical unit of resistance to be that of a

column of 1 sq. mm. in cross-section and 106·3 cm. long at 0° C.; the cross-section being determined by weighing, and the density of mercury being taken as 13·5956 and that of water as unity.

According to the above decision

1 B.A. unit = 0·9859 ohm,

and 1 ohm = 1·014 B.A. unit.

6. Change of Units.—We have seen (§ 1) that the numerical value n of a length l is given in terms of the unit-length L by the equation—

$$n=\frac{l}{L} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here we notice, in the first place, that the numerical value of a concrete quantity varies *directly* as the quantity itself, and *inversely* as the unit employed in measuring it. From equation (1) we have—

$$l=nL \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This second equation gives us a complete expression for the length l, an expression which consists of two parts—the first being a number (n), and the second a quantity (L) of the same kind as that under consideration, and which we call the unit. Our everyday expressions for all physical magnitudes are, in fact, phrases which consist of a numerical and denominational part; thus we speak of a length of *ten yards*, and we say that *ten yards* are equal to *thirty feet*. This last statement involves a change of units,—a process which is perfectly easy when we have only to deal separately with units of length, mass, or time; but which becomes more difficult when two or more of the units have to be simultaneously changed. In dynamical problems which involve a change of units, it is usual to change the units one at

a time; but this process becomes very laborious when the fundamental units are involved in a complex manner in those derived from them, as is the case with most electrical units. In proceeding with the general theory of units, we shall consider first, as a simple example, the principle involved in the statement that

$$10 \text{ yards} = 30 \text{ feet}.$$

Let l denote, as before, the length to be measured and n its numerical value when L is the unit of length. We wish to find its numerical value n' when the unit-length is L'. Now

$$n = \frac{l}{\mathsf{L}}, \text{ and } n' = \frac{l}{\mathsf{L}'},$$

$$\therefore \; l = n\mathsf{L} = n'\mathsf{L}',$$

or

$$n' = n \times \frac{\mathsf{L}}{\mathsf{L}'}\cdot$$

The value of the quantity $\frac{\mathsf{L}}{\mathsf{L}'}$, which is the ratio of the first unit-length to the second, is called the *change-ratio* from the first system to the second. It is the factor by which the numerical value of the quantity in the first system must be multiplied in order to obtain its numerical value in the second system. In the case under consideration

$$\frac{\mathsf{L}}{\mathsf{L}'} = \frac{\text{yard}}{\text{foot}} = \frac{3 \text{ feet}}{1 \text{ foot}} = 3,$$

and

$$\therefore \; n' = n \times \frac{\mathsf{L}}{\mathsf{L}'} = n \times 3 = 10 \times 3 = 30.$$

DIMENSIONS OF PHYSICAL QUANTITIES.

7. Velocity.—We shall next consider how the measure of a velocity changes when the units by which it is measured are changed (using, as above, thick letters to represent units, and italics to represent the concrete quantities). Let V denote the unit of velocity based

upon L and T as the units of length and time, V′ the unit of velocity in a second system in which the units of length and time are L′ and T′; and let v denote a concrete velocity such that a space l is described in the time t.

If n denote the measure (or numerical value) of this velocity in terms of the unit V, and n' its value in terms of the second unit V′, then

$$n=\frac{v}{V}, \text{ and } n'=\frac{v}{V'} \quad . \quad . \quad . \quad . \quad (1)$$

or

$$v=nV=n'V.$$

Now the measure of a velocity is the number of units of space$\left(\frac{l}{L}\right)$ described in unit time,

$$\left.\begin{aligned} \therefore\ n&=\frac{l}{L}\div\frac{t}{T}, \text{ or } n=\frac{l}{L}\cdot\frac{T}{t}, \\ \text{and} \quad n'&=\frac{l}{L'}\div\frac{t}{T'}, \text{ or } n'=\frac{l}{L'}\cdot\frac{T'}{t} \end{aligned}\right\} \quad . \quad . \quad (2)$$

From equations (1) and (2) we have

$$V=v\cdot\frac{L}{l}\cdot\frac{t}{T}, \text{ and } V'=v\cdot\frac{L'}{l}\cdot\frac{t}{T'}.$$

But

$$v=nV=n'V',$$

$$\therefore\ nv\cdot\frac{L}{l}\cdot\frac{t}{T}=n'v\cdot\frac{L'}{l}\cdot\frac{t}{T'},$$

which may be written in the form

$$n\cdot\frac{L}{T}=n'\frac{L'}{T'} \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) enables us to find the measure (n') of the velocity in the second system, when the relations between the fundamental units L and L′, T and T′ are known. Comparing it with the equation

$$nV=n'V' \quad . \quad . \quad . \quad . \quad (4)$$

we see that the unit of velocity varies *directly* as the

unit of length, and *inversely* as the unit of time. This is usually expressed by saying that the *dimensions* of the unit of velocity are of one degree in length, and minus one degree in time; or that the dimensions of velocity are $\frac{L}{T}$ or LT^{-1}.

It should be noticed that $\frac{L}{T}$ is not a number, or a ratio in the strict Euclidean sense. Nor does the symbol indicate division in the usual sense; for we cannot divide a length by a time. The symbol $\frac{L}{T}$ rather indicates the idea which is implied in our usual use of the word *per*; as when we speak of a velocity of 30 feet *per* second. When, therefore, we write

$$V=\frac{L}{T},$$

we mean that the unit of velocity (V) is such that the space L is described per time T.

8. Acceleration.—Proceeding with this reasoning we shall find that in acceleration time is involved twice. For acceleration is measured by the increase of velocity per unit time; so that if A denote the unit of acceleration, A is equal to V per T, or $=V/T$, and since we have already seen that

$$V=\frac{L}{T},$$

it follows that

$$A=\frac{V}{T}=\frac{L}{T^2}.$$

We can arrive at the same result more formally as follows:—

Let A, V, L, and T represent respectively the units of acceleration, velocity, length and time in one system, A', V', L', and T' the corresponding quantities in a second system; and let a denote an acceleration such that the

velocity v is generated in the time t. If n be the measure of the quantity in the first system,

$$n=\frac{a}{\mathsf{A}}.$$

But the measure of an acceleration is the number of units of velocity generated per unit of time, so that

$$n=\frac{v}{\mathsf{V}}\div\frac{t}{\mathsf{T}}=\frac{v}{\mathsf{V}}\cdot\frac{\mathsf{T}}{t},$$

$$\therefore \mathsf{A}=a\frac{\mathsf{V}}{v}\cdot\frac{t}{\mathsf{T}},$$

and similarly

$$\mathsf{A}'=a\frac{\mathsf{V}'}{v}\cdot\frac{t}{\mathsf{T}'}.$$

But since the quantity measured in both systems is the same, we have,

$$a=n\mathsf{A}=n'\mathsf{A}' \quad . \quad . \quad . \quad . \quad (5)$$

and

$$na\frac{\mathsf{V}}{v}\cdot\frac{t}{\mathsf{T}}=n'a\frac{\mathsf{V}'}{v}\cdot\frac{t}{\mathsf{T}'},$$

$$\therefore n\frac{\mathsf{V}}{\mathsf{T}}=n'\frac{\mathsf{V}'}{\mathsf{T}'},$$

or

$$n\cdot\frac{\mathsf{L}}{\mathsf{T}^2}=n'\frac{\mathsf{L}'}{\mathsf{T}'^2} \quad . \quad . \quad . \quad . \quad (6)$$

A comparison of equations (5) and (6) shows that the unit of acceleration varies directly as the unit of length and inversely as the *square* of the unit of time; in other words, that its dimensions are $\frac{\mathsf{L}}{\mathsf{T}^2}$ or LT^{-2}.

The following example will illustrate the way in which these equations are applied :—

Ex. 1. *Express the acceleration due to gravity in terms of the mile and the hour as the units of length and time, its value being* 32 *when the foot is the unit of length, and the second the unit of time.*

From (6) we have directly

$$n' = n \cdot \frac{L}{L'} \cdot \frac{T'^2}{T^2},$$

an equation which gives us the required measure (n') in the new system, when the relations between the fundamental units in the old and new systems are known. In the example $n = 32$,

$$\frac{L}{L'} = \frac{\text{foot}}{\text{mile}} = \frac{1 \text{ foot}}{(1760 \times 3) \text{ feet}} = \frac{1}{5280},$$
$$\frac{T'}{T} = \frac{\text{hour}}{\text{second}} = \frac{3600 \text{ seconds}}{1 \text{ second}} = 3600,$$

$$\therefore n' = 32 \times \frac{1}{5280} \times (3600)^2 = 78545 \cdot \dot{4}\dot{5}.$$

9. Force, Work, and Power.—Before proceeding to give the dimensions of other derived units in mechanics, it may be well to point out the considerations which determine the choice of any new unit based upon the fundamental quantities or upon derived units which have already been fixed. We shall take the unit of force as our example.

According to the second law of motion, *force* is measured by the change of momentum which it produces, *i.e.*

$$f \propto (\text{rate of change of } mv),$$
$$\therefore f \propto ma,$$

(where a denotes acceleration), or

$$f = k \cdot ma.$$

The units of mass and acceleration are already fixed, but we may make the unit of force whatever we please, and it will obviously be most convenient to choose it so that the constant multiplier k shall be equal to unity. Our equation will now become

$$f = ma.$$

Now suppose m and a to be each equal to unity; then f will also be equal to unity. Thus our unit of force (F) is defined as being that force which produces unit acceleration in unit mass. We may therefore write

$$F = M \cdot A,$$

but

$$A = \frac{L}{T^2},$$

$$\therefore F = \frac{ML}{T^2} = MLT^{-2},$$

an equation which gives the dimensions of force.

Work is measured by the product of force into the distance through which the force acts. Hence the dimensions of work will be those of force multiplied by length, or

$$W = MLT^{-2} \times L = ML^2T^{-2}.$$

The *power* (or activity) of an agent is measured by the rate at which it does work; hence the dimensions of power are

$$\frac{ML^2T^{-2}}{T} = ML^2T^{-3}.$$

10. Change of Units.—Knowing the dimensions of these quantities, we can perform the change of units without going through the lengthy reasoning of §§ 7 and 8; we shall indicate the general method to be followed, but it will be best understood by reference to the actual examples given. [See also equations (5) and (6) in § 8.]

Let q be any concrete quantity, and let its measure be n in terms of the unit Q, which is based upon the fundamental units M, L, and T; we wish to find its measure n' in a new system in terms of the unit Q′ which is based upon M′, L′, and T′. Since the quantity measured in both systems is the same,

$$q = nQ = n'Q' \quad . \quad . \quad . \quad . \quad (7)$$

Let the dimensions of Q be $M^xL^yT^z$; substituting for Q and Q′ in equation (7) we have

$$q = n \cdot M^x L^y T^z = n' \cdot M'^x L'^y T'^z,$$

and $$\therefore \frac{n'}{n} = \left(\frac{M}{M'}\right)^x \left(\frac{L}{L'}\right)^y \left(\frac{T}{T'}\right)^z,$$

or $$n' = n \cdot \left(\frac{M}{M'}\right)^x \left(\frac{L}{L'}\right)^y \left(\frac{T}{T'}\right)^z \quad . \quad . \quad . \quad (8)$$

EX. 2. *Find the number of dynes in a poundal (the poundal being the British absolute unit of force, based upon the pound, foot, and second).*

Referring to equations (7) and (8) we see that, since $n=1$, the required number n' is the change-ratio or multiplier for changing from British to G.C.S. units of force. The dimensions of force are MLT^{-2}, so that $x=1$, $y=1$, and $z=-2$. T and T′, the units of time, are the same (one second) in both systems.

$$\frac{M}{M'} = \frac{\text{pound}}{\text{gramme}} = 453{\cdot}6,$$

$$\frac{L}{L'} = \frac{\text{foot}}{\text{centimetre}} = 30{\cdot}48,$$

$$\therefore n' = 453{\cdot}6 \times 30{\cdot}48 = 13825{\cdot}8.$$

EX. 3. *Find the value of a horse-power in watts, a horse-power being equivalent to* 550 *foot-pounds per second, and the value of g being* 32·18.

As the foot-pound is a gravitation unit, we shall first have to reduce to the corresponding absolute unit by multiplying by g—

550 foot-pounds $= 550 \times 32{\cdot}18$ foot-poundals.

The dimensional equation for finding the equivalent rate of working in G.C.S. units (ergs per second) is,

$$550 \times 32{\cdot}18 \times ML^2T^{-3} = n' \times M'L'^2T'^{-3}.$$

The units of time (T and T') are the same in both systems; and, as in Example 2,

$$\frac{M}{M'}=453 \cdot 6, \text{ and } \frac{L}{L'}=30 \cdot 48,$$

$$\therefore n'=550 \times 32 \cdot 18 \times 453 \cdot 6 \times (30 \cdot 48)^2$$
$$=745 \cdot 8 \times 10^7.$$

Thus 1 horse-power $=745 \cdot 8 \times 10^7$ ergs per second, or (since 1 watt $=10^7$ ergs per second)

$$=745 \cdot 8 \text{ watts.}$$

11. Magnetic and Electrical Units.—The dimensions of the most important of these are given below, and it will be useful practice for the student to deduce them from the corresponding physical laws, as we have done in the preceding articles.

Dimensions of Magnetic Units.

Magnetic pole (or quantity of magnetism).	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$
Magnetic moment of magnet . . .	$M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$
Strength of magnetic field . . .	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$

Ex. 4. *The dimensions of magnetic intensity (or strength of field) are* $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$, *and the horizontal intensity of the earth's magnetic force at Aberystwyth is* 0·1774 *in C.G.S. units: what is its value in British (foot-grain-second) units?*

The intensity is the same, whatever units we employ to measure it. Let x be its numerical value in the British system, in which the unit of field intensity is H', the corresponding unit in the C.G.S. system being H; then

$$0 \cdot 1774\, H = xH',$$

or

$$0 \cdot 1774\, M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} = xM'^{\frac{1}{2}}L'^{-\frac{1}{2}}T'^{-1},$$

and $$\therefore x = 0{\cdot}1774 \left(\frac{\mathsf{M}}{\mathsf{M}'}\right)^{\frac{1}{2}} \left(\frac{\mathsf{L}}{\mathsf{L}'}\right)^{-\frac{1}{2}} \left(\frac{\mathsf{T}}{\mathsf{T}'}\right)^{-1}.$$

Now since 1 gramme = 15·43 grains, and 1 foot = 30·48 centimetres,

$$\frac{\mathsf{M}}{\mathsf{M}'} = \frac{\text{gramme}}{\text{grain}} = 15{\cdot}43, \text{ and } \frac{\mathsf{L}}{\mathsf{L}'} = \frac{\text{centimetre}}{\text{foot}} = \frac{1}{30{\cdot}48},$$

and the units of time (T and T') are the same in both systems.

Thus $$x = 0{\cdot}1774 \times (15{\cdot}43)^{\frac{1}{2}} \times (1/30{\cdot}48)^{-\frac{1}{2}},$$

$$= 0{\cdot}1774 \times \sqrt{15{\cdot}43 \times 30{\cdot}48} = 3{\cdot}847.$$

Ex. 5. *Assuming Coulomb's law (the law of inverse squares) to find the dimensions of the unit of quantity in the electrostatic system.*

According to the law of inverse squares, the force exerted between two bodies charged with quantities q and q' of electricity, and situated at a distance d from one another, is proportional to the product of the charges and inversely proportional to the square of the distance. Choosing our unit of quantity in accordance with the definition of § 4, we may write this in the form

$$f = \frac{qq'}{d^2}.$$

If we suppose that $q' = q$, we have $q^2 = d^2 f$, or $q = d\sqrt{f}$, so that the dimensions of the unit of quantity are $\mathsf{L} \times \sqrt{\mathsf{MLT}^{-2}}$ or $\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{3}{2}}\mathsf{T}^{-1}$.

Dimensions in Electrostatic System.

Quantity of electricity . . .	$\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{3}{2}}\mathsf{T}^{-1}$
Electrostatic potential . . .	$\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{1}{2}}\mathsf{T}^{-1}$
Capacity	L
Strength of current . . .	$\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{3}{2}}\mathsf{T}^{-2}$
Resistance	$\mathsf{L}^{-1}\mathsf{T}$

Dimensions in Electro-magnetic System.

Strength of current	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$
Quantity of electricity	$M^{\frac{1}{2}} L^{\frac{1}{2}}$
Potential or E.M.F.	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}$
Resistance	$L T^{-1}$
Capacity	$L^{-1} T^{2}$

12. Suppressed Dimensions of Electrical Quantities.—In the preceding article it has been assumed that *specific inductive capacity* and *magnetic permeability* are of zero dimensions, *i.e.* that they are merely abstract numbers. This, however, is an arbitrary assumption, and is open to the obvious objection, among others, that it implies a knowledge of the nature of these quantities which we do not as yet possess. Professor Rücker (*Proc. Phys. Soc.* x. 37) prefers to regard these as quantities of which the dimensions are unknown or undetermined, and to retain in the dimensional equations symbols indicating these unknown dimensions. If we denote specific inductive capacity by K, and magnetic permeability by μ, we obtain dimensions such as the following:—

	In terms of M, L, T and K	In terms of M, L, T and μ.
Quantity of magnetism or magnetic pole .	$M^{\frac{1}{2}} L^{\frac{1}{2}} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} \mu^{\frac{1}{2}}$
Magnetic field	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$
Quantity of electricity . .	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} \mu^{-\frac{1}{2}}$
Electric current	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$
Potential or E.M.F . .	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} \mu^{\frac{1}{2}}$
Resistance	$L^{-1} T K^{-1}$	$L T^{-1} \mu$
Capacity	$L K$	$L^{-1} T^{2} \mu^{-1}$

From the above the dimensions in the usual electrostatic and electro-magnetic systems (as in § 11) can be derived by suppressing K in the one case and μ in the other.

13. Approximate Calculations.—Arithmetical working may often be abbreviated by devices such as the contracted methods of multiplication and division of decimals. The degree of approximation to which the calculation must be carried out depends upon the accuracy of the data given. Physical measurements are never absolutely correct. If, then, we have to calculate out the results of an experiment made by a method which is liable to an error of (say) one in a thousand, it would be labour thrown away to carry out the calculation to more than four or five significant figures. Now it frequently happens, in working out the results of physical experiments, that the "uncorrected result" has to be multiplied by one or more correcting factors (each nearly equal to unity) in order to obtain the "corrected result"; and it is to the manipulation of these factors that the student's attention is now directed.

Suppose that the experiment under consideration consists in measuring the distance between two points by means of a steel metre scale, the length of which at 0° C. is known to be 1·00057 metre; and suppose further that the measurement is carried out at a temperature of 15° C. The steel scale expands on heating, and its length at 15° is greater (Chap. IV.) than its length at 0° in the ratio of 1·00018 to 1. If the uncorrected distance, as determined by direct measurement, is d, then the true distance (corrected for error of scale *and* error through temperature) will be

$$d' = d(1+\alpha)(1+\beta),$$

where $1+\alpha = 1{\cdot}00057$, and $1+\beta = 1{\cdot}00018$.

Now $(1+\alpha)(1+\beta) = 1+\alpha+\beta+\alpha\beta$; and since $\alpha = 0{\cdot}00057$ and $\beta = 0{\cdot}00018$, $\alpha\beta = 0{\cdot}000{,}000{,}1026$, so that the error caused by neglecting this last term would

only be 1 in 10,000,000. The measurement itself would probably not be correct to 1 in 1,000,000, so that we may safely adopt the approximation and write

$$d' = d(1+\alpha+\beta) = d \times 1{\cdot}00075. \quad (q.p.)$$

Again, suppose that in reducing our observations we have to multiply the uncorrected result by $\frac{1+\alpha}{1+\beta}$. (This correcting factor occurs in reducing observed barometric heights to 0° C., and other examples of its use are given in Chap. IV.) By ordinary algebraical division we have

$$\frac{1+\alpha}{1+\beta} = 1+\alpha-\beta+\left(\frac{\alpha\beta+\beta^2}{1+\beta}\right).$$

We have already seen that if both α and β are small quantities compared with unity, their product may be neglected; and the same is true for α^2 and β^2.

Thus $$\frac{1+\alpha}{1+\beta} = 1+\alpha-\beta. \quad (q.p.)$$

Ex. 6. *Find correctly to three decimal places the value of*

$$15{\cdot}24 \times \frac{1{\cdot}00217}{1{\cdot}00192}.$$

$$\frac{1{\cdot}00217}{1{\cdot}00192} = 1 + 0{\cdot}00217 - 0{\cdot}00192, \quad (q.p.)$$
$$= 1{\cdot}00025, \text{ and therefore}$$
$$15{\cdot}24 \times \frac{1{\cdot}00217}{1{\cdot}00192} = 15{\cdot}24 \times 1{\cdot}00025,$$
$$= 15{\cdot}24 + (15{\cdot}24 \times 0{\cdot}00025),$$
$$= 15{\cdot}24 + 0{\cdot}000381 = 15{\cdot}24381.$$

The answer, correctly to three decimal places, is 15·244.

The student can easily verify for himself the following results, which are approximately correct when the quantities α and β are small compared with unity (so

that their squares and higher powers may be neglected).

TABLE OF APPROXIMATE RELATIONS.

$$(1+\alpha)(1+\beta)=1+\alpha+\beta$$
$$(1+\alpha)(1-\beta)=1+\alpha-\beta$$
$$(1+\alpha)^2=1+2\alpha \qquad (1-\alpha)^2=1-2\alpha$$
$$(1+\alpha)^3=1+3\alpha \qquad (1-\alpha)^3=1-3\alpha$$
$$\sqrt{1+\alpha}=1+\tfrac{1}{2}\alpha \qquad \sqrt{1-\alpha}=1-\tfrac{1}{2}\alpha$$
$$\frac{1}{1+\alpha}=1-\alpha \qquad \frac{1}{1-\alpha}=1+\alpha$$
$$\frac{1+\alpha}{1+\beta}=1+\alpha-\beta.$$

14. Use of Logarithms.—It is proved in treatises on algebra that different powers of any fixed number can be multiplied by adding together the indices of those powers. We may assume that a list can be drawn up, giving the indices of the powers of some fixed number which are equal to every whole number, say from 1 to 10,000. Such a list is called a table of logarithms, and the fixed number is called the *base* of the system of logarithms; we may therefore define the logarithm of a number to a given base as being the index of that power of the base which is equal to the given number. Thus if $a^x=n$, then x is called the logarithm of n to the base a.

From motives of convenience the number 10 is chosen as the base of the common system of logarithms, and a table will be found at the end of this book giving the decimal parts (to four places) of the common logarithms of numbers from 1 to 9999.

To find the Logarithm of a Number from the Table.—The first two figures of the number are to be found in the left-hand column, and the third in the first series of figures (0 to 9) in the top column; the number opposite the first two figures, and below the third, is the decimal part of the logarithm. When the number whose loga-

rithm is required contains four figures, the fourth figure is to be looked for in the second series of figures (1 to 9) in the top column; the proportional part, which is found opposite the first two figures and below the fourth, is to be added to the part of the logarithm already found, the right-hand figure of the proportional part being added to the right-hand figure of the logarithm.

The integral part of a logarithm is called the *characteristic*; the decimal part is called the *mantissa*.

The characteristic of the logarithm of a number may be determined by inspection. For

$$10^0 = 1, \quad 10^1 = 10, \quad 10^2 = 100, \text{ etc.,}$$

and it therefore follows that the logarithm of any number between 1 and 10 is a positive decimal fraction: the logarithm of any number between 10 and 100 lies between 1 and 2, and the logarithm of any number between 100 and 1000 lies between 2 and 3. Hence the rule:—

(1.) *The characteristic of the logarithm of a number greater than unity is one less than the number of integral figures in that number.* Thus

$$\begin{aligned} \log\ 3{\cdot}14 &= 0{\cdot}4969 \\ \log\ 31{\cdot}4 &= 1{\cdot}4969 \\ \log\ 314 &= 2{\cdot}4969 \\ \log\ 3140 &= 3{\cdot}4969. \end{aligned}$$

Again, since $10^0 = 1$, and $10^{-1} = 0{\cdot}1$, it follows that the logarithm of any number between 0 and 0·1 is a negative decimal fraction, and may therefore be written in the form—

$$-1 + \text{a decimal fraction.}$$

Similarly the logarithm of any number between 0·1 and 0·01 (*i.e.* between 10^{-1} and 10^{-2}) may be written in the form—

$$-2 + \text{ a decimal fraction,}$$

the decimal part being always kept *positive*. Hence the rule :—

(2.) *The characteristic of the logarithm of a number less than unity is negative, and is one more than the number of ciphers immediately following the decimal point.*

Thus the logarithm of 0·314 is $-1 + 0{\cdot}4969$, which is abbreviated thus: $\bar{1}{\cdot}4969$; the logarithm of 0·0314 is $\bar{2}{\cdot}4969$, and so on.

The operation of multiplication is performed by adding together the logarithms of the numbers which are to be multiplied: the sum is the logarithm of their product. Division is performed by subtracting the logarithm of the divisor from that of the dividend: the remainder is the logarithm of the quotient. The manner in which these operations are carried out, and the method of finding a number when its logarithm is given, will be best explained by an example.

EX. 7. *To find the value of* $\dfrac{453{\cdot}6 \times 30{\cdot}48}{(2{\cdot}54)^2 \times 13600}$ *by the use of four-place logarithms.*

$$\begin{array}{ll}
\log 453{\cdot}6 & = 2{\cdot}6567 \\
\log 30{\cdot}48 & = 1{\cdot}4840 \\
\hline
\text{log dividend} & = 4{\cdot}1407 \\
\text{log divisor} & = 4{\cdot}9431 \\
\hline
\text{log quotient} & = \bar{1}{\cdot}1976
\end{array}
\qquad
\begin{array}{ll}
\log (2{\cdot}54)^2 & \\
\quad = 2 \log 2{\cdot}54 & \\
\quad = 2 \times 0{\cdot}4048 & = 0{\cdot}8096 \\
\log 13600 & = 4{\cdot}1335 \\
\hline
\text{log divisor} & = 4{\cdot}9431
\end{array}$$

0·1976 is not one of the logs given in the table: the next lower one is 0·1959, which is the log of 1·57. Now $0{\cdot}1976 - 0{\cdot}1959 = 0{\cdot}0017$. Looking along the row (in which the log is given) for 17, we find that it stands in a column headed by the figure 6, and this is the fourth figure of the number. Lastly, by rule (2), we see that $\bar{1}{\cdot}1976 = \log 0{\cdot}1576$. The value of the fraction is therefore 0·1576.

A full account of the methods of logarithmic calculation will be found in Chambers's *Mathematical Tables*, and these may be used for more accurate work; but the table of four-place logarithms at the end of this book will be found sufficient for working out most of the problems given.

The student is advised to practise the methods of approximation indicated in § 13, and to make himself thoroughly familiar with the use of logarithms, as a large amount of arithmetical calculation will thus be avoided.

In working out examples he should aim at something more than merely getting a correct numerical answer: diagrams or rough sketches should be given wherever they render the solution more intelligible, and formulæ should not be quoted without explanation unless the relations which they express are perfectly well known and easily remembered. In particular, every step in the reasoning should be carefully thought out and clearly explained, for the solution of problems is not so much an end itself as a means of acquiring a thorough and intimate acquaintance with physical laws.

CHAPTER I

DYNAMICS

Note.—In all the examples, excepting where otherwise stated, the numerical value of g is taken as 981 when the centimetre and second are the units of length and time, and as 32 when the foot is the unit of length.

The abbreviation *cm.* is used for *centimetre(s)*.
„ *gm.* „ *gramme(s)*.
„ *c.c.* „ *cubic centimetre(s)*.

In examples on change of units, the following (approximate) relations may be assumed—

1 foot = 30·48 cm.
1 inch = 2·54 cm.
1 pound = 453·6 gm.

1. State and discuss Newton's First Law of Motion, and show that it provides us with a definition of force.

2. Enunciate Newton's Second Law; state the exact meaning of the phrase "change of motion" as used by him, and explain how the law enables us to measure forces.

3. Starting from Newton's Second Law of Motion, show how to obtain a definition of the C.G.S. unit of force (the dyne). If the weight of a gramme be taken as the unit of force, what is the unit of mass?

4. A force of 25 units acts upon a mass 10: find the acceleration produced, and the space described in 30 seconds from rest.

5. A force of 100 dynes acts upon a mass of 25 grammes for 5 seconds: what velocity does it generate?

6. A constant force acting upon a mass of 30 grammes causes it to move through 10 metres in 3 seconds, starting from rest: what is the value of the force in dynes?

7. A force of 1,000,000 dynes acts upon a body for 10 seconds, and gives it a velocity of a metre per second: find the mass of the body in grammes.

8. How long must a force of 5 units act upon a body in order to give it a momentum of 3000 units? (The unit of momentum is that of a gramme moving at the rate of one centimetre per second.)

9. During what time must a constant force of 60 dynes act upon a kilogramme in order to generate in it a velocity of 3 metres per second?

10. What force acting upon a mass of 50 grammes for one minute will produce a velocity of 45 centimetres per second?

11. A body moving with uniform velocity in a circle is commonly said to be acted on by "centrifugal force." Discuss the correctness of this expression, stating whether the quantity referred to is really a force. Is its action centrifugal?

12. State and explain Newton's Third Law of Motion, and give examples of its application. If the earth attracts the moon with a force F, what is the attraction exerted by the moon upon the earth?

13. The mass of a gun is 2 tons, and that of the shot is 14 lbs. The shot leaves the gun with a velocity of 800 feet per second: what is the initial velocity of the recoil?

14. A 56-lb. shot is projected with velocity v from a gun, the mass of which (together with its carriage) is 6 tons. Express, in terms of v, the velocity of recoil of the gun.

15. Do you consider *weight* to be an essential property of matter? State clearly what distinction you would draw between *mass* and *weight*; and illustrate your remarks by reference to the force required (1) to open a large iron gate, well balanced and swinging upon good hinges, and (2) to lift up the same gate when lying on the ground.

16. Explain what is meant by "the acceleration due to gravity." If its numerical value be 32 when the unit of length is the foot and the unit of time the second, what is its value when the unit of length is the yard and the unit of time the minute? (See § 8, Ex. 1.)

17. A 4-oz. weight is suspended from a spring-balance which is carried in a balloon; what will be its apparent weight as shown by the index (1) when the balloon is ascending with an uniform acceleration of 8 feet per second, (2) when it is descending with an acceleration of 16 feet per second?

18. What do you understand by the phrase "weight of a pound"? The British unit of force (called a poundal) is defined as being that force which, acting upon a pound mass for one second, generates in it a velocity of one foot per second: how many poundals are there in a pound weight?

19. Explain the distinction between gravitation measure and the absolute measure of force. Show also how the one may be expressed in terms of the other, finding, for example, the number of dynes in a gramme weight.

20. Express the weight of 10 kilogrammes in dynes, and the value of a dyne in terms of a gramme weight.

21. Calculate the value of a pound weight in dynes. (See note on p. 25.)

22. A force of 980 dynes acts vertically upwards upon a body of mass 5 grammes, at a place where $g = 981$: find the acceleration of the body.

23. A force equal to a weight of 10 lbs. acts upon

a mass of 25 lbs.: what is the acceleration produced, and what momentum will be generated in 5 minutes?

24. At the top of a pit, 1024 feet deep, a stone is thrown vertically upwards with a velocity of 64 feet per second: when will it reach the bottom?

25. A body of mass 6 lbs. is acted upon by a force of 30 poundals: find its velocity and momentum at the end of half a minute.

26. A spring-balance is carried in a balloon which is ascending vertically. Find the acceleration of the balloon when a half-pound weight hung upon the spring-balance is found to indicate 9 ounces.

27. By what number would the acceleration due to gravity be expressed, if the day and the mile were the units of time and length?

28. If the unit of length were the yard, the unit of time the minute, and the unit of mass a mass of 10 lbs., what would be the value of the unit of force in terms of the poundal?

29. A certain force acts upon a mass of 150 grammes for 10 seconds, and produces in it a velocity of 50 metres per second: compare the force with the weight of a gramme.

30. A force equal to the weight of one pound acts upon a ton: what acceleration is produced, and what will be the velocity at the end of ten seconds?

31. Compare the momentum of a body of mass 7 lbs., moving with a velocity of 24 feet per second, with that of a body of mass 3 cwt., moving with a velocity of 10 yards per minute.

32. Taking as units the pound, foot, and second, express the momentum of a mass of 1 cwt. that has been falling from rest for 5 seconds [$g=32$.]

33. A certain force acts upon a mass m and generates in it an acceleration a: find the weight which the force would statically support.

34. A spring-balance is graduated at a place where $g = 32 \cdot 3$; at another place where $g = 32$, a body is tested and the balance indicates 8 oz.: what is the correct mass of the body?

35. A certain force acting on a mass of 14 pounds for 10 seconds generates in it a velocity of 128 feet per second. Compare the force with the weight of one pound, and determine the acceleration which it would produce in a mass of one ton.

36. An 18-ton truck is moving at the rate of 30 miles per hour: what is its momentum? (Take the foot and pound as the units of length and mass.)

37. Compare the momentum of a 15-lb. cannon-ball moving at the rate of 300 feet per second, with that of a 3-oz. bullet which has a velocity of 700 yards per second.

38. In what time will a velocity of 45 miles per hour be generated in a train of 80 tons, if the engine exerts upon it a pull equal to a weight of 2 tons?

39. If a body of mass 10 kilogrammes be acted upon for one minute by a force which can statically support 125 grammes, what momentum will it acquire?

40. An acceleration of 1 foot per minute is expressed by the number $0 \cdot 1$ in a certain system in which the inch is the unit of length. What is the unit of time in this system?

41. A body of mass 4 lbs. is observed to be moving at a rate of 8 feet per second; at this instant a constant force begins to act upon it in the direction of its motion, and after 20 seconds its velocity has increased to 24 feet per second. Determine the magnitude of the force, and explain clearly what unit of force you employ in your solution.

42. Compare the amounts of momentum in (1) a 56-lb. weight which has fallen for 2 seconds from rest, and (2) a cannon-ball of 12 lbs. moving with a velocity of 900 feet per second.

43. If the mile be taken as the unit of length, and

the acceleration caused by gravity as the unit of acceleration, what will be the unit of velocity?

44. In a certain system the unit of mass is a kilogramme, the unit of length is 10 cm., and the unit of time 100 secs. Prove that in this system the unit of force is equal to that in the C.G.S. system (the dyne).

45. A 7-lb. weight hanging over the edge of a smooth table drags a mass of 49 lbs. along it: find the acceleration, and the distance moved through in 5 seconds from rest.

46. A falling weight of 162 grammes is connected by a string to a mass of 1800 grammes lying on a smooth flat table: find the acceleration, and the tension of the string.

47. A mass of 3 lbs. is drawn along a smooth horizontal table by a mass of 6 oz. hanging vertically: calculate the space described in 3 seconds.

48. A force of 30 dynes acts for 12 seconds upon a body resting on a smooth horizontal plane, and imparts to it a velocity of 120 centimetres per second: what is the mass of the body?

49. A mass of 15 lbs. lying on a smooth flat table is acted upon by a force of 60 poundals: how far will it move in 6 seconds?

50. A body of mass 10 is connected with another body of mass 6 by a string passing over a frictionless pulley: find the acceleration and the distance moved through in 4 seconds. Show how such an arrangement could be employed for finding the value of g, and explain why the method would be better than that of experimenting with a freely falling body.

51. Weights of 14 and 21 lbs. are hung on the ends of a rope passing over a pulley: find the tension in the rope in pounds weight and in poundals.

52. Two masses of 100 and 120 grammes are attached to the extremities of a string passing over a smooth

pulley: if the value of g is 975, what will be the velocity after 8 seconds?

53. Two unequal masses are attached to the ends of a string passing over a smooth peg: find the ratio between them in order that each may move through 16 feet in 2 seconds, starting from rest.

54. Two buckets, each weighing 28 lbs., are suspended from the ends of a rope passing over a windlass; a gallon (10 lbs.) of water is poured into one of the buckets: find how far it will descend in 10 seconds, neglecting friction.

55. The length of an inclined plane of inclination 30° to the horizontal is 32 ft. Find the time in which a body will slide down it, and its velocity at the foot of the plane.

56. The upper end of a smooth straight wire of length l is attached to the top of a vertical pole with which the wire makes an angle of 30°. A bead is allowed to slip along the wire from top to bottom. What will be its velocity when it reaches the free end?

57. A train weighing 80 tons, travelling on the level with a velocity of 60 ft. per second, is brought to rest by a force acting in an opposite direction in 30 seconds. Find the force in lbs. weight.

58. A train starts from rest on a level line and moves through 1200 feet in the first minute. It then begins to ascend an uniform incline, up which it is found to run with uniform velocity: find the inclination of this portion of the line on the supposition that the engine exerts a constant pull.

59. Describe Atwood's machine, and explain how it may be used to prove—

(*a*) That when different forces act upon the same mass the accelerations observed are proportional to the forces.

(*b*) That when the force is constant the accelerations are inversely proportional to the masses.

(*c*) That the space described in n seconds from rest is proportional to n^2.

What other experimental method has been devised for testing the last proposition?

60. The sum of the two weights in an Atwood's machine is 2 lbs., and the difference between them is an ounce: find the acceleration and the space described in the first second.

61. The two masses in an Atwood's machine are each 24 grammes. One of them is loaded with an excess weight of 1 gramme. With what acceleration will it descend? and through what distance will it move in the fourth second after starting? (Take $g = 980$.)

62. In an experiment with Atwood's machine the masses were 520 and 480 grammes; in 2 seconds from rest the heavier mass descended 76 centimetres. What value does this give for the acceleration of gravity? If your result differs from the usual value, suggest any cause for the difference.

63. The two equal masses in an Atwood's machine are each 100 grammes; what excess weight must be placed upon one of them in order that, at the end of 3 seconds, it may be descending with a velocity of 243 centimetres per second?

64. A smooth pulley is supported by a hook, and over it there passes a flexible string, to the ends of which are attached two masses of 55 and 45 grammes respectively. Show that when the masses are free to move, the pull on the hook is equal to the weight of 99 grammes.

65. By means of an Atwood's machine a force equal to the weight of 10 grammes was made to act upon a mass of 500 grammes, and it was found that an acceleration of 19·6 cm. per second was produced. Find the value of g.

66. The suspended masses in an Atwood's machine are 15 lbs. and 17 lbs. After it has been in motion for

8 seconds, 8 lbs. are removed from the larger mass. What time will elapse before the directions of motion of the masses are reversed?

67. A body of mass m moves with uniform velocity v in a circle of radius r. Prove that a force $\frac{mv^2}{r}$ is required to keep it in its circular path, and that this force is directed along the radius and towards the centre.

68. A body of mass 2 lbs. is attached to the end of a string a yard long, and is whirled round at an uniform rate, making twenty revolutions in a minute: what is the tension in the string?

69. A mass of a kilogramme is connected to a fixed point by a string one metre in length, and whirls round in a circle once a second: find the tension of the string in terms of the weight of a gramme.

70. A person skating on ice at the rate of 16 feet per second describes a circle of 15 feet radius. What is his inclination to the vertical?

71. Assuming that there are 86,164 seconds in a sidereal day, and that the earth's mean equatorial radius is 3962 miles; calculate (in feet per second) the acceleration of a point on the equator.

72. Starting from the result of the preceding problem, discuss the effect of the earth's rotation upon a spring-balance which is used to weigh the same body (1) at the pole, (2) at the equator; and show that if the earth revolved about seventeen times as fast as it now does, a body on the equator would have no apparent weight.

73. Prove, by any method, that the time of a complete oscillation of a simple pendulum is $2\pi\sqrt{\frac{l}{g}}$ when the amplitude of oscillation is indefinitely small.

The bob of a pendulum consists of a jar containing mercury. If more mercury is poured in will the time of oscillation be increased or diminished?

74. Find the value of g at a place where the length of the seconds pendulum is 0·994 metre.

[*N.B.*—A seconds pendulum is one which makes half a complete oscillation in a second.]

75. A pendulum 10 feet in length makes ten complete oscillations in 35 seconds: what is the value of g at the place?

76. Supposing a pendulum to be constructed to beat seconds at a place where $g=950$; how would its length have to be altered in order to make it beat seconds on the surface of the moon, where $g=150$?

77. Show that a pendulum of one metre in length would beat seconds if the value of g were 987.

78. What is the value of g at Greenwich, where the length of the seconds pendulum is found to be 39·14 inches, and what is the length of a pendulum which loses 10 minutes a day at this place?

79. The bob of a pendulum can be raised by means of a screw which has thirty threads to the inch; if the pendulum loses 5 minutes a day, how many turns of the head of the screw must be made in order to correct it? (Assume that the pendulum keeps correct time when its length is 39 in.)

80. A balloon ascends with a constant acceleration of 1 ft./sec.^{2} Find how many seconds per hour a clock carried with it will gain.

81. If the length of the seconds pendulum were taken as the unit of length, what would be the value of g?

82. Enunciate the law of universal gravitation, and give an account of the method of measuring the attraction between two spheres, devised by Mitchell, and carried out by Cavendish.

83. How did Newton prove that the weight of a body is proportional to its mass? Describe the nature of his experiment, and explain how he deduced his conclusions.

84. Assuming the preceding proposition, and the third

law of motion, show that it follows immediately that the attraction between two gravitating masses is directly proportional to the product of these masses.

85. *Prove that a spherical shell exerts no attraction upon a particle placed within it. You may assume

(*a*) That the area of a transverse section of a cone of small aperture varies as the square of the distance from the vertex.

(*b*) That a transverse section has a smaller area than an oblique section at the same distance, in the proportion of the cosine of the angle between them.

86. *Prove that a uniform spherical shell attracts an external particle as if its whole mass were condensed at its centre.

5

Work.—When the point of application of a force F moves through a distance *s* in the direction of the force, the work (W) done is

$$W = Fs.$$

If the force is measured in *dynes* (see § 3) and the distance in *centimetres*, the work will be expressed in *ergs*.

If the force is measured in poundals and the distance in feet, the work will be expressed in foot-poundals. (A poundal, or British absolute unit of force, is that force which, acting upon a mass of one pound, generates in it an acceleration of one foot per second in a second.)

When the unit of force is one which depends upon the intensity of gravity, the work is expressed in gravitation-units, whose value varies from place to place. By way of distinction, the erg and the foot-poundal are called absolute units. The engineer's unit of work—the foot-pound—is a gravitation-unit; it represents the work

* The asterisk is used to indicate the more difficult questions.

done at any place in raising a pound weight vertically through a distance of one foot at that particular place. A foot-pound is equal to g foot-poundals; or, taking $g=32$,

$$1 \text{ foot-pound} = 32 \text{ foot-poundals.}$$

Since half an ounce is $\frac{1}{32}$ of a pound, the foot-poundal about corresponds to the work done in raising half an ounce through a vertical distance of one foot.

The kilogramme-metre (which is the French engineer's unit of work) is the work done in raising the weight of a kilogramme through a vertical distance of one metre against the force of gravity. It is open to the same objections as the foot-pound, viz. that its value varies from place to place and from level to level.

The gramme-centimetre is the work done in raising a gramme weight through a vertical distance of one centimetre; at a place where $g=981$, the weight of a gramme is 981 dynes, and a gramme-centimetre is equal to 981 ergs.

87. Find the work done by a force of 50 dynes acting through a distance of 2 metres.

$$\text{Here } F=50,$$
$$s=2 \text{ metres} = 200 \text{ cm.}$$

The work done is

$$W=Fs=50 \times 200 = 10{,}000 \text{ ergs.}$$

88. How much work is done in raising a weight of one ton through a vertical height of 5 yards?

$$1 \text{ ton} = 2240 \text{ lbs.,}$$
$$\text{and } 5 \text{ yards} = 15 \text{ feet.}$$

Expressed in foot-pounds, the work done is

$$2240 \times 15 = 33600.$$

Expressed in foot-poundals, the work is

$$2240 \times 15 \times 32 = 1075200 \text{ (taking } g=32).$$

89. The weight of a tram-car is 8 tons, and the

resistance due to friction encountered in moving it is equal to one-sixteenth of the weight of the car: how much work is done in a run of 4 miles?

$$\text{The resistance to motion} = \text{weight of } \tfrac{8}{16} \text{ ton,}$$
$$= \text{weight of } 1120 \text{ pounds.}$$

The distance through which this force is overcome is

$$4 \text{ miles} = 4 \times 1760 \times 3 \text{ feet,}$$
$$= 21120 \text{ feet.}$$

The work done (expressed in gravitation-units) is

$$1120 \times 21120 \text{ foot-pounds} = 23{,}654{,}400 \text{ foot-pounds.}$$

90. Assuming that a person walking on level ground does work equivalent to the raising of his own weight vertically upwards through one-twentieth of the distance walked, find (in foot-tons) the average daily work done by Weston in his walk of 5000 miles in 100 days, his weight being 9 stone 2 lbs.

The average daily walk was 50 miles, and the average daily work was equivalent to the raising of his own weight through

$$\frac{50}{20} \text{ miles} = \frac{50 \times 1760 \times 3}{20} \text{ feet} = 50 \times 88 \times 3 \text{ feet.}$$

The weight raised was 9 stone 2 lbs. = 128 lbs. Hence the average daily work, in foot-tons, was

$$\frac{50 \times 88 \times 3 \times 128}{112 \times 20} = \frac{5280}{7} = 754\tfrac{2}{7}.$$

91. A mass of 12 kilogrammes is raised through a vertical height of 8 metres: express the work done in gramme-centimetres, and convert this into ergs.

92. A man can pump 30 gallons of water per minute to a height of 16 feet: how many foot-pounds of work does he do in an hour?

93. An agent A exerts a force equal to a weight of 50 lbs. through a distance of 120 feet, and another agent

B exerts a force equal to a weight of 180 lbs. through 90 feet. What is the ratio between A's work and B's?

94. A ladder 20 feet long rests against a vertical wall and is inclined at the angle of 30° to it: how much work is done by a man weighing 13 stone in ascending it?

95. A body of mass 3 lbs. is projected vertically upwards with a velocity of 640 feet per second: how much work has been done against gravity when it has ascended to half its maximum height?

96. How much work would be done in lifting 8 kilogrammes to a height of 12 metres above the surface of the moon, where g is 150?

97. Two masses M and M_1 are acted upon by the same force for the same time: find the relation between (1) the amounts of momentum generated in the masses, (2) the amounts of work done upon them.

98. A body of mass 12 lbs. rests upon a horizontal plane, the coefficient of friction between it and the plane being 0·14: find the work done in moving the body through a distance of 4 yards along the plane.

99. If the plane in the preceding question were inclined at an angle of 30° to the horizontal, how much work would have to be done in order to move the body 3 yards along it?

100. Calculate, in foot-tons, the work done in moving a railway train weighing 120 tons through a distance of 2 miles along a level line, assuming that the resistances amount to 12 lbs. for every ton in motion.

101. If we change from a foot-pound-second to a yard-pound-minute system, in what ratio must we alter the unit of work?

102. Calculate the work done in drawing a train of 140 tons one mile along a level line, the coefficient of friction being $\frac{1}{160}$.

103. An engine is running at the rate of 80 feet per second, on level ground, when the steam is shut off. Assuming that the frictional resistances are equivalent to

a weight of 14 lbs. per ton, find how far the engine will run, and for how long. ($g = 32$.)

104. A train is started from rest on level ground and the resistances to motion amount to 12 lbs. per ton. What horse-power must the engine exert in order to get up a velocity of 30 miles per hour in 5 minutes?

105. A body slides down an inclined plane 18 ft. long, inclined at 30° to the horizontal. If it starts from rest, find its velocity when it reaches the lower end of the plane—(1) if the plane is smooth, (2) if rough, the coefficient of friction being $\sqrt{3}/4$. ($g = 32$.) ✓

106. A locomotive and train together weigh 128 tons, and the coupling chain between them can just stand a pull equal to the weight of 8 tons. What is the shortest time in which a speed of 45 miles an hour can be worked up on a frictionless level line?

107. A body of mass 28 lbs. is moving with a velocity of 30 yards per minute. What is the magnitude of the retarding force which will just bring it to rest in 7 seconds?

108. An engine exerting 90 H.-P. draws a train, the weight of which (including that of the engine) is 120 tons, along a level line with a uniform velocity of 20 miles an hour: find the coefficient of friction.

109. A train moves from rest with uniform acceleration for 4 minutes, and in this time it passes over 2 miles.

(1) In what time will it pass over the next 6 miles?

(2) If the weight of the train be 120 tons, and the resistances (from the air, friction, etc.) $11\frac{2}{3}$ lbs. per ton, compare the force of the engine with the weight of 1 lb.

110. A train starts from rest and proceeds with uniform acceleration for 70 seconds, at the end of which time its velocity is 30 miles an hour: how far has it travelled?

It now proceeds with uniform velocity for 10 miles, when the brakes are applied and bring it to rest (with

uniform retardation) in 220 yards. What is the whole distance traversed, and how long does the journey take?

111. Find the work done in drawing a carriage of 20 tons up an incline one mile in length and rising 1 in 120, the coefficient of friction being $\frac{1}{140}$.

112. A Venetian blind consists of 30 wooden slips each weighing 100 grammes, and when the blind is down the slips are 5 cm. apart. Find the work done in drawing it up, assuming, for the purposes of your calculation, that when drawn up all the slips may be regarded as being raised to the level of the top slip.

113. The cylinder of a steam-engine has a diameter of 6 inches, and the piston moves through a distance of 10 inches: find the work done per stroke, assuming the pressure of the steam in the cylinder to be constant and equal to 30 lbs. per square inch.

114. Two bodies of 80 lbs. and 60 lbs. are raised through heights of 100 feet and 50 feet respectively. Calculate the total amount of work done, and show that it is equal to the work which would be done in raising the sum of the weights through a vertical distance equal to that through which their centre of gravity is raised.

115. Assuming the result indicated in the preceding question, and taking the weight of one cubic foot of water as 62·5 lbs.: find how much work must be done in order to empty a well 10 feet in diameter and 200 feet deep, filled to the brim with water.

Energy.—The energy of a body is the power which it possesses of doing work. When it possesses this power in virtue of its position (as when it is raised above the level of the ground) the energy is called statical or potential energy. When it possesses the power of doing work in virtue of its motion, the energy is called kinetic energy (K.E.).

The weight of a body of mass m grammes is mg dynes; if the body be raised to a height of h centimetres

above the level of the ground, the work which it can do in falling is mgh ergs, or

The potential energy of a body of mass m, raised to a height h, is mgh.

If m is expressed in pounds and h in feet, the product mgh will be the measure of the energy in foot-poundals; but expressed in foot-pounds the measure of the energy will be mh simply.

Now if a body be projected vertically upwards with a velocity v, it will rise to a height h, such that

$$v^2 = 2gh.$$

Multiplying each side of this equation by $\frac{m}{2}$, we have

$$\tfrac{1}{2}mv^2 = mgh.$$

But mgh represents the work which would have to be done in order to raise the body to the height h; and this amount of work is done by the body in virtue of the energy of motion, or kinetic energy, which it possessed on starting; hence

The kinetic energy of a body of mass m moving with a velocity v is $\frac{1}{2}mv^2$.

[It should be noticed that since

$$\tfrac{1}{2}(2) \times 1^2 = 1,$$

the unit of kinetic energy is that possessed by *two* units of mass moving with unit velocity (*not* that possessed by unit mass moving with unit velocity).]

116. A reservoir contains water at a height of 200 feet above the ground: what is the potential energy of the water in foot-pounds per gallon?

The potential energy of each pound of water in the reservoir is 200 foot-pounds, and one gallon of water = 10 lbs. Hence the potential energy per gallon is $10 \times 200 = 2000$ foot-pounds.

117. What is the potential energy of a mass of 25 kilogrammes raised to a height of 40 metres above the ground?

$$\begin{aligned}\text{Its energy} &= 25 \times 40 \text{ kilogramme-metres,}\\ &= 1000 \text{ kilogramme-metres,}\\ &= 1000 \times 10^5 \text{ gramme-centimetres,}\\ &= 981 \times 10^8 \text{ ergs.}\end{aligned}$$

118. A stone of mass 6 kilogrammes falls from rest at a place where $g = 980$: what will be its kinetic energy at the end of 5 seconds?

The velocity acquired in 5 seconds will be

$$v = g \times 5 = 980 \times 5 = 4900,$$

and since 6 kilos. $= 6000$ gm.,

$$\text{K.E.} = \tfrac{1}{2} \times 6000 \times (4900)^2 = 7 \cdot 203 \times 10^{10} \text{ ergs.}$$

119. What is the K.E. of a body of mass 16 lbs. moving with a velocity of 50 feet per second?

Expressed in foot-poundals the K.E. of the body is

$$\tfrac{1}{2} \times 16 \times (50)^2 = 20{,}000.$$

Since one foot-pound $= 32$ foot-poundals, the K.E. in foot-pounds is

$$\frac{20{,}000}{32} = 625.$$

[We might have commenced by *defining* kinetic energy as being the value of the product $\frac{1}{2}mv^2$; then proceeding, as follows, to show that this quantity is a measure of the work which a body can do in virtue of its motion :—

Suppose a body of mass m moving with a velocity u to be acted upon by a force F in the direction of its motion, and let the acceleration produced by this force be $a = \frac{F}{m}$. After the body has moved through a space s its velocity v will be given by the equation

$$v^2 = u^2 + 2as.$$

Multiplying each term in this equation by $\frac{m}{2}$, we have

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mu^2 + mas.$$

But since $F = ma$, $mas = Fs$, and Fs is the work done by the force F acting through the space s. Thus the work done by the force is measured by the increase of kinetic energy which it produces; or,

[K.E. at any time] = [initial K.E.] + [work done by acting force].

If the force acts upon the body in a direction opposite to that of its motion, it resists its motion and diminishes its kinetic energy: the + sign in the equation must be changed to –, and we now have

[K.E. at any time] = [initial K.E.] – [work done against resistance].

Both theorems are expressed in the statement that the work done by the force is equal to the change of kinetic energy which it produces.

One case of the second theorem is of special importance. Suppose the body to be brought to rest by the resistance, the final K.E. = 0,

$\therefore$ 0 = [initial K.E.] – [work done against resistance],

or,

[Initial K.E.] = [work done against resistance].

Thus we have shown that the kinetic energy ($\tfrac{1}{2}mv^2$) of a body is a measure of its capacity for doing work.]

120. A train is moving at the rate of 15 miles an hour when the steam is cut off. Supposing the resistance due to friction, etc. to amount to $\frac{1}{64}$ of the weight of the train, find how far it will travel before it comes to rest.

15 miles an hour = 22 feet per second.

If the mass of the train be m lbs., its kinetic energy (in foot-poundals) is

$$\frac{mv^2}{2}=\frac{m}{2}\times(22)^2.$$

The resistance is equal to the weight of $\frac{m}{64}$ pounds

$$=(m/64)\times g \text{ poundals,}$$
$$=m/2 \text{ poundals (taking } g=32).$$

If the train travels x feet, the work done is $(m/2)\times x$ foot-poundals; and, since it is brought to rest by the resistance, this must be equal to its kinetic energy,

$$\therefore (m/2)\times x=(m/2)\times(22)^2,$$

and

$$x=484.$$

Thus the train travels 484 feet before coming to rest.

121. A bullet of 100 grammes is discharged with a velocity of 400 metres per second from a rifle, the barrel of which is one metre in length. Calculate the energy of the bullet when it leaves the muzzle, and the mean force exerted by the powder upon it.

The kinetic energy of the bullet is

$$\tfrac{1}{2}\times 100\times(40{,}000)^2=8\times 10^{10} \text{ ergs.}$$

The energy acquired by the bullet is equal to the work done upon it (by the expansion of the powder) as it travels down the barrel. Let F denote the mean force, in dynes, exerted by the powder; then, since $Fs=\frac{1}{2}mv^2$,

$$F\times 100=8\times 10^{10},$$

or

$$F=8\times 10^8 \text{ dynes.}$$

122. A girl weighing 6 st. $6\frac{3}{4}$ lbs. skips 6 inches high fourteen times. Show that the energy thus spent would suffice to stop a thief weighing $13\frac{1}{2}$ stones and running at the rate of 10 miles an hour.

$13\frac{1}{2}$ st. $= 189$ lbs., and 10 miles an hour $= 44/3$ feet per second.

Thus the kinetic energy of the thief (in foot-poundals) is

$$\tfrac{1}{2} \times 189 \times (44/3)^2 = \tfrac{1}{2} \times 21 \times 11^2 \times 4^2.$$

Again the work done by the girl is

$$14 \times [\text{work done in raising } 90\tfrac{3}{4} \text{ lbs. through } \tfrac{1}{2} \text{ foot}]$$

$$= 14 \times \tfrac{1}{2} \times 363/4 \text{ foot-pounds,}$$

or, in foot-poundals (taking $g = 32$)

$$= 7 \times 363 \times 8 = 7 \times 3 \times 11^2 \times \tfrac{1}{2} \text{ of } 4^2 = \text{K.E. of thief.}$$

123. Express (1) in foot-pounds and (2) in foot-poundals the potential energy of a mass of 5 tons raised to a height of 10 yards above the ground.

124. A sack of flour weighs $2\frac{1}{2}$ cwt.: to what height must it be raised in order that its potential energy may be 9240 foot-pounds?

125. A 4-oz. bullet is projected vertically upwards with a velocity of 800 feet per second: what is its potential energy when it has ascended to its maximum height?

126. A mass of 24 kilogrammes is raised to a height of 16 metres: find its energy (in ergs).

127. A kilogramme weight is suspended from the lower end of a string 2 metres long so as to form an approximately simple pendulum: calculate, in ergs, the energy of the bob of this pendulum when it is held so that the string makes an angle of 60° with the vertical.

128. What is the energy of a mass of 5 kilogrammes moving with a velocity of 50 metres per second?

129. A cannon ball of 10 kilogrammes is discharged from a gun with a velocity of 300 metres per second: express its kinetic energy in ergs.

130. What is the K.E. of a mass of 1 cwt. after it has fallen from rest for 4 seconds?

131. A mass of 1 lb. hanging vertically draws a mass of 15 lbs. along a smooth horizontal plane. Find the

K.E. of the system when each mass has moved over 4 ft.

132. Calculate the momentum and the K.E. of a mass of 5 cwt. after it has fallen through a vertical distance of 8 feet.

133. A stone of mass 3 lbs. is thrown vertically upwards with a velocity of 96 feet per second: what is its kinetic energy at the end of 2 seconds?

134. A 5-lb stone is thrown vertically up and at the end of the first second is moving upwards at a rate of 64 feet per second: calculate its kinetic energy at the moment when it reaches the ground.

135. A mass of 50 lbs. starts from rest under the action of a constant force and acquires a velocity of 12 feet per second in 2 seconds: what force acts upon it, and what will be the kinetic energy acquired at the end of the fifth second?

136. A 100-gramme bullet strikes an iron target with a velocity of 400 metres per second, and falls dead: how much kinetic energy is lost?

137. A body of mass m is moving with a velocity such that its K.E. is e; show that its momentum is $\sqrt{2me}$.

138. Find in foot-tons the K.E. of a train of 64 tons running at the rate of 45 miles an hour. The brakes are suddenly applied to the train and it stops within 11 yards: find the average force exerted by the brakes.

139. The two masses in an Atwood's machine are 255 and 245 grammes. Calculate, by considering the work done, what should be the velocity at the end of 3 seconds, assuming that $g=980$ and that there is no loss by friction, etc. If the actual velocity is only 48·8 cm. per sec., how much energy has been wasted?

140. A mass of 50 kilogrammes starts from rest under the action of a force, and some time afterwards is observed to be moving with a velocity of 10 metres per second: how many ergs of work have been done upon it?

141. How many foot-pounds of work must be done on

a mass of one ton in order to give it a velocity of 15 miles an hour?

142. The mass of a pendulum-bob is 100 grammes, and the string is a metre long. The bob is held so that the string is horizontal, and is then allowed to fall: find its kinetic energy when the string makes an angle of 30° with the vertical.

143. A system A consists of a light bar 20 cm. long pivoted at its centre and having a mass of 90 grammes attached to each end of the bar. A similar system B consists of a bar 50 cm. long having a mass of 40 grammes attached to each end. Compare the kinetic energies of the two systems when A is rotating 100 times and B 120 times per minute.

144. A shot travelling at the rate of 200 metres per second is just able to pierce a plank 4 cm. thick: what velocity is required to pierce a plank 12 cm. thick?

Assuming that the resistance offered by the plank is uniform it follows from the equation

$$Fs = \tfrac{1}{2}mv^2$$

that the thickness which the shot can penetrate is proportional to the square of its velocity. If a shot moving with velocity v can pierce a plank of thickness t, and a shot moving with velocity v' can pierce a plank of thickness t', then

$$t : t' :: v^2 : v'^2.$$

In the above example

$$v'^2 = (200)^2 \times 12/4 = 120{,}000,$$

and therefore the required velocity is 346·4 metres per second.

145. If a bullet moving with a velocity of 150 metres per second can penetrate 2 cm. into a block of wood, through what distance would it penetrate when moving at the rate of 450 metres per second?

146. What is the energy of a train of 40 tons moving

at the rate of 30 miles an hour? What force, acting for 30 seconds, would be sufficient to bring the train to rest?

147. A stone of mass 3 lbs. falls from rest for 2 seconds, when it comes in contact with a flat roofing-slate, which it smashes, thereby losing two-thirds of its velocity: how much energy does it lose by breaking the slate?

148. An ounce bullet strikes a target one inch thick with a velocity of 400 feet per second and emerges with a velocity of 200 ft. per second. Compare the average pressure on the target with the weight of 1 lb.

149. A stone is projected along smooth ice with a velocity of 15 ft. per sec., and comes to rest after travelling 75 yards: what is the coefficient of friction of the stone on the ice?

150. The bore of a cannon is 2·4 metres long; with the normal charge of powder it is able to project a 12-kilogramme cannon-ball with a velocity of 500 metres per sec. Calculate the average value of the pressure exerted on the ball as it travels along the bore.

151. One locomotive engine draws a train of total weight 200 tons and can get up a speed of 45 miles an hour in 5 minutes, starting from rest: another can in the same time get up a speed of 30 miles an hour in a train of 300 tons. Compare the strength of the first engine with that of the second.

152. A body is acted upon by a constant force for 8 secs. During this time 480 units of work are done upon it, and it acquires 120 units of momentum. Determine the mass of the body and also its velocity at the end of the given time.

153. A 2 oz. bullet is fired from a gun, the barrel of which is 3 ft. long, and leaves the muzzle with a velocity of 1000 ft. per sec. Find the average pressure exerted by the powder on the bullet.

154. Compare the amount of kinetic energy in (1) a

boulder of 1 cwt. which has fallen for 1 second from rest, and (2) a 1-lb. projectile moving with a velocity of 800 feet per second.

155. A bullet of 90 grammes leaves the muzzle of a gun with a velocity of 500 metres per second. If the barrel be 120 centimetres long, find the mean pressure exerted by the powder upon the bullet.

156. A railway carriage contains forty passengers, whose average weight is 140 lbs. If the carriage itself weighs 6 tons, and is moving at a rate of 30 miles an hour, what is its kinetic energy?

157. A body of mass m moves under the action of a force F through a space s in a straight line, which is inclined at an angle θ to the direction of the force: if v be the velocity generated, show that $Fs \cos \theta = \frac{1}{2}mv^2$.

158. A 20-lb cannon ball falls through a vertical distance of 1600 feet: what is its energy? With what velocity would it have to be projected from a cannon in order to possess an equal amount of energy?

159. Two inelastic balls moving in opposite directions come into collision; the one has a mass 10 and velocity 50, the other a mass 50 and velocity 10: what is their total kinetic energy before and after impact, and what has become of the energy apparently destroyed?

160. A body of mass 56 lbs. starts from rest under the action of a constant force, and acquires a velocity of 64 feet per second while moving through a space of 160 feet: find the acting force and the work done by it.

161. A constant force acts upon a body for 20 seconds, doing 10 units of work upon it, and generating in it during the same time 30 units of momentum: find the mass of the body and the velocity which it will have acquired.

162. The bob of a simple pendulum is let go when the pendulum is inclined at an angle of 60° to the vertical. Compare its kinetic energy after describing an arc of 30° with its K.E. at its lowest point.

163. A tricyclist working steadily at the rate of $\frac{1}{11}$ of a horse-power can just do 10 miles an hour on a level road. Prove that the resistance of the road is just equal to a weight 3·41 lbs.

164. An 81-ton gun discharges a shell of mass half a ton, and the recoil of the gun is checked by a device which may be regarded as exerting a constant pressure equal to a weight of 8 tons. Prove that if the gun moves backwards through a distance of 8 ft., the muzzle-velocity of the shell must be 1152 ft. per sec.

165. *A hammer moving at the rate of 12 ft. per sec. hits a nail and drives one inch of its length into a board. The mass of the hammer-head is half a pound. Assuming it to be inelastic and to come to rest, find the average resistance which it encounters.

Power.—The power (or activity) of an agent is the rate at which it can do work, and is measured by the number of units of work done per unit of time. The unit of power commonly used by engineers in this country is the horse-power, which is defined as being the power of doing 33,000 foot-pounds of work per minute, or 550 foot-pounds of work per second.

166. Assuming that the pressure within the cylinder of a steam-engine remains constant throughout the whole of the stroke, find the horse-power developed in each cylinder of an engine, having given—

A = area of piston in square inches.
P = pressure upon the piston in pounds per square inch.
S = length of stroke in feet.
R = number of revolutions per minute.

Here P denotes the *intensity* of the pressure on the piston in pounds weight per square inch. The total pressure on the piston is the weight of AP pounds. This is the acting force, and the distance through which it moves in each stroke is S feet.

Thus the work done in each stroke is SAP foot-pounds.
Since there are two strokes for each revolution, the number of strokes per minute is 2R, and the work done per minute is 2SRAP foot-pounds. Thus the horse-power developed is

$$\text{H.-P.} = 2\text{SRAP}/33{,}000.$$

167. Water is supplied to a hydraulic motor at a pressure of 100 lbs. per square inch. Express the potential energy of the water in the reservoir in foot-pounds per gallon; and calculate the maximum H.-P. which can be developed by the motor if the rate of supply is 50 gallons per minute.

Let h = height in feet of the reservoir. The pressure in pounds weight per square foot $= h\rho$, where ρ = number of pounds in a cubic foot of water $= 62 \cdot 5$,

$\therefore h \times 62 \cdot 5/144$ = pressure in pounds weight per square inch $= 100$,

and $h \times 14400/62 \cdot 5 = 230 \cdot 4$.

Thus the potential energy of one pound of water is 230·4 foot-pounds, and the potential energy of one gallon (or 10 lbs.) is 2304 foot-pounds.
(Notice that since the pressure is supposed constant in the question, we must also assume that the level of the water in the reservoir is kept constant.)
The work done by a supply of 50 gallons per minute is

$$2304 \times 50 \text{ ft.-lbs. per min.}$$

The power developed is

$$\text{H.-P.} = 2304 \times 50/33{,}000 = 3\frac{27}{55} = 3 \cdot 49.$$

168. The nominal value of a horse-power is 33,000 foot-pounds per minute. Express this (1) in kilogramme-metres per minute and (2) in ergs per second.

169. A five H.-P. engine is employed to pump water from the bottom of a mine 100 feet deep. How many cubic feet of water will it raise in 24 hours? (1 cub. ft. of water $= 62\frac{1}{2}$ lbs.)

170. What should be the indicated H.-P. of an engine that is intended to pump 200 gallons of water per minute to a height of 50 yards? (1 gal. = 10 lbs.)

171. A 300 H.-P. engine draws a train of 180 tons, the resistance due to friction being 12 lbs. per ton: find its maximum velocity along a level line. $HP = \frac{R \cdot v}{550}$

172. Find the H.-P. of an engine that should be employed for raising coal from a pit 200 feet deep, the average daily yield being 1782 tons.

173. A vertical shaft of depth 500 feet and cross section 12 square feet is filled with water. What is the horse-power of an engine which will empty it in 12 hours? (1 cubic foot of water = 62·5 lbs.)

174. Find the horse-power exerted by an engine which draws 150 tons up an incline of 1 in 200 at the rate of 15 miles per hour, the resistance due to friction, etc. being equivalent to a weight of 14 lbs. for every ton in motion.

175. Determine the rate at which an engine is working when it drives a train of 150 tons at a rate of 30 miles an hour, the resistance to motion being equal to a weight of 16 lbs. for every ton.

176. The mass of a train is 200 tons, and the resistances to its motion amount to 20 lbs. per ton on a level line: find the horse-power of an engine which can just keep it going at the rate of 45 miles an hour.

177. A steam-engine supplies 1000 houses with 100 gallons of water each, working 12 hours per day: if the mean height to which the water has to be raised is 80 feet, at what rate does the engine work?

178. What alteration would be produced in the unit of work if the units of mass, length, and time were each increased ten-fold? If a horse-power be represented by 550 under the old system, what would be its numerical value in the new?

179. *A belt transmits power from a wheel $3\frac{1}{2}$ feet in diameter which revolves 180 times per minute. The difference between the tensions on the tight and slack sides of the belt is equal to 200 lbs. weight. What horse-power does the belt transmit? ($\pi = 22/7$.)

180. A band is stretched as a friction dynamometer over the upper half of the fly-wheel of an engine: from one end of the band is hung a weight of W_1 lbs., and the other end is attached to a spring balance. The radius of the fly-wheel is R feet, and it makes N revolutions per minute. If the spring balance indicates a pull of W_2 lbs., show that the engine is working at the rate of $2\pi RN(W_1 - W_2)/33{,}000$ horse-power.

EXAMINATION QUESTIONS[1]

181. A bird can fly in still air at 20 miles an hour. When a wind is blowing straight from the north at 10 miles an hour, in what direction must the bird aim in order that it may fly from east to west, and at what speed will it travel relative to the earth's surface?

Illustrate your answer by a careful figure.

Matric. 1895.

182. A parachute, weighing 1 cwt., falling with a

[1] The following abbreviations are used in marking the sources from which the examples at the end of each chapter are taken:—

London University Examinations:
- Matric. = Matriculation.
- Int. Sc. = Intermediate Science (or 1st B. Sc.)
- B. Sc. = Final B. Sc.
- Prel. Sc. = Preliminary Scientific.

S. K. = Examinations of the Science and Art Department (South Kensington: Advanced Stage).
Camb. Schol. = Cambridge Entrance Scholarship Examinations.
Camb. B.A. = Cambridge General and Special B.A. Exams.
N. S. Tripos = Natural Science Tripos.
M. Tripos = Mathematical Tripos.
Vict. Int. = Intermediate B.A. and B.Sc. Exams., Victoria University.
Ind. C. S. = Indian Civil Service.

uniform acceleration from rest, descends 16 feet in the first 4 seconds. Find the resultant vertical pressure of the air on the parachute. Matric. 1895.

183. A body, whose mass is 100 lbs., is projected along a horizontal surface with a velocity of 10 feet per second. A constant horizontal retarding force acts on the body, and brings it to rest after the body has passed over 100 feet. Find the magnitude of the retarding force. Matric. 1895.

184. A body is thrown up in a lift with a velocity equal to u relative to the lift and the time of flight is found to be t. Show that the lift is moving up with an acceleration $(2u - tg)/t$, and find the actual velocity of the body just before it strikes the hand of the observer if the lift has been in motion for the time T.

Camb. Schol. 1891.

185. What is the "absolute" unit of force, and why is it so called?

If a train whose weight is 200 tons is moving at the rate of a mile per minute, find (in tons weight) the constant force which would stop it (1) in one minute, (2) in one mile. Vict. Int. Sc. 1890.

186. Explain the term *relative velocity.*

A ship A was 20 miles north of another ship B at noon. A was sailing due south at the rate of 12 miles per hour, B due east at 9 miles per hour. At what time were they nearest to each other; and how far apart were they then? Camb. Schol. 1893.

187. The velocity of a body is observed to increase by four miles per hour in every minute of its motion: compare the force acting on it with the force of gravity.

B.Sc. 1891.

188. If two weights w and $2w$ are connected by a string passing over a smooth weightless pulley, which is attached to a third weight $3w$ by a string passing

over a smooth fixed pulley, prove that the weight $3w$ descends with an acceleration $\frac{g}{17}$. Camb. Schol. 1882.

Let α denote the acceleration of the weight $2w$ downwards *relative to the pulley P*, and let β denote the acceleration of P upwards, or of the weight $3w$ downwards.

The actual acceleration of $2w$ in space (or relative to the fixed pulley P′), is $\alpha - \beta$. Let T be the tension in the string connecting w and $2w$. Considering the motion of $2w$, we see that the force causing motion is $2wg - T$, and the mass moved is $2w$, and the acceleration produced is $\alpha - \beta$.

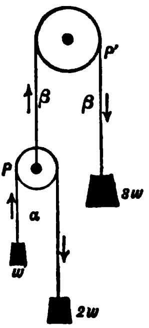

Fig. 1.

Therefore

$$2wg - T = 2w(\alpha - \beta).$$

The acceleration of w upwards is $\alpha + \beta$, and the force acting upon w is $T - wg$; hence

$$T - wg = w(\alpha + \beta).$$

The tension in the string connecting the third weight with the pulley P is $T' = 2T$, and the equation for the motion of $3w$ is

$$3wg - 2T = 3w\beta.$$

Solving these three equations, we find that the weight $3w$ descends with an acceleration $g/17$.

189. State Newton's Laws of Motion, and discuss the bearing of the second law on the use of the word force.

A velocity of one foot per second is changed uniformly in one minute to a velocity of one mile per hour. Express numerically the acceleration when a yard and a minute are the units of space and time. Camb. Schol. 1894.

190. What force must be applied for one-tenth of a second to a mass of 10 tons in order to produce in it a velocity of 3840 ft. per minute? [*N.B.*—A numerical answer is meaningless unless the unit intended is also stated.] Matric. 1891.

191. A ball thrown up is caught by the thrower 7 seconds afterwards. How high did it go, and with what

speed was it thrown? How far below its highest point was it 4 seconds after its start? Matric. 1891.

192. How does the time of a simple pendulum, oscillating under gravity alone, vary with the force of gravity? If gravity is $\frac{1}{200}$ greater at the North Pole than at the Equator, how many seconds a day will a second's pendulum at the North Pole lose when taken to the Equator? Vict. Prel. 1890.

193. Richer discovered, in the 17th century, that a pendulum-clock, regulated at Paris, lost 2 minutes 28 seconds daily when taken to Cayenne; deduce the conclusion that the value of gravity is smaller at Cayenne than at Paris by 0·11 of a foot-second unit approximately. B. Sc. 1892.

194. The measure of a certain acceleration is f when a foot and a second are the units of space and time: find its measure when l feet and t seconds are the units.

What are the units of mass, length, and time when the weight of one ton is the unit of force, the acceleration due to gravity the unit of acceleration, and a velocity of one mile an hour the unit of velocity? Camb. Schol. 1891.

195. Give a definition of acceleration which will apply to any case, and show that if a particle move uniformly in a circle the acceleration is towards the centre and equals v^2/r, where v is the uniform velocity and r the radius of the circle.

A weightless string has attached to one end a mass of 1 lb. which lies on a smooth horizontal table, the length of the string on the table is one foot, the remainder of the string hangs vertically through a small hole in the table, and has a certain weight attached to it; find what this weight must be in order that the mass of 1 lb. having been started may continue to describe the horizontal circle of one foot radius at the rate of one revolution per minute. Camb. Schol. 1891.

196. A bullet, moving at the rate of 1100 feet per

second, passes through a thin plank and comes out with a velocity of 1000 feet a second; if it then passes through another plank exactly like the former, with what velocity will it come out of this second plank? S. K. 1895.

197. A mass of 5 lbs. resting on a smooth plane inclined at 30 degrees to the horizon is connected by a fine thread, which passes over a pulley at the summit of the plane, with a mass of 3 lbs. hanging vertically. Compare the pull in the thread when the mass on the plane is held fixed and when it is let go. If the thread is severed or burnt 3 seconds after this mass has been let go, find how far it will rise on the plane before falling back. Int. Sc. 1891.

198. Two equal masses are at rest side by side. One moves from rest under a constant force F while at the same instant the other receives a blow I. Show that they will be again side by side after a time $2I/F$. ?

Camb. B.A. 1893.

199. A uniform force equal to the weight of 20 lbs. acts upon a body which is initially at rest, and causes it to move through 24 feet in the first second. Find the mass of the body. Matric. 1894.

200. Distinguish between the momentum and the energy of a moving body. A 30-ton mass is moving on smooth level rails at 20 miles an hour: what steady force can stop it, (*a*) in half a minute, (*b*) in half a mile? Specify the force completely. Matric. 1892.

201. Assuming that the earth turns once in 86,164 seconds, that the equatorial radius is 20,900,000 feet, and that the acceleration due to gravity at the equator is 32·1, find what part of the weight of a body is used in keeping the body on the equator. [*N.B.*—Take $\pi^2 = 9{\cdot}87$.] S. K. 1895.

202. A bullet, weighing 1 oz., is fired horizontally from a height of 16 feet. When it strikes the ground the vertical velocity is 1/20th of the horizontal velocity. Find the energy, in foot-lbs., possessed by the bullet at the instant of projection. Matric. 1895.

203. Define "work" and "energy." Calculate the energy, in foot-pounds, of a mass of 180 lbs., moving at the rate of 10 feet a second. How high would such a mass move under gravity if projected upwards with this velocity? Prel. Sc. 1891.

204. A set of railway carriages start from rest down an incline a mile long with a gradient of 1 in 100; find how many yards they will travel on the level after leaving the incline, before they come to rest, the frictional resistance to their motion being 10 lbs. per ton. Find also, in miles per hour, the greatest velocity they will acquire. Int. Sc. 1891.

205. A 2-oz. bullet moving horizontally with a velocity of 1000 feet per second strikes and remains embedded in a piece of wood whose mass is 20 lbs., and which is hung up by a long string. Through what vertical height will the piece of wood be caused to rise by the blow? Camb. Schol. 1891.

206. A train weighing 60 tons, and running at the rate of 40 miles an hour, is stopped by an obstacle in 10 yards. What is the average force applied by the obstacle, and what amount of heat will be produced? Int. Sc. 1890.

207. A stone is projected along a rough horizontal plane, coefficient of friction $\frac{1}{3}$, with a velocity of 10 feet per second. After moving for 1 yard it begins to ascend a smooth plane inclined at 30° to the horizon. How far will it rise? Glasg. M. A. 1890.

208. A man who weighs 168 lbs. walks up a mountain path at a slope of 30° to the horizon at the rate of one mile per hour; compare his rate of working in raising his own weight with a horse-power. Int. Sc. 1891.

209. A train of mass M is travelling with uniform velocity on a level line; the last carriage, whose mass is m, becoming uncoupled, the driver discovers it after travelling on a distance l, and then shuts off steam. Show that when both parts come to rest the distance

between them is $Ml/(M - m)$, if the resistance to motion be uniform and proportional to the weight and the pull of the engine be constant. Camb. Schol. 1891.

210. A locomotive draws a load of 200 tons. Find the pull needed (1) at a constant speed if the friction is 0·05 of the load; (2) if the friction is the same, and the speed rises from 30 ft./sec. to 60 ft./sec. in one minute ($g = 32$ ft./sec.^{2}). Matric. 1895.

211. An engine takes a train of 60 tons in all up an incline of 1 in 100 at a maximum speed of 30 miles per hour, and it can take a train of 150 tons on the level at the same speed; find the frictional resistance of the road in pounds per ton, and also the rate, in horse-power, at which the engine works when running at this speed.

B. Sc. 1891.

212. What is meant by an impulsive force? How is such a force measured?

A billiard ball whose weight is 5 oz. is hit so as to move off with a velocity of 16 inches per second. If the time during which the blow lasts be $\frac{1}{100}$ of a second, find the average value of the force during impact.

Glasg. M. A. 1890.

213. A ball whose mass is $1\frac{1}{2}$ lb. is struck by a stick when at rest, and a velocity of 120 feet per second is imparted to it. If the ball and stick only remain in contact for $\frac{1}{100}$ of a second, find the mean force which the stick exerts upon the ball. Camb. Schol. 1891.

214. *Discuss the principles on which depend the results of the collision of two smooth uniform spheres.

A hammer whose mass is 10 lbs. is used to drive a nail, whose mass is 4 ounces, into a board. The surfaces of the hammer and nail are inelastic, and the hammer when it strikes the nail has a velocity of 10 feet per second. Show that in order that each blow may drive the nail 1 inch into the board the resistance, supposed constant, of the latter must not exceed a weight of 183 lbs. M. Tripos. 1890.

215. *State Newton's second law of motion, and de-

duce from it a relation between the force acting on a body, the mass of the body, and its acceleration. Explain the corresponding law for angular motion about an axis, and apply it to the following problem: A constant couple of 2 foot-lb. acts upon a fly-wheel for 5 seconds, and produces an angular velocity of 5 revolutions per second. Find the moment of inertia of the wheel.

N. S. Tripos. 1890.

216. Find in foot-pounds the least energy required to drive a golf ball weighing $1\frac{1}{2}$ oz. a distance of 200 yards up an incline of 30°, the resistance of the air being neglected. B.Sc. 1893.

217. State the laws of impact, and explain on what foundations they rest.

Two small equal spheres hang by vertical strings, 2 feet and 3 feet long respectively, so as to be just in contact with their centres at the same level. The former sphere is drawn aside till its string makes an angle of 60° with the vertical, and is then released, and the second sphere is observed after the impact to swing through an arc of 30°. Find the coefficient of restitution.

B.Sc. 1893.

218. A heavy particle falls down a rough inclined plane which makes an angle of 45° with the horizon; the velocity of the particle after falling down the plane is one-half the velocity which would have been acquired by the particle if it had fallen freely through the vertical height of the plane. Find the coefficient of friction between the particle and the plane, assuming that this is independent of the velocity of the particle.

B.Sc. 1893.

CHAPTER II

STATICS

1. Two forces act at a point. The magnitude of the first is $10\sqrt{3}$ and its direction is due north; the magnitude of the second is 10 and its direction is due west: what is the magnitude and direction of their resultant?

2. Six forces, of 1, 2, 3, 4, 5, and 6 lbs. respectively, act at a point in directions represented by the six sides of a regular hexagon taken in order. Find the direction and magnitude of their resultant.

3. ABCD is a parallelogram such that the diagonal AC is perpendicular to the side BC. Three forces act at the point A, and are represented in magnitude and direction by AB, CA, and AD. Show that they are in equilibrium.

4. Four forces, A, B, C, and D, act at a point. B is double of A, and acts at right angles to it. C is equal to the sum of A and B, and acts at right angles to their resultant. D is equal to the sum of A, B, and C, and acts at right angles to their resultant. Show that the resultant of A, B, C, and D is equal to $5A\sqrt{2}$. (Reckon all the right angles in the same direction.)

5. A beam, whose length is 4 ft. and weight 20 lbs., has weights of 4 lbs. and 8 lbs. suspended from its extremities. Find the position of a point about which it will balance.

6. Two men carry a load of 1 cwt. suspended from a

horizontal pole 12 ft. long. The pole weighs 24 lbs., and its ends rest on their shoulders. If the load is suspended from a point 3 ft. from one end of the pole, what weight is borne by each man?

7. An uniform pole weighs 20 lbs. and is 10 ft. long. At a distance of 2 ft. from one end is attached a 7-lb. weight, and at a further distance of 1 ft. a second weight of 6 lbs. At what point will the pole balance?

8. An uniform plank, 24 ft. long, rests on the top of two walls 12 feet apart. The walls are of the same height, and the plank projects equally beyond each. A man starts walking from the centre towards one end of the plank. If his weight is 12 stone, and that of the plank 6 stone, find how far beyond the wall he can walk without tipping up the plank.

9. An iron bar weighs 4 lbs. per foot length. It balances about a point 2 ft. from one end when a weight of 5 lbs. is suspended from that end. How long is the bar?

10. A picture of mass 3 lbs. hangs vertically from a nail by a cord passing through two rings in the frame. The parts of the string form an equilateral triangle. Find the tension of the string.

11. A mass of 30 grammes is suspended by a string: what horizontal force is required to displace it until the string makes an angle of 30° with the vertical?

12. A mass of 3 lbs. is suspended by two strings, one horizontal and the other making an angle of 30° with the vertical: find the tension in each string.

13. A picture weighing 7 lbs. hangs in the ordinary way by a cord passing through two hooks and over a smooth nail. The hooks are 18 in. apart and the cord is 4 feet long. What is the tension of the cord?

14. If the cord in the last question is lengthened to 6 feet, how will the tension in it be altered?

15. A, B, and C are three smooth pegs in a vertical wall, A being the highest. AB and AC make angles of 30° and 60° respectively with the vertical on opposite

sides. A string carrying two weights of 12 lbs. each hangs over the pegs with the ends free. Find the pressures on the pegs.

16. A rod 80 cm. long is suspended by a string a metre long, the ends of which are attached to the ends of the rod, while the middle of it is slung over a hook. If the rod weighs 150 gm. what is the tension in the string?

17. A weight of 30 lbs. is suspended by a knot from two strings attached to a horizontal beam. These strings make angles of 45° and 60° respectively with the vertical. Find their tensions.

18. A rod weighing 1 lb. and 5 ft. in length is hung from a nail by two strings tied to its ends. One of the strings is 3 ft. long and the other is 4 ft. Find the tension in each.

19. A cord is stretched horizontally between two posts 6 ft. apart. What is the tension in it, and how much longer does it become when a 7-lb weight suspended from the middle makes it droop 3 inches?

20. A sphere of diameter 1 foot hangs against a smooth vertical wall by a string 6 in. long fastened to its circumference and to the wall: find the tension of the string.

21. A wire AD 15 in. long is bent upwards at right angles 4 in. from A and 6 in. from D. Prove that if it be suspended from A it will rest with the second bend vertically below A.

22. A metre scale (the mass of which is negligible) is divided into decimetres, and at the successive divisions are attached weights of 1, 2, 3 . . . 10 kgm. Where is the centre of mass of the system?

23. A wheel of mass $1\frac{1}{2}$ cwt. is attached to one end of a straight uniform axle 4 ft. long and of mass $\frac{1}{2}$ cwt. Find the centre of mass.

24. A stone pillar consists of two cylinders placed one on top of the other with a common axis. The

lengths of the cylinders are as 1 to 4 and their masses are as 3 to 2. Where is the centre of mass of the pillar?

25. From a square plate whose side is a foot a square of 3 in. side is cut away at one corner. Find the centre of gravity of the remainder.

26. Find the centre of mass of the complete figure formed of a right-angled triangle, and of three squares described upon its sides as bases.

27. An iron shaft consists of three portions. The first is 2 ft. long and weighs 21 lbs. The second (central) part is 4 ft. long. The third is to be of such diameter that it weighs 6 lbs. per ft.-length: how long must it be in order that the whole may balance about the middle of the central part?

28. A circular table weighing 60 lbs. stands on four vertical legs, situated at equidistant points on its rim. What is the maximum weight which can be placed on the rim half way between any two consecutive legs?

29. Find the horizontal force required to support a weight of 60 lbs. on a smooth inclined plane which has a gradient of 5 in 13.

30. The average force that has to be applied at the end of a pair of nut-crackers in order to crack a nut is equal to the weight of 0·5 kgm., the nut-crackers being 14 cm. long and the nut placed 3·5 cm. from the hinge. Find the force (in dynes) which would crack the nut directly.

31. Draw a diagram of a set of pulleys in which 3 lbs. will balance 93 lbs., the weights of the pulleys being negligible. If each pulley weighed 1 lb., what weight would the 3 lbs. support?

32. A step-ladder has the form of a letter A of semi-vertical angle α. It rests on a smooth horizontal plane and its legs are kept from slipping by a cord connecting them together half-way up. How much greater does the tension in the cord become when a weight W is placed on top of the ladder?

33. The jib of a crane is 20 ft. long, and its upper end is secured by a chain at right angles to it to a fixed point vertically above its lower end. A weight of 2 tons hangs from the upper end of the jib, which is 16 ft. higher than the lower. Find the tension in the chain (1) when the weight of the jib is negligible, (2) when it is 5 cwt., the jib being of uniform section.

34. A ladder rests against a smooth vertical wall at an inclination of 45° to the horizon. Show that the friction exerted by the ground is half the weight of the ladder.

35. A light rod is hinged at one end and loaded at the other end with a weight of 6 lbs. The rod is supported in a horizontal position by a string which is attached to the loaded end and which makes an angle of 30° with the rod. Find the tension of the string and the thrust on the hinge.

36. The lower end of a pole rests on the ground and a point 2 ft. from its upper end rests against a smooth rail, the pole being inclined at an angle of 60° to the horizon. If its whole length is 7 ft. and weight 21 lbs., find the direction and magnitude of the action of the ground on the pole.

37. A mass of 20 lbs. rests on rough ground ($\mu = 1/\sqrt{3}$). Prove that the least force which will move it is equal to the weight of 10 lbs. and acts at an angle of 30° to the horizon.

38. The total length of a spade is 4 ft. and it weighs 7 lbs. Its centre of gravity is at the upper edge of the blade, which is rectangular and 1 ft. long. A man raises 10 lbs. of earth, uniformly distributed over the blade, taking hold of the handle with his right hand and placing his left hand at the middle of the spade. What force does he exert with each hand?

39. The resultant of two forces A and B is C. The resultant is doubled when A is doubled; and is also doubled when A is reversed. Prove that

$$A : B : C = \sqrt{2} : \sqrt{3} : \sqrt{2}.$$

40. A curtain hangs from a horizontal rail by rings, the coefficient of friction for rail and ring being $\cdot 577$ $(=1/\sqrt{3})$. Prove that no force will draw the curtain aside unless the curtain edge be inclined at more than 30° to the vertical.

41. The post AB of a crane ABC (Fig. 2) is 2·6 metres high; the jib BC is 6 metres long, and is movable about B. The tie consists partly of a rod CR and partly of a double chain which can be shortened or lengthened at will. It is shortened so as to drop the load (3000 kgm.) at a point D, 4·5 metres from B. Find the tension along the tie and thrust on the jib.

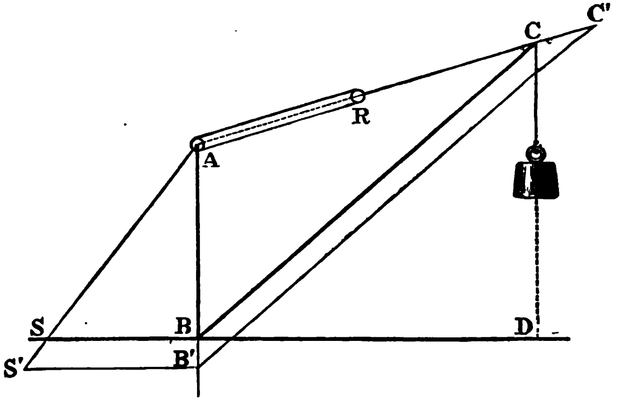

Fig. 2.

Mark off the position of the point D 4·5 cm. from B along the horizontal line. Draw the vertical line DC, and on it find a point C such that BC = 6 cm. The vertical line BA, 2·6 cm. long, represents the post. Join AC: this represents the length and position of the tie under the conditions given.

Produce AB to B′, making AB′ = 3 cm. Through B′ draw

B′C′ parallel to BC, and meeting AC produced at C′. The sides of the triangle AB′C′ represent in direction, and therefore also in magnitude, the forces acting at C on a scale of 1 cm. to 1000 kgm. By measurement we find that AC′ = 5·4 cm., and hence the tension along the tie is equal to a weight of 5400 kgm. (*q.p.*) B′C′ is 7 cm. long, and hence the thrust on the jib is equal to a weight of 7000 kgm.

42. A plank 70 cm. long is inclined so that one end

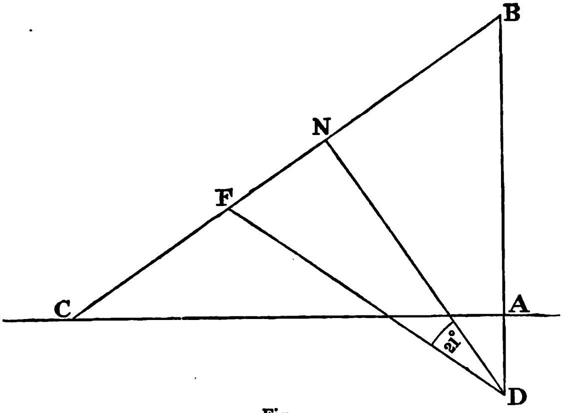

Fig. 3.

is 40 cm. higher than the other, and a block of wood weighing 5 kgm. is placed on it. Find what force applied to the block along the inclined plane thus formed will be just sufficient to move the block up along it, having given that the limiting angle of friction for the block and plank is 21°.

Draw the vertical line AB (Fig. 3) 4 cm. long. Through A draw a horizontal line, and from B draw a line meeting this at C, so that BC = 7 cm.

Produce BA to D, making BD = 5 cm. DB represents the weight of the block on a scale of 1 cm. to 1 kgm. Draw

DN perpendicular to BC. NB represents the resolved component of the weight of the block acting downward along the plane. DN represents the pressure normal to the plane. Through D draw DF, making with DN an angle of 21° and meeting BC at F. FN represents the friction (which also acts downward along the plane). The resultant of these two forces is represented by FB, and by measurement we find that the length of FB is 4·5 cm. Hence the force acting along the inclined plane which will just produce motion is equal to a weight of 4·5 kgm.

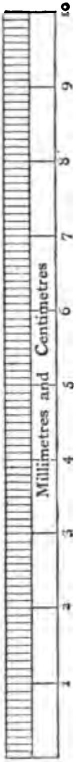

Fig. 4.

[*Note.*—In solving Examples 43-45 make use of the graphical method, which is illustrated in the solutions of the two preceding problems. The degree of correctness of the answers obtained depends upon the care and accuracy with which the scale-diagrams are drawn. But in any case the student should endeavour to acquire some knowledge of graphical methods, inasmuch as they are largely used in practice, and can be applied where ordinary methods of calculation would be of little use.

It should, however, be distinctly understood that the solutions above given are merely intended to illustrate the method, and that the size of the diagrams is limited by the size of the page. For anything like accurate work the scale-drawing should be at least three or four times as large as Fig. 2 or Fig. 3.]

43. Find graphically the resultant of the forces in Ex. 4.

44. Taking the data given in Ex. 42, find what force applied parallel to the plank will just suffice to move the block up along it when the plank is inclined so that one end of it is 30 cm. higher than the other.

45. Find the moment of the force tending to snap the post AB in Ex. 41 off at B. If this tendency is

counteracted by a single stay AS, fixed so that BS = 2 metres, find the strain on the stay produced by the given load.

46. A rod of length r is supported horizontally, and to its extremities are attached the ends of a string of length s. A heavy ring of weight W is slung on the string: prove that the rod experiences a thrust equal to $Wr/2\sqrt{s^2 - r^2}$.

EXAMINATION QUESTIONS

47. Show by the aid of a figure, or of figures, how the degree of stability of a coach or cart depends on the height of the centre of gravity.

The wheels of a coach are 5 feet apart, and the centre of gravity is 10 feet from the line of contact of the wheels and ground on either side. To what height may the wheels on one side run up a bank before the coach is upset? Matric. 1894.

48. A uniform beam, 14 feet long and weighing 120 lbs., is attached to two props, one of which is 3 feet, and the other 5 feet, from its centre: calculate the forces on the props when a weight of 100 lbs. is placed first at one end and then at the other end of the beam.

Matric. 1891.

49. A uniform beam 12 ft. long and weighing 56 lbs. rests on and is fastened to two props 5 ft. apart, one of which is 3 ft. from one end of the beam. A load of 35 lbs. is placed (*a*) on the middle of the beam, (*b*) at the end nearest a prop, (*c*) at the end farthest from a prop: calculate the weight each prop has to bear in each case. Matric. 1891.

50. If G is the centre of mass of a uniform triangular lamina ABC, show that forces represented in magnitude by GA, GB, GC will be in equilibrium. Matric. 1895.

51. ABCD is a quadrilateral, G the centre of the parallelogram formed by joining the middle points of

its sides. Show that the resultant of forces represented by OA, OB, OC, OD, is 4OG, O being any point in the plane. Edinb. M.A. 1893.

52. A and B are two fixed points not in the same horizontal line; the two ends of a thread of given length are tied together, and the thread is hung on to A and B; if a weight is hung by a smooth hook from the thread, so as to draw the thread into the form of a triangle ABC, show how to construct the triangle, and prove that the pressures on the fixed points and the weight are in the same ratio as cos (A/2), cos (B/2), and cos (C/2). S. K. 1895.

53. Prove that three forces in a plane are in equilibrium if the algebraical sum of their moments about each of three points, which lie in the plane but not in the same straight line, vanish.

Six forces acting along the sides of a regular hexagon ABCDEF are in equilibrium. Those acting along AB, BC, CD, are 1 lb., 2 lbs., and 3 lbs. respectively: find the other three forces. Vict. Int. Sc. 1890.

54. A uniform beam, 24 ft. long and weighing 200 lbs., is supported on two props, one 6 ft. from one end, the other 9 feet from the other end of the beam; calculate the pressure on each prop, when a man weighing 180 lbs. stands as near this latter end as he can without upsetting the beam. Matric. 1890.

55. A beam is suspended by two parallel strings; when the strings are tied on at the ends of the beam the tensions are 10 lbs. and 20 lbs., but when the strings are tied on at distances of 1 ft. from the ends the tensions are respectively 9 lbs. and 21 lbs.: find the length of the beam. Vict. Int. Sc. 1890.

56. Prove that the resultant of two forces P and $2P$, inclined at an angle of 60° to one another, makes with the resultant of other two P and P acting in the same directions an angle whose tangent is $1/3\sqrt{3}$.

Glasg. M.A. 1890.

57. State and prove the "Triangle of Forces," and its converse.

O is any point in the plane of a triangle ABC; D, E, F are the middle points of the sides. Prove that the resultant of forces represented in magnitude and direction by OE, OF, DO is represented by OA.

Camb. Schol. 1891.

58. Draw ABC an equilateral triangle with the base AB horizontal and C downwards; let a weight at C be tied by threads AC, BC to fixed points at A and B; if the thread BC is cut, show that the tension of AC is suddenly increased by one-half. S. K. 1895.

59. A heavy rectangular block, resting on a very rough horizontal plane, is to be upset by a force applied by means of a cord of given length, of which one end is attached to the vertical face of the block, and the other is in the plane; prove that the pull will be most effective when the cord is inclined at an angle of 45° to the vertical, the height of the block being supposed great enough to allow this inclination. Int. Sc. 1889.

60. A weight of 20 lbs., suspended by a string from a peg P, is pulled aside by another string knotted to the first at a point K, and pulled horizontally. Find the force necessary to pull it until PK is 60° from the vertical; and find, at the same time, the force on the peg. Matric. 1892.

61. Two equal uniform ladders, hinged together at one extremity, rest on a smooth plane, and are connected by a rope joining their middle points. Draw a diagram to show the forces acting on the system, and show how to find the tension of the rope. Edinb. M.A. 1890.

62. A slab is lying on a plane inclined at 30° to the horizon, and the coefficient of friction is $\frac{1}{2}$. Find the least force which will pull the slab upwards.

Edinb. M.A. 1891.

63. A wire, 40 inches long, is stretched with a force of 50 lbs.: find roughly the force which must be applied

to the middle point to deflect it laterally half an inch out of the line joining the ends. Int. Sc. 1890.

64. Prove that if the arms of a balance are unequal, the true weight of a body is equal to the geometric mean of the apparent weights, when it is weighed, first in one scale-pan, and then in the other. Has inequality in the weights of the pans themselves any influence?

Prove the following graphical method of finding the true weight and the ratio of the arms: Lay off lengths AB, BC in a line, proportional to the apparent weights; describe a semicircle on AC as diameter, and erect the ordinate BD: then BD represents the true weight, and $CD : AD$ is the ratio of the arms. Hence show that when the arms are very nearly equal, we may take the arithmetic instead of the geometric mean of the apparent weights. Int. Sc. 1890.

65. ABCD is a thread suspended from points A and D, and carrying a weight of 10 lbs. at B, and a weight W at C; the inclinations to the vertical of AB and CD are 45° and 30° respectively, and ABC is an angle of 165°: find, by construction or calculation, W and the tension of BC. S.K. 1891.

66. ABC is an equilateral triangle formed of three weightless rods joined together at the ends. It is hung up by the point A, and a weight W is fastened to a point P in BC, such that $4 . BP = BC$: find the position in which the triangle comes to rest, the stresses in the rods, and which rods are in compression and which in extension. S.K. 1891.

67. Explain the meaning of the terms "Limiting Friction," "Angle of Friction."

A ladder, whose weight is 50 lbs., rests at an inclination of 45° against a vertical wall. If the angles of friction at the ground and the wall be respectively $\tan^{-1}\frac{1}{9}$, $\tan^{-1}\frac{1}{4}$, find the least horizontal force which, applied at the lower end, will keep it from slipping.

Glasg. M.A. 1890.

68. From an isosceles right-angled triangle ABC (right angle C), another isosceles triangle ABP is cut out. If the C.G. of the remainder be at P, show that the sides of the triangle ABP must be in the ratio $\sqrt{5} : 4$.

Glasg. M.A. 1891.

69. State the laws of friction.

A ladder 70 ft. long and weighing 4 cwt. is equally inclined to a wall and to the ground. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$, and for the ground $\frac{1}{2}$. How far can a man weighing 12 stone carry a 56-lb. weight up the ladder before it begins to slip?

Oxf. Schol. 1892.

70. A heavy rod rests with one end on a smooth inclined plane making an angle of 30° with the horizontal; a string attached to the other end of the rod passes over a smooth pulley and supports a weight equal to the weight of the rod. Prove that in the position of equilibrium the inclination of the rod to the horizontal is $\tan^{-1} 2/\sqrt{3}$. Camb. Schol. 1891.

71. Explain the terms "true," "sensible," "stable" as applied to the common balance; and find the requisite conditions for sensibility.

A false balance, the weight of whose beam AB may be neglected, has scale pans of unequal weight P and Q hanging from A and B respectively, but the beam is horizontal with no weights in the scale pans. A body is placed in the scale pan hanging from B and balanced by known weights in P, and the corrected weight of the body is found to be W; but the scale pans had been accidently changed, so that the body was really placed in scale pan P; prove that the correction to be applied to W to obtain the true weight of the body is $(Q^2 - P^2)/P$.

Camb. Schol. 1891.

72. An uniform log of timber is carried by five persons, of whom one takes hold of an end of the log and the others take hold of the ends of two cross-bars on which the log rests. Prove that the bars may be

arranged in various ways so that the load shall be equally distributed between the bearers; also that of these ways the one in which the middle point of support is at greatest distance from the nearest of the other two is that in which the three points are at equal intervals of $5/12$ of the length of the log. Int. Sc. 1891.

73. Two equal frictionless circular discs are suspended from a peg by threads of equal length attached to their circumferences, and a third equal disc is supported by these two, the whole system being confined to the same vertical plane. Prove that in the position of equilibrium of the discs the lower ones are not in contact, and the depths of the centres below the level of the peg are in the ratio of 3 to 2. Prove also that the third disc cannot be supported in this manner if the length of a thread exceeds $4 \cdot 29$ times the radius of a disc. Int. Sc. 1891.

CHAPTER III

HYDROSTATICS

In solving the examples in this chapter, the following facts may be assumed :—

The mass of a cubic foot of water is 1000 ounces or $62\frac{1}{2}$ pounds.

The specific gravity of mercury is 13·6.

In examples on fluid pressure, the pressure of the atmosphere may be neglected, unless the contrary is expressly stated.

The normal atmospheric pressure is that due to a column of mercury 76 centimetres in height. When the inch is taken as the unit of length, the normal barometric height may be assumed as 30 inches.

Geometrical Relations.—The ratio of the circumference of a circle to its diameter is 3·1416 (or approximately $\frac{22}{7}$), and is denoted by the Greek letter π.

The circumference of a circle of radius r is $2\pi r$, and its area is πr^2.

The area of the surface of a sphere is $4\pi r^2$.

The volume of a sphere is $\frac{4}{3}\pi r^3$.

1. Define a fluid, and explain what is meant by the pressure of a fluid at any point within it. Prove that the pressure at any point of a fluid at rest is the same in every direction.

2. State and illustrate the law of transmission of

pressures in liquids, and explain how it is applied in the construction of hydraulic presses.

3. Distinguish between pressure and intensity of pressure; and find the dimensions of each of these quantities in terms of the fundamental units of length, time, and mass.

4. Define density and specific gravity, pointing out the essential distinction between them. Explain how it is that the density of a substance has, in the C.G.S. system, the same numerical value as its specific gravity, or density relative to water.

5. A block of mahogany 2 in. long, $1\frac{1}{2}$ in. broad, and $\frac{7}{8}$ in. thick is found to weigh[1] 461 grains: express its density in grains per cubic in.

The volume of the block is $2 \times \frac{3}{2} \times \frac{7}{8} = \frac{21}{8}$ cubic inches, and its mass is 461 grains. The density, in grains per cubic inch, is

$$\Delta = \frac{\text{mass}}{\text{volume}} = 461 \bigg/ \frac{21}{8} = 175 \cdot 6.$$

6. A ton of clay is found to occupy a volume of 18 cubic ft.: what is its density in pounds per cubic ft.?

7. The density of water is $62\frac{1}{2}$ pounds per cubic ft.: express this in ounces per cubic in.

8. Basalt is three times as heavy as water: what is its density in ounces per cubic in.?

9. A statuette of marble (density $= 2 \cdot 7$) weighs $8 \cdot 1$ kgm. What is its volume in cubic centimetres?

10. A ton of chalk occupies a volume of $15\frac{1}{2}$ cubic ft.: what is its *specific gravity* referred to water as the standard substance?

The mass of a cubic ft. of the chalk is $(2240 \div \frac{31}{2})$ pounds, and the mass of a cubic ft. of water is $62\frac{1}{2}$ pounds. Hence the

[1] In this and the succeeding chapters the term weight is occasionally used instead of mass, when there is no danger of confusion between the two. When a body is said to be weighed in air, the weight of the displaced air may be neglected, unless the contrary is expressly stated.

required specific gravity (or ratio between the masses of equal volumes) is

$$\left(2240 \times \frac{2}{31}\right) \div \frac{125}{2} = \frac{2240 \times 2 \times 2}{31 \times 125} = 2\cdot312.$$

11. A cubic in. of a substance weighs half a pound: what is its specific gravity?

12. The specific gravity of lead is 11·4: what is the weight of a cube of lead 3 in. in the side?

13. A mahogany block of the same dimensions as that in Example 5 is found to weigh 30·35 grammes: express its density in grammes per cubic centimetre.

14. The specific gravity of iron is 7·7: what is the weight of an iron rod $2\frac{1}{2}$ in. broad, 2 in. thick, and 18 ft. long?

15. A body weighs 1000 pounds, and its density is five times that of water: what is its volume?

16. Prove that the number obtained on multiplying the density of a substance by 0·75 expresses approximately the mass of a cubic yard of the substance in tons.

17. How many grammes of mercury will be required to fill a cylindrical glass tube, the length of which is 70 centimetres, and internal diameter 0·8 centimetre?

The cross-section of the tube is $\pi r^2 = \frac{22}{7} \times (0\cdot4)^2 = \frac{22}{7} \times 0\cdot16$ square centimetre, and its internal volume is $70 \times \frac{22}{7} \times 0\cdot16 = 35\cdot2$ cubic centimetres. The specific gravity of mercury is 13·6 approximately, and this number also represents its density in the C.G.S. system: *i.e.* 1 cubic centimetre of mercury = 13·6 grammes.

Thus $35\cdot2 \times 13\cdot6 = 478\cdot72$ grammes are required to fill the tube.

18. A tube 120 centimetres long holds 600 grammes of mercury: find its cross-section and internal diameter.

19. Find the mass of a piece of copper wire 5 metres in length and 1·8 millimetre in diameter, given that the density of copper is 8·8.

20. A cylindrical tube 1 metre in length and 1 centi-

metre in internal diameter weighs 100 grammes when empty, and 210 grammes when filled with a liquid: find the specific gravity of the liquid.

21. Find the diameter of a cylindrical kilogramme weight made of brass (density 8·4), its height being 7·5 centimetres.

22. The radii of two spheres are 2 centimetres and 3 centimetres, and their masses are 200 grammes and 250 grammes respectively: compare their densities.

23. A litre of hydrogen gas weighs 0·0896 gramme, and the density of carbon-dioxide is twenty-two times that of hydrogen: how much carbon-dioxide is required to fill a gas-bag which holds 10 litres?

24. The mean radius of the earth is $6 \cdot 37 \times 10^8$ centimetres and its mean density is 5·6. Express its mass in grammes.

25. One litre of a liquid of specific gravity 1·4 is mixed with 2 litres of a liquid of specific gravity 0·96, and the mixture occupies nine-tenths of the volume of its components: what is its specific gravity?

The volume of the mixture is

$$V = \frac{9}{10} \times 3000 = 2700 \text{ c.c.}$$

If Δ denote its density, then its mass in grammes is

$$V\Delta = (1000 \times 1 \cdot 4) + (2000 \times 0 \cdot 96) = 1400 + 1920 = 3320.$$

Thus $$\Delta = \frac{3320}{2700} = 1 \cdot 23.$$

26. If the specific gravities of two liquids be 4 and 5 respectively, find the specific gravity of a mixture containing 3 parts by weight of the former to 4 parts of the latter.

27. What would have been the specific gravity of the mixture in the last example if the proportions had been taken by volume instead of by weight?

28. Equal volumes of two liquids whose specific

gravities are s and $2s$ are mixed together, and the mixture occupies four-fifths of the sum of the volumes of its components: what is its specific gravity?

29. The density of fire-damp is one-half that of air: what is the density of foul air containing 15 per cent of fire-damp?

30. Equal weights of two liquids whose specific gravities are 0·9 and 0·7 are mixed together, and a contraction of 10 per cent occurs in the volume: what is the specific gravity of the mixture?

31. The densities of three liquids are as 1 : 2 : 3. Equal volumes of the three are mixed together: the mixture is separated into three equal parts, and to each part is then added an equal volume of one of the original liquids. Compare the densities of the mixtures thus prepared.

32. Equal volumes of three fluids are mixed. The specific gravity of the first is 1·55, that of the second 1·75, and that of the mixture is 1·6: find the specific gravity of the third.

Fluid Pressure.—At a depth z below the surface of a heavy liquid of density ρ the pressure is

$$p = g\rho z,$$

neglecting atmospheric pressure.

When g, ρ, and z are all expressed in the C.G.S. system, p denotes the pressure in dynes per square centimetre; if the factor g be omitted, p will represent the pressure in grammes weight per square centimetre. What we have here called pressure is really intensity of pressure, or pressure per unit area; and the pressure on a surface of area a is $P = pa$, if the intensity of the pressure over the surface is uniform.

Taking into account the pressure due to the atmosphere, the pressure at a depth z is

$$g\rho z + \Pi,$$

where Π denotes the atmospheric pressure on unit area of the surface of the liquid.

33. Find the pressure due to a column of mercury 1 metre high.

Here $z=100$, $\rho=13 \cdot 6$, and taking $g=981$, we have

$$p=981 \times 100 \times 13 \cdot 6$$
$$=1 \cdot 334 \times 10^6 \text{ dynes per sq. cm.}$$
$$=1 \cdot 334 \text{ megadynes per sq. cm.}$$

Expressed in grammes weight per square centimetre the pressure would be $100 \times 13 \cdot 6=1360$.

34. What is the pressure at a depth of 50 feet below the surface of the sea, the specific gravity of sea-water being 1·025?

The pressure in pounds per square foot is $p=\rho z$, where

ρ = density of sea-water in lbs. per cub. ft.,
and z = depth below the surface in feet.

Now 1 cubic foot of water weighs $62\frac{1}{2}$ pounds, and therefore 1 cubic foot of sea-water weighs $62 \cdot 5 \times 1 \cdot 025=64 \cdot 06$ pounds; thus the pressure is $50 \times 64 \cdot 06=3203$ pounds per square foot.

35. Find the pressure due to a column of water 1 metre in depth. Express your result (1) in grammes weight per square centimetre, and (2) in dynes per square centimetre.

36. Calculate the total pressure (in grammes weight) upon the base of a cylindrical vessel one decimetre in diameter, filled with mercury to a height of 40 centimetres.

37. What must be the height of a column of mercury to exert a pressure of 1 kilogramme per square centimetre?

38. The specific gravity of sea-water is 1·025; calculate the pressure in grammes weight per square centimetre at a depth of 40 metres below the surface of the sea.

39. Mercury is poured into a vessel until the layer is 10 centimetres deep, and then water is added until the depth of the water-column is 75 centimetres: determine the pressure on the base in dynes per square centimetre.

40. What must be the height of a column of water in order that the pressure at its base may be a megadyne (10^6 dynes) per square centimetre?

41. Find the equivalent in dynes per square centimetre of a pressure of 1000 kilogrammes weight per square metre.

42. Determine the pressure in grammes weight exerted upon a horizontal area of 2 square decimetres sunk to a depth of 75 centimetres below the surface of oil of specific gravity 0·85.

43. If the inch be the unit of length, and the second the unit of time, what must be the density of the standard substance in order that the equation $p = g\rho z$ may give the pressure in pounds weight?

44. Express in pounds weight per square foot the pressure at the bottom of a lake 300 feet deep.

45. Determine the available water-pressure (in pounds per square inch) in a laboratory which is supplied from a tank at a height of 40 feet.

46. To what depth must a surface be sunk in water in order that the pressure upon it may be 60 pounds per square inch?

47. If a cubic foot of sea-water weighs 64 pounds, what is the pressure at a depth of a mile under the surface of the sea?

48. If the atmospheric pressure be 15 pounds per square inch, what is the pressure at the bottom of a pond 30 feet deep?

49. At what depth below the surface of water will the pressure be equal to two atmospheres, if the atmospheric pressure is 1 megadyne per square centimetre?

50. Show that if two liquids which do not mix meet

in communicating tubes, their heights above the common surface of separation will be inversely as their densities.

If the heights of the two liquids above the surface of separation are 15 and 18 inches, and the density of the first is 1·08, what is the density of the second?

51. I wish to discover what the water pressure at a particular tap is, and, in order to do so, I connect it with a mercury manometer; on opening the tap the mercury is forced to a height of 110 centimetres: express the "head of water" in feet.

52. Calculate the available water pressure in pounds per square inch in Example 51.

53. An uniform U-tube is about half filled with water: how much oil of specific gravity 0·8 must be poured into one limb in order to make the water rise 4 inches in the other?

54. The limbs of a V-tube are inclined at an angle of 60°. A length l of the tube is filled with water; it is then held upright with its limbs equally inclined to the vertical, and as much oil of specific gravity s is poured into one side as fills a length l' of it: find the difference of level of the liquids in the two sides.

[In Examples 55-62 the result is to be expressed in grammes weight, neglecting atmospheric pressure.]

55. What is the pressure on an area of 10 square centimetres immersed in water, the centre of gravity of the area being at a depth of 15 centimetres?

56. Calculate the pressure on a circular disc 16 centimetres in diameter immersed in mercury, the centre of the circle being at a depth of 25 centimetres below the surface.

57. A rectangular plate 30 centimetres long and 10 centimetres broad is immersed horizontally at a depth of 3 metres in brine of density 1·1: what is the pressure upon its surface?

58. What would be the pressure upon the plate in the preceding example, if it were held in a vertical plane

with its lower and higher corners 60 and 50 centimetres respectively below the surface?

59. A cube, the edge of which is one decimetre, is suspended in water with its sides vertical and its upper surface at a depth of 1 metre below the surface: find the pressure on each of its faces.

60. Prove that if a cubical box be filled with water, the total pressure to which it is subjected is equal to three times the weight of the water which it contains.

61. Compare the pressures on the top, bottom, and sides of a rectangular box 2 ft. by 2 ft. by 4 ft., with its long sides vertical, the base being at a depth of 10 ft. below the surface of a trough of liquid.

62. A hole, 1 ft. square, is made in a lock-gate at a depth of 15 ft. below the surface of the water: what force must be exerted to keep back the water by holding a piece of wood against the hole?

63. Prove that a body immersed in a liquid sustains an upward pressure which is equal to the weight of the liquid displaced: and describe any practical applications of this fact.

64. Describe how you would demonstrate experimentally the truth of the principle of Archimedes; and explain what is meant by the *apparent weight* of a body in water.

A body weighs 62 grammes in vacuo and 42 grammes in water: find its volume and specific gravity.

65. What will be the apparent weight in water of a piece of rock-crystal (density 2·7) which weighs 35 grammes in vacuo?

66. A bar of aluminium (density 2·6) weighs 54·8 grammes in vacuo: what will be the loss of weight when it is weighed in water?

67. An irregular solid is found to weigh 98 grammes in vacuo and 64 grammes in water: what is its volume?

68. A glass stopper is suspended from one arm of a balance, and is tared by pouring sand in the pan sus-

pended from the other arm. It is then immersed successively in chloroform and in water: in the first case 12·78 gm. and in the second case 8·31 gm. is required to restore equilibrium. What is the specific gravity of the chloroform?

69. A hollow stopper made of glass of density 2·6 is found to weigh 23·4 gm. in air[1] and only 3·9 gm. in water. What is the volume of the internal cavity?

70. A solid cube, 4 inches in the side, is formed of a substance of specific gravity 12·5: what will its apparent weight in water be?

71. A body which weighs 24 grammes in air is found to weigh 20 grammes in water: what will be its apparent weight in alcohol of specific gravity 0·8?

72. A body which weighs 35 grammes in air is found to weigh 30 grammes in one fluid and 25 grammes in another: what will be its weight when immersed in a mixture containing equal volumes of the two fluids?

73. Two bodies are in equilibrium when suspended in water from the arms of a balance: the mass of the one body is 28 and its density is 5·6; if the mass of the other is 36, what is its density?

The volume of the first body is

$$v_1 = \frac{28}{5 \cdot 6} = 5,$$

and if d_2 be the density of the second body, its volume is

$$v_2 = \frac{36}{d_2}.$$

When immersed in water, the apparent weight of the first body is

$$28 - 5 = 23,$$

and of the second

$$36 - \frac{36}{d_2}.$$

[1] See note on p. 76.

Since these are equal,

$$23 = 36 - \frac{36}{d_2},$$

$$\text{and} \therefore d_2 = \frac{36}{13} = 2{\cdot}77.$$

74. Two masses m_1 and m_2 balance each other when weighed in water; the specific gravity of the one being s_1, what is that of the other?

75. A piece of silver (specific gravity = 10·5) weighing 20 grammes and a piece of tin (specific gravity = 7·3) are fastened to the two ends of a string passing over a pulley, and hang in equilibrium when entirely immersed in water: determine the weight of the tin.

76. What should be the weight of the piece of tin in the preceding example, in order that both bodies might hang in equilibrium when immersed in a liquid of specific gravity 1·5?

77. A string is passed over a pulley so that one end hangs in a beaker of water and the other in a beaker containing salt solution of specific gravity 1·281. A lump of copper (specific gravity 8·9) is hung from one end of the string so as to be immersed in the water: what weight of lead of specific gravity 11·4 must be attached to the other end of the string so as to keep the copper in equilibrium when the lead is completely immersed in the salt solution?

78. A body of density δ is dropped gently on the surface of a layer of liquid of depth d and density δ' (δ' being less than δ). Show that it will reach the bottom of the liquid layer after a time $\sqrt{2d\delta/g(\delta - \delta')}$.

79. Find the acceleration with which a stone of specific gravity 2·8 would sink in water, and the time which it would take to get to the bottom of a pool 16 feet deep.

80. A 7-lb. iron weight (specific gravity 7·3) is suspended from one end of an equal-armed lever and

immersed in water: what weight must be hung from the other end of the lever in order to counterpoise it?

81. A cube of gutta-percha (specific gravity 0·96), two centimetres in the side, is suspended from the short pan of a hydrostatic balance, and a basin of water is placed beneath: what weight must be placed upon the pan in order that there may be equilibrium when the gutta-percha is completely immersed?

82. State the conditions which must be fulfilled in order that a body may float in equilibrium in a liquid; and show that if a homogeneous body of volume v and specific gravity s floats in a liquid of specific gravity s', the volume of the part immersed will be $\frac{v.s}{s'}$.

83. What volume of water will be displaced by a kilogramme of wood of specific gravity 0·75 floating in equilibrium?

84. A piece of cork of mass 300 grammes and density 0·25 is placed in a vessel full of water: how much water will overflow?

85. The specific gravity of ice is 0·918 and that of sea-water is 1·03: what is the total volume of an iceberg which floats with 700 cubic yards exposed?

86. A sphere of density 0·95 and volume 100 cubic centimetres floats on water. Oil of density 0·9 is poured upon the water, the layer of oil being deep enough to cover the sphere: how much of it will now be immersed in the water?

87. A solid body floats in water with just half its volume immersed. When it floats in a mixture of equal volumes of water and another liquid, one-third of it is immersed: find the specific gravities of the solid and liquid.

88. A sphere of glass (density 2·8) is dropped into a vessel containing mercury and water: find its position of equilibrium.

89. A cube of wood of specific gravity 0·8 and 2 centimetres in the side is gently placed in a beaker filled with water so as to float on it. How much water will run over?

90. An iceberg of density 0·917 floats in sea-water of density 1·025. Find the ratio of the volume exposed to the volume submerged.

91. An uniform rod XYZ can turn about a pivot at the end X, and it rests in equilibrium with the portion YZ immersed in water: prove that the specific gravity of the rod is $1-(XY/XZ)^2$.

92. A mass m of iron is placed on top of a cubical block of wood floating in water, and the block is found to sink until the top of it is just level with that of the water. The iron is now removed and another piece of iron of mass m' is attached to the bottom of the cube of wood: it is again found that the upper surface of the cube is just level with the water. Taking the specific gravity of iron as 7·7, find the ratio between m and m'.

93. A solid weighs 100 grammes in air and 64 grammes in a liquid of specific gravity 1·2: what is its specific gravity?

94. A body weighs 124 grammes in vacuo, 108 grammes in water, and 98 grammes in another liquid: what is the specific gravity of the liquid?

95. A solid which weighs 120 grammes in air is found to weigh 90 grammes in water and 78 grammes in a strong solution of zinc sulphate: what is the specific gravity of the solution?

96. A specific gravity bottle weighs 14·72 grammes when empty, 39·74 grammes when filled with water, and 44·85 grammes when filled with a solution of common salt: what is the specific gravity of the solution?

97. A specific gravity bottle when filled with water weighs 61·485 grammes, and when filled with methylated

spirit 53·462 grammes. If the bottle weighs 15·063 grammes what is the specific gravity of the spirit?

98. In a determination of the specific gravity of a saturated solution of zinc sulphate with the same bottle, the weight of the bottle filled with water was 61·460 grammes, and, when filled with a solution at 14°·6 C., 81·559 grammes: find the specific gravity of the solution at this temperature.

99. Find the specific gravity of a mixture of methylated spirit and water from the following data:—

Weight of empty bottle	= 15·056	gm.
" bottle + water	= 61·500	"
" bottle + mixture	= 57·450	"

100. A specific gravity bottle was found to weigh 39·74 grammes when filled with water. Some iron nails weighing 8·5 grammes were introduced, and the bottle was again filled up with water. The weight now was 47·12 grammes: show that the specific gravity of the iron nails was 7·59.

101. After removing the nails from the bottle, 40·37 grammes of lead shot was introduced, and after filling up with water, the weight of the whole was 76·54 grammes: what was the specific gravity of the shot?

102. The apparent weight of a piece of platinum when weighed in water is 20·6 gm. When immersed in mercury its apparent weight is only 8 gm. Calculate the density of the platinum, assuming that of the mercury to be 13·6: and find also the volume of the platinum.

103. Find the specific gravity of monoclinic sulphur from the following data:—

Weight of sulphur taken . . .	0·5260	gm.
" sp. gr. bottle + water (full)	49·9598	"
" bottle + sulphur + water .	50·2158	"

104. A solid, lighter than water, and weighing 25

grammes in air, is fastened to a piece of metal, and the combination is found to weigh 36 grammes in water: if the piece of metal weighs 45 grammes in water, what is the specific gravity of the light solid?

105. Find the specific gravity of a given solid from the following data:—

Weight of solid in air	0·5	gm.
" sinker in air	4·0	"
" solid and sinker in water .	3·375	"
Specific gravity of sinker	8·0	"

106. A sinker weighing 38 grammes is fastened to a piece of cork weighing 10 grammes, and the two together just sink when placed in water: find the specific gravity of the sinker, taking that of cork as 0·25.

107. A Nicholson's hydrometer, when floating in water, required a weight of 0·15 gramme to be placed upon the upper dish in order to make it sink to a fixed mark on the stem; and 5·72 grammes had to be placed upon the dish in order to make it sink to the same mark in a solution of salt. If the hydrometer weighed 94·47 grammes, what was the specific gravity of the solution?

108. 60·3 gm. has to be placed in the pan of a hydrometer to sink it to the mark in water, and 6·8 gm. only in alcohol. If the hydrometer weighs 200 gm., what is the specific gravity of the alcohol?

109. In order to determine the specific gravity of a piece of fluor-spar, it was placed upon the upper pan of a Nicholson's hydrometer, and weights were added until the instrument sank to a fixed mark in the stem. On removing the fluor-spar an additional weight of 9·85 grammes had to be added. This additional weight was next taken off and the spar placed in the lower pan (immersed in water), when it was found that 3·06 grammes had to be placed in the upper pan to sink the hydrometer to the same mark: what was the specific gravity of the spar?

110. When a small solid is placed in the upper cup 6·32 grammes, and when in the lower cup 8·53 grammes are required to sink a Nicholson's hydrometer to the mark, the liquid being distilled water. If another liquid is used, the weights required are respectively 6·00 and 7·69 grammes. Find the specific gravity of the liquid.

111. The density of the brass weights used with a Nicholson's hydrometer is 8·4. What weights must be placed in the lower pan to produce the same effect as 18·5 gm. placed in the upper pan?

112. Walker's specific gravity balance consists of a lever with unequal arms; a fixed weight is hung from the shorter arm, and the longer arm is graduated in inches and tenths of an inch. The lever rests on a knife-edge at X, and is horizontal when a body is suspended in air from a point Y on the long arm; when it is immersed in water the point of suspension has to be moved to Z in order to obtain equilibrium: show that the specific gravity of the body is

$$XZ/(XZ-XY).$$

113. The specific gravity of a piece of solder was determined by Walker's balance. When weighed in air the length of the arm (or distance of the point of suspension from the knife-edge) was 11·05 inches; when weighed in water the length of the arm was 12·55 inches: show that the specific gravity of the solder was 8·37.

114. In order to find the specific gravity of a mineral two experiments were made, the position of the weight on the short arm being altered after the first determination.

		Length of Arm.
Exp. I.	Substance weighed in air . .	5·78 in.
	,, ,, water .	9·07 ,,
Exp. II.	,, ,, air . .	9·17 ,,
	,, ,, water .	14·42 ,,

Show that the mean result of the two experiments is 2·751.

115. The lever of a Walker's balance is in equilibrium when a body is suspended from a point Y on the long arm, the fulcrum being at X. When the body is immersed in water the point of suspension is at Z, and when it is immersed in another liquid the point of suspension is at Z′. Prove that the specific gravity of the liquid is

$$XZ(XZ' - XY)/XZ'(XZ - XY).$$

116. A pebble was suspended from the long arm of the balance, and weighed successively in air, in water, and in methylated spirit, the lengths of the arms being 5·78 inches, 9·1 inches, and 8·27 inches respectively: find the specific gravity of the pebble and of the spirit.

117. The area of the smaller piston of a Bramah press is $2\frac{1}{2}$ square inches, and the area of the larger piston is 200 square inches. A force equal to a weight of 3 pounds is applied to the smaller piston: what pressure will be communicated to the larger one?

118. The lever of a hydraulic press gives a mechanical advantage of 6; the sectional area of the smaller plunger is half a square inch, and that of the larger plunger 15 square inches. A 56-lb. weight is hung from the handle: what weight will the large plunger sustain?

119. If the diameters of the pistons of a Bramah press are in the ratio of 10 to 1, and a power of 14 pounds applied to the smaller piston produces a pressure of 1 ton on the larger, what is the ratio of the arms of the lever used for working the piston?

120. The two pistons of a hydraulic press have diameters of a foot and an inch respectively: what is the pressure exerted by the larger piston when a weight of 56 pounds is placed on the smaller one? If the stroke of the smaller piston is $1\frac{1}{2}$ inch, through what distance will the larger piston have moved after ten strokes?

121. The small piston of a hydraulic press is $\frac{3}{4}$ inch

in diameter, and is worked by a lever: the distance from the piston to the fulcrum is $3\frac{1}{2}$ inches, and from the fulcrum to the power is 35 inches. If the large piston is 9 inches in diameter, what is the mechanical advantage?

122. Give a clear explanation of the manner in which the column of mercury in a barometer is supported. Why is mercury the liquid generally employed? How is the height of the mercurial column affected (1) by changes of temperature; (2) by the narrowness of the tubes; (3) by differences in the value of g?

123. What is meant by the "capacity error" of a cistern barometer? Give a brief description of Fortin's barometer, explaining the adjustment by which the mercury in the cistern is kept at a constant level.

124. How will the reading of a barometer be affected if its tube is not in a vertical position? Calculate, as an example, the length (or apparent height) of the mercurial column in a barometer, the tube of which is inclined at 30° to the vertical, the actual barometric height being 30 inches.

125. What is the height of a water barometer when a mercurial barometer reads 30 inches?

126. During a storm a mercurial barometer falls from 30 to 29 inches: through what distance would a water barometer fall under the same conditions?

127. Find the height of a glycerine barometer when the water barometer stands at 32 feet. (Density of glycerine = 1·27.)

128. When the barometric height is 76 centimetres, a glycerine barometer is found to stand at 810 centimetres. Calculate from this the specific gravity of glycerine referred (1) to mercury; (2) to water, as the standard substance.

129. What would be the height of a barometer con-

taining oil of specific gravity 0·86, when the height of a mercurial barometer is 75 centimetres?

130. Given that the atmospheric pressure is 15 pounds per square inch, calculate its value in kilogrammes per square metre.

Atmospheric pressure per unit area.—If the barometric height h be given, the numerical value of the atmospheric pressure in units of force per unit area may be found as follows:—

Let Π denote the pressure in dynes per square centimetre, and ρ the density of mercury $= 13 \cdot 596$, then

$$\Pi = h\rho g,$$

or, taking the normal barometric height (76 centimetres),

$$\Pi = 76 \times 13 \cdot 596 \times 981 = 1{,}013{,}663 \ (q.p.).$$

Thus the normal atmospheric pressure is somewhat greater than a megadyne (one million dynes) per square centimetre. Expressed in grammes weight per square centimetre the pressure is

$$\Pi = h\rho = 76 \times 13 \cdot 596 = 1033.$$

This corresponds to a weight of 10,330 kilogrammes per square metre.

Suppose we require to express the atmospheric pressure in pounds weight per square inch, when the barometer stands at 30 inches. Consider a barometer-tube the cross-section of which is 1 square inch; the pressure at the base of the column of mercury is equal to the weight of the column itself. Now we know that 1 cubic foot (or 1728 cubic inches) of water weighs 1000 ounces or 62·5 pounds. Thus 1 cubic inch of water weighs $\frac{62 \cdot 5}{1728}$ pounds, and 1 cubic inch of mercury weighs $\frac{62 \cdot 5}{1728} \times 13 \cdot 596$ pounds. The weight of a column 30 inches high and 1 square inch in cross-section will therefore be

$$\frac{62 \cdot 5}{1728} \times 13 \cdot 596 \times 30 = 14 \cdot 75 \text{ lbs.}$$

Hence the pressure is equal to a weight of 14·75 pounds on the square inch.

131. Prove that a pressure of 1 megadyne (10^6 dynes) per square centimetre corresponds almost exactly to a barometric height of 75 centimetres.

132. Calculate, in dynes per square centimetre, the atmospheric pressure when the barometer stands at 78 centimetres.

133. What is the atmospheric pressure, in pounds per square inch, when the barometric height is 28·5 inches?

134. Determine the value of the atmospheric pressure in ounces per square foot, when the height of the water barometer is 30 feet.

135. Express in grammes weight per square centimetre, and also in kilogrammes weight per square metre, the pressure due to the atmosphere when the barometric height is 74 centimetres.

136. The mercury in a cistern barometer stands at 75·8 centimetres; calculate, in absolute measure, the pressure on the free surface of the cistern, the area of which is 40 square centimetres.

137. What change in the atmospheric pressure, on a square inch, is indicated by a fall of 1 inch in the height of the barometric column?

138. State Boyle's law, and explain the nature of the limitations to which it is subject. Give an account of Regnault's investigations upon the accuracy of the law.

139. The volume of a quantity of gas is measured when the barometer stands at 72 centimetres, and is found to be 646 cubic centimetres; what would its volume be at the normal pressure?

The normal pressure is that due to a column of mercury 76 centimetres high; and if p and v denote the original pressure and volume, and p' and v' the final pressure and volume, the relation

$$pv = p'v'$$

expresses Boyle's law.

Thus the volume under the normal pressure would be

$$v' = v \times p/p' = 646 \times 72/76 = 612 \text{ c.c.}$$

140. At what pressure would the gas in the preceding question have a volume of 580 cubic centimetres?

The required pressure is given by the equation

$$p' = p \times v/v';$$

and is therefore equal to the pressure due to a column of mercury $72 \times 646/580 = 80 \cdot 19$ centimetres high.

141. A piston is situated in the middle of a closed cylinder, 10 inches long, and there are equal quantities of air on each side of it. The piston is pushed until it is within an inch of one of the ends: compare the pressures on each side.

142. The same quantities of air are contained in two hollow spheres of radii r and r' respectively: compare the pressures within the two spheres.

143. A quantity of hydrogen gas was found to measure 146·8 cubic centimetres when the barometer stood at 78 centimetres. On the next day the volume had increased to 159·2 cubic centimetres: what was now the barometric height?

144. A cylindrical receiver, fitted with a piston, contains air at a pressure of 15 pounds per square inch when the piston is 2 feet from the bottom of the receiver. What will be the pressure of the contained air when the piston is forced down until it is 1 inch above the base: and how far should the piston be pushed in order to increase the pressure to 100 pounds per square inch

145. Compare the weights of equal volumes of air at pressures of 62·5 centimetres and 81·6 centimetres respectively.

146. A litre of air weighs 1·293 gramme at the normal pressure: find the weight of the air contained in a litre flask when the barometer stands at 82 centimetres.

147. Two closed vessels, A and B, are connected by a stop-cock: A is vacuous, while B contains air at a pressure of six atmospheres. If A is twice as large as B, find the common pressure in both the vessels after the stop-cock is opened.

148. In the preceding question, if A originally contained air at a pressure of 120 centimetres of mercury, and B hydrogen gas at a pressure of 50 centimetres, what would be the pressure after opening the stop-cock?

149. The volume of an air-bubble increases ten-fold in rising from the bottom of a lake to its surface. If the height of the barometer is 30 in., what is the depth of the lake? (Density of mercury = 13·6.)

150. Find the pressure exerted by a gramme of hydrogen placed in a vessel of 5·55 litres capacity at 0° C., assuming that the mass of a cubic centimetre of hydrogen the same temperature and at atmospheric pressure is [illegible] 10^{-5} grammes.

151. The mouth of a vertical cylinder, 18 inches high, [illegible] piston whose weight is 6 pounds and area [illegible] uare inches. If the pist[illegible]llowed to fall, how far it descend, supposing the atmospheric pressure to 14 pounds per square inch when the pist[illegible] is inserted?

152. The same quantities of air are contained in two hollow spheres whose radii are r and r' respectively: compare the whole pressures on the spherical surfaces.

153. A tube, 6 feet in length, closed at one end, is half filled with mercury, and is then inverted with its open end just dipping into a mercury trough. If the barometer stands at 30 inches, what will be the height of the mercury inside the tube?

The air in the tube expands and depresses the mercury until the pressure of the column of mercury and the pressure of the contained air (diminished by expansion) are together equal to the atmospheric pressure (30 inches). Suppose the mercury to fall until it stands x inches higher inside the tube than in the trough; then the pressure on the air inside is $(30-x)$ inches. The volume of the air is proportional to the length of the tube which it occupies; this was 3 feet = 36 inches when the pressure was 30 inches, but under the diminished pressure of $(30-x)$ inches the length occupied is $(72-x)$ inches. Hence, by Boyle's law,

$$30 \times 36 = (30-x)(72-x).$$

The roots of this quadratic equation are 90 and 12; and on inspection we find that the latter root alone satisfies the conditions of the problem: the root 90 is the solution of a different problem, the algebraical statement of which would lead to the same equation.

154. A glass tube, 1 metre in length, and open at both ends, is plunged into a deep cistern of mercury until a length of 90 centimetres is submerged. The top is now closed by the finger, and the tube is raised until a length of 90 centimetres stands out of the mercury. If the barometric height at the time is 75 centimetres, at what height will the mercury stand inside the tube?

155. To what height should the tube in the preceding example have been raised in order that the air might occupy a length of exactly 30 centimetres?

156. A cylindrical tube 50 cm. long, and open at both ends, is lowered vertically into a mercury-trough so that half its length is immersed, the barometric height at the time being 76 cm. The upper end is now closed by the finger and the tube is slowly raised. Find the length of the column of air when the bottom of the tube is just level with the surface of the mercury in the trough.

157. The tube of a barometer has a cross-section of 1 square centimetre, and when the mercurial column stands at 77 centimetres the length of the vacuous space above it is 8 centimetres: how far will the

mercurial column be depressed if 1 cubic centimetre of air is passed up into the tube?

Suppose the mercury to be depressed through x centimetres; then x measures the pressure to which the enclosed air is exposed. Since the section of the tube is 1 square centimetre, the volume of the air under the pressure x centimetres is $(8+x)$ cubic centimetres. Applying Boyle's law,

$$77 \times 1 = x\,(8+x),$$

or

$$x^2 + 8x - 77 = 0,$$

the positive solution of which is $x = 5 \cdot 65$. Thus the mercury is depressed through 5·65 centimetres, and the barometric height, as indicated by this faulty barometer, is now $77 - 5 \cdot 65 = 71 \cdot 35$ centimetres.

158. What is the true barometric height when the faulty barometer in the preceding example indicates 65 centimetres?

159. When a barometer stands at 75 centimetres, the volume of the empty space above the mercury is 8 cubic centimetres, and its length is 13 centimetres: how far would the column fall if you introduced 5 cubic centimetres of air into the tube?

160. A quantity of air is collected in a barometer tube, the mercury standing 10 inches higher inside the tube than outside, and the correct barometric height being 30 inches. If the volume of the air under these conditions is 1 cubic inch, what would be its volume at the atmospheric pressure?

161. The reading of the pressure of the atmosphere by a cylindrical barometer of 1 sq. cm. section is 75 cm. of mercury on a certain day. The tube is pushed down into the trough until the mercury just fills it, and then some bubbles of air are passed up it. If the level of the mercury then falls to 50 cm., find the volume of the air inserted in it at atmospheric pressure.

162. A barometer which has not been completely freed from air reads 76 centimetres when the correct

barometric height is 77 centimetres, the free internal height of the top of the barometer tube above the surface of the mercury in the cistern being 85 centimetres: show that when this defective barometer reads 75 centimetres a true barometer would stand at 75·9 centimetres.

163. The tube of a barometer has a cross-section of 1 square centimetre, and when the mercury stands at 75 centimetres the volume of the vacuum above it is 12 cubic centimetres: what would be the volume at the same pressure (75 centimetres) of the air which, when introduced into this barometer, would depress the mercury (1) to 30 centimetres, and (2) to 0 centimetre?

164. A cylindrical diving-bell, 7 feet in height, is lowered until the top of the bell is 20 feet below the surface of the water. If the height of the mercury barometer at the time is 30 inches, how high will the water rise inside the bell?

Let x be the height of the space occupied by the air inside the bell; the height of the column of water exerting pressure upon this is $(20+x)$ feet. The pressure of the atmosphere is equal to that of a column of mercury 30 inches high, or of a column of water whose height is

$$30 \times 13{\cdot}6 = 408 \text{ in.} = 34 \text{ ft.}$$

(This may be expressed by saying that the height of the water barometer is 34 feet.)

The total pressure is equal to that of a column of water whose height is $34+20+x=(54+x)$ feet. Since the section of the bell is uniform, the original and final volumes are as 7 to x; and hence, applying Boyle's law,

$$34 \times 7 = (54+x)x,$$

or

$$x^2 + 54x - 238 = 0.$$

The positive value of x is $4{\cdot}1$; thus the water rises $2{\cdot}9$ feet within the bell.

Note.—In Examples 165-171 the height of the mercury barometer may be taken as 30 inches and that of the water barometer as 34 feet.

165. A cylindrical diving-bell, 9 feet high, is immersed in water so that its top is 27 feet below the surface: find how high the water rises within it.

166. A cylindrical diving-bell, 8 feet in height, is lowered to the bed of a river 48 feet deep: to what height will the water rise within the bell?

167. To what depth must a diving-bell 6 feet high be immersed so that the water may rise 4 feet within it?

168. If the cross-section of the diving-bell in Example 164 was 14 square feet, how much air at the atmospheric pressure would have to be introduced in order to keep the water from entering?

The bell originally contained $7 \times 14 = 98$ cubic feet of air, under a pressure $= 34$ feet of water. When the bell is filled with air and its top is 20 feet below the surface, the total pressure on the air is $34 + 20 + 7 = 61$ feet of water. Hence, by Boyle's law, if v cubic feet of air at atmospheric pressure have to be introduced,

$$34\,(98 + v) = 61 \times 98,$$

and

$$\therefore v = 98 \times 27/34 = 77{\cdot}82 \text{ cub. ft.}$$

169. Find how much air would have to be pumped in so as to keep the water from entering if, in the preceding example, the bell had been lowered until the top was 50 feet below the surface of the water. What would be the height of a mercury barometer within the bell when the latter was filled with air?

170. An inverted test-tube, 6 inches long, and of uniform cross-section, is just immersed in mercury of specific gravity 13·6: how high does the liquid rise inside the test-tube?

171. A cylindrical gas-jar, 1 foot in length, is plunged mouth downwards in sea-water of density 1·026: to what depth must it be sunk in order that the water may rise half way up the jar?

172. An empty bottle is floated on water. Into it

water is gradually poured until it just begins to sink, and this happens when it is one-third full of water. The bottle is now emptied, immersed bottom upwards in the water, and gradually pushed down in that position. Taking the height of the water-barometer as 34 feet, and neglecting the thickness of the bottle, show that it will just float freely when at such a depth that the level of the water inside it is 17 feet below the surface, that above this depth it tends to rise, and that it will sink if lowered further.

173. The top of a cylindrical gas jar, 8 cm. in diameter, is ground flat and is closed by a glass plate weighing 400 gm. which fits on it air-tight. The barometric pressure at the instant when it is closed is 78 cm.: how low must the barometer fall in order that the plate may just be lifted?

174. The volume of the receiver of an air-pump is R, and that of the barrel is B: prove that if the density of the air in the receiver before exhaustion is D, the density after n complete strokes is

$$D_n = \{R/(R+B)\}^n D.$$

175. The receiver of an air-pump has three times the volume of the barrel: find the density after ten complete strokes. (Use logarithms: see § 14.)

176. After three complete strokes the density of the air in the receiver of an air-pump was to its original density as 125 : 216: show that the volume of the receiver was five times that of the barrel.

177. The receiver of an air-pump has a volume of 1350 cubic centimetres, and it contains air of density 0·001293 at the normal pressure. The pump is now worked until the density of the air is reduced by 25 per cent: find the mass of air that has been removed.

178. Two bodies, whose volumes are 600 cubic centimetres and 30 cubic centimetres respectively, are sus-

pended from the arms of a balance, and exactly balance each other in vacuo: find what weight must be attached to the larger body in order that they may balance each other in air. (1 cubic centimetre of air = 0·0013 gm.)

179. The receiver of an air-pump has a capacity of 1 litre, and the pressure of the air within it is 76·5 cm. If the barrel has a capacity of 500 c.c., what will be the pressure after four strokes?

180. After four strokes the density of the air in the receiver of an air-pump is found to bear to its original density the ratio of 256 to 625. What is the ratio of the volume of the barrel to that of the receiver?

181. If w denote the weight of a body in vacuo, and w' its weight in water, prove that its weight in air of density δ will be

$$w - (w - w')\delta.$$

182. The weights in vacuo of two portions of the same substance are w and w'. The first portion is placed in the upper cup of a Nicholson's hydrometer in presence of air of density δ, and the hydrometer is immersed to the same depth as when the second portion is placed in the lower cup: show that the density of the substance is

$$(w' - w\delta)/(w' - w).$$

183. The volume of a balloon filled with coal-gas is 1,000,000 litres, and its mass (including the car) is 500 kilogrammes. If the density of the air is 1·28, and that of the coal-gas 0·52, both being in grammes per litre, find what additional weight the balloon can sustain in the air.

184. Find the lifting power of a balloon, filled with coal-gas, from the following data: volume of balloon = 100 cubic metres; weight of balloon and car = 60 kgm. [A litre of coal-gas weighs 0·193 gm.; a litre of air weighs 1·293 gm.]

185. A litre of hydrogen weighs 0·089 gm., and a litre of air 1·293 gm. at 0°. If a balloon of 1000 cubic metres capacity is filled with hydrogen at this temperature, what weight can it carry?

EXAMINATION QUESTIONS

186. Define density or specific gravity, and calculate the mass of 1 cubic centimetre of a certain solid from the following data: a mass of 720 grammes hanging from one pan of a balance is totally immersed in water, and found to be counterpoised by a weight of 645 grammes in the other pan. Matric. 1891.

187. The specific gravity of lead is 11·3; how many lbs. will a cubic foot of lead weigh in water, if a cubic foot of water weighs 1000 ounces? What weight of lead will be required to sink a cubic foot of cork (s.g. ·25) in water? Camb. B.A. 1891.

188. Define specific gravity. Find the specific gravity of a solid substance from the following data:—A flask which, when filled with water weighs altogether 410 grammes, has 80 grammes of a solid introduced, and being then filled up with water weighs 470 grammes. What is the volume of a kilogramme of the solid? Int. Sc. 1890.

189. Find the specific gravity of a piece of wood from these data: weight of wood = 230 grammes; weight of a piece of iron in water = 580 grammes; weight of the wood and iron together in water = 465 grammes. Vict. Prel. 1890.

190. The depth of water on one side of a vertical dock-gate of rectangular shape is 25 feet, and on the other 15 feet, measured in each case from the bottom of the gate. Determine the position of the point of application of the resultant force on the gate. N. S. Tripos. 1891.

191. Show that the units may be chosen so that the

specific gravity and the density of a substance are identical.

A nugget of gold mixed with quartz weighs 12 ounces, and has a specific gravity 6·4; given that the specific gravity of gold is 19·35, and of quartz is 2·15, find (to one place of decimals) the quantity of gold in the nugget. B.Sc. 1891.

192. A thin uniform rod six feet long of specific gravity ·75 floats partly immersed in water, its upper end being supported by a string 3 feet long whose other end is attached to a hook $4\frac{1}{2}$ feet above the surface of the water. Find the magnitudes and directions of the forces acting on the stick and the angle the stick makes with the surface of the water. Draw a diagram to scale showing the position of equilibrium of the system. Camb. Schol. 1891.

193. Explain how Archimedes' principle enables us to compare the densities of liquids and solids.

A piece of lead weighs 7·88 grammes in air, 7·19 in water, and 7·33 in alcohol; a piece of oak weighs 13·21 grammes in air, and the oak and lead together weigh 4·87 grammes in water: find the specifie gravities of lead, oak, and alcohol. Camb. Schol. 1893.

194. A balloon is filled with a gas whose specific gravity is $\frac{1}{10}$th of that of air at the pressure of 760 mm. of mercury at 0° C. Compare the lifting power of the balloon in air when the height of the barometer is 750 mm. with its lifting power when the barometer stands at 760 mm. The temperature in both cases is 0° C., and the volume of the balloon is supposed to remain unaltered. Matric. 1894.

195. A common hydrometer whose weight is 100 grammes floats immersed up to a certain point on its stem in a liquid whose specific gravity is ·9. What weight must be attached to the top of the stem in order that the hydrometer when floating in water may be

immersed up to the same depth? Will the stability of the hydrometer be the same in the two cases?

Camb. Schol. 1891.

196. State Boyle's Law, and explain the term "height of the homogeneous atmosphere."

An accurate barometer reads 30 inches when one containing air above the mercury reads 24 inches. If the tube of the latter be raised 3 inches, the reading becomes 25 inches. Find what length of the tube the air would occupy if brought to atmospheric pressure.

Vict. Int. Sc. 1890.

197. A flask when empty weighs 120 grammes, when full of air it weighs 121·3 grammes, and when full of water 1220 grammes: calculate the density of the air.

Explain whether it is or is not necessary to take account of the weight of air displaced. Matric. 1891.

198. What do you know about the density of gases in relation to temperature and pressure? Describe experiments which show that the density of a gas at constant temperature is proportional to its pressure. A uniform tube closed at top, open at bottom, is plunged into mercury, so that it contains 25 c.c. of gas at atmospheric pressure 76 cm.; the tube is now raised until the gas occupies 50 c.c.: how much has it been raised?

Matric. 1891.

199. Explain the construction of a barometer, what it measures, and how it measures it. Translate pressure measured in terms of the height of a barometer mercury column (say either 27 inches or 60 centimetres) into absolute units of pressure. Matric. 1891.

200. A glass bulb will just stand an excess of inside over outside pressure of 200 gm. per sq. cm. It is sealed up at a place where the barometer stands at 75 cm., and then taken uphill till it bursts. What is the height of the barometer at the place where this occurs?

Matric. 1895.

201. A certain quantity of a gas has a volume of 5

cubic feet, and its pressure is equal to that which is due to 28 inches of mercury; a certain quantity of another gas has a volume of 4 cubic feet, and its pressure is equal to that which is due to 25 inches of mercury. When they are mixed their volume is 8 cubic feet; find from first principles the pressure of the mixture, the temperature of the gases and of the mixture being the same. S. K. 1895.

202. A barometer reads 30 inches at the base of a tower, and 29·8 inches at the top, 180 feet above. Find the average mass of a cubic foot of air in the tower, taking the specific gravity of mercury as 13·5, and the mass of a cubic foot of water as 62·4 lbs. Matric. 1894.

203. When the barometer is standing at 30 inches, a uniform straight tube, closed at one end, is partly filled with mercury and held in a vertical position with its closed end upwards and with its open end in a tank of mercury. When the closed end is 25 inches above the surface of the mercury in the tank the surface of the mercury inside the tube is one inch from the closed end. How far must the tube be pulled up in order that the top of the mercury inside it may be 6 inches from the closed end? Camb. B.A. 1891.

204. What must be the least diameter given to a soap-bubble inflated with hydrogen gas so that it may just float in air; the temperature being 15° C. and the pressure 770 mm.; the thickness of the soap-bubble film being ·002 cm.?

[Density of mercury, 13·596 at 0° C. Density (at 0° C. and under a pressure of 1 megadyne) of air ·0012752; of hydrogen ·00008837 grammes per c.c.]

Camb. Schol. 1891

CHAPTER IV

HEAT—EXPANSION

1. Expansion of Solids

Definition.—The coefficient of linear expansion of a substance may be defined in any of the following ways :—

1. If a bar of a substance be heated through one degree, its length will increase by a certain fraction, and this fraction is called the coefficient of linear expansion of the substance.
2. The coefficient of linear expansion is the ratio of the increase of length produced by a rise of 1° to the original lengths.
3. The coefficient of linear expansion is numerically equal to the increase of length produced in unit length of the substance by a rise in temperature of 1°.

Let a denote the coefficient of linear expansion of a body whose length at 0° is l_0 : starting with any one of the above definitions you will easily see—

(1) that the expansion produced by heating the body through 1° is $l_0 a$,

(2) that the expansion produced by heating it to t is $l_0 at$, and hence

(3) that its length l_t at $t°$ is given by the equation

$$l_t = l_0 + l_0 at = l_0(1 + at) \quad . \quad . \quad . \quad (1)$$

Again $\qquad l_0 at = l_t - l_0$

and $$\therefore a = \frac{l_t - l_0}{l_0 t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or, – (coeff. of exp.) $= \dfrac{\text{total expansion}}{(\text{original length}) \times (\text{interval of temperature})}$.

No solid or liquid expands with absolute uniformity, but the statements above made may be regarded as approximately

accurate. In the case of a body which expands irregularly we may take equation (2) as defining its *mean* coefficient of expansion. In all three definitions given above it would be more strictly accurate to use the expression "from 0° to 1°."

APPROXIMATE COEFFICIENTS OF LINEAR EXPANSION.

Glass	0·0000086
Platinum	0·0000086
Iron and Steel	0·000012
Copper	0·000017
Brass	0·000019
Zinc	0·000029

These values may be assumed in solving the examples in this chapter.

1. Find the length at 200° of a zinc rod whose length at 0° is 128 cm.

If the length at 200° be denoted by l_{200}, then

$$\begin{aligned} l_{200} &= l_0(1+200\alpha) \\ &= 128(1+\overline{200\times 0\cdot 000029}) \\ &= 128\times 1\cdot 0058 \\ &= 128\cdot 7424 \text{ cm.} \end{aligned}$$

2. A piece of brass wire is exactly 3 metres long at 250°: what will be its length at 0°?

Using the same system of notation, we have, by equation (1),

$$\begin{aligned} l_0 &= l_{250}/(1+250\alpha) \\ &= 300/(1+\overline{250\times 0\cdot 000019}) \\ &= 300/1\cdot 00475 = 298\cdot 582 \text{ cm.} \end{aligned}$$

3. The distance between two marks on a brass bar is found to be 90 cm. at 10°. What will be the distance between the marks at 90°?

The expansion of the brass on heating through 80° will be $l\alpha t = 90\times 0\cdot 000019\times 80 = 0\cdot 1368$ cm., and hence the distance between the marks at 90° will be 90·1368 cm.

Note.—The solution here given assumes that if l_t be the length of a bar at any temperature $t°$, its length $l_{t'}$ at any other temperature $t'°$ is

$$l_{t'} = l_t(1+\alpha\,.\,\overline{t'-t}) \quad . \quad . \quad . \quad . \quad (3)$$

This may be taken as being very approximately correct. It is not absolutely correct if, as is usual, the coefficient of expansion is measured and expressed with reference to the length of the bar at 0°.

The straightforward method of solving the problem would be to find first the length l_0 at 0° from the equation

$$l_t = l_0 (1 + at),$$

and then to calculate the length at t'° from the corresponding equation,

$$l_{t'} = l_0 (1 + at').$$

This would involve a lot of arithmetical work, and would give a result differing only slightly from the other. For it follows directly from these two equations that

$$\frac{l_{t'}}{l_t} = \frac{1 + at'}{1 + at},$$

and

$$\therefore l_{t'} = l_t \cdot \frac{1 + at'}{1 + at}.$$

Suppose that x and y are quantities which are very small compared with unity, so that their squares can be neglected in comparison with it. Then the fraction $(1 + x)/(1 + y)$ is approximately equal to $1 + x - y$. (This can be proved by dividing algebraically, and showing that the "remainder" is negligible. Cp. § 13). Now the coefficients of expansion of metals are very small; and unless t and t' are very high temperatures, at and at' will also be small compared with unity. It therefore follows that

$$\frac{1 + at'}{1 + at} = 1 + at' - at,$$

and $$l_{t'} = l_t (1 + a . \overline{t' - t}), \text{ approximately} \quad . \quad . \quad (4)$$

Observe that all this depends on the smallness of a. In dealing with the cubical expansion of liquids (where the coefficients are large) the method only gives a rough approximation. The coefficients of cubical expansion of gases are still larger (about 0·00366); here all volumes must be referred to the standard temperature of 0°, and the above method of approximation is *quite inadmissible.*

---

4. A steel metre-scale was carefully measured at 10°

and its length was found to be 99·981 cm. At 40° its length was found to be 100·015 cm.: calculate the coefficient of expansion of the steel, and find the temperature at which the scale is exactly 1 metre long.

By equation (3) we have

$$\begin{aligned} a &= (l_{t'} - l_t)/l_t(t' - t) \\ &= (100{\cdot}015 - 99{\cdot}981)/99{\cdot}981 \times 30 \\ &= 0{\cdot}034/2999{\cdot}43 = 0{\cdot}0000113. \end{aligned}$$

If $x°$ be the temperature at which the scale is correct,

$$100 = 99{\cdot}981\{1 + 0{\cdot}0000113(x - 10)\},$$

$$\therefore x - 10 = 0{\cdot}019/0{\cdot}0000113 \times 99{\cdot}981,$$

and $$x = 10 + 16{\cdot}82 = 26°{\cdot}82.$$

5. An iron steam-pipe is 40 feet long at 0°: what will be its length when steam at 100° passes through it?

6. An iron yard-measure is correct at the temperature of melting ice: express, as a fraction of an inch, its error at the temperature of boiling water.

7. What is meant by saying that the coefficient of expansion of steel is 0·000012? Assuming the highest summer temperature to be 40° C., and the lowest winter temperature −20° C., what allowance should be made for expansion in one of the 1700-feet spans of the Forth Bridge?

8. The height of the barometer appears to be 76·4 cm., according to a brass scale which is correct at 0°. If the temperature at the time of reading is 20°, what is the actual height of the mercury column?

9. The length of the iron railway bridge across the Menai Straits is about 461 metres. Find the total expansion of this iron tube between −5° and 35°.

10. A copper rod, the length of which at 0° is 2 metres, is heated to 200°: what will its length now be? At what temperature will its length be 200·51 centimetres?

11. The length of a glass tube at 100° is 154 cm. What would its length at 0° be?

12. What is the length of a brass wire which on heating through 200° increases in length by 1 centimetre?

13. The distance between two points appears to be 87·2 cm., when measured at 28° on a brass scale which is right at 0°: what is the real distance?

14. What must be the length at 50° of a brass standard yard-measure, in order that it may be exactly correct at the freezing-point?

15. A rod which is exactly $2\frac{1}{4}$ metres long at 10° is heated to 160°, when its length is found to be 2·277 metres: what is its coefficient of expansion? At what temperature will its length be 2·295 metres?

16. A copper wire is found to be 0·034 cm. longer at 25° than it is at 5°. Calculate accurately what its length at 0° would be.

17. A metal scale was examined at 32° F. and 76° F., and was found to have expanded 0·014 inch between these temperatures: find what change would be produced in its length by an elevation of temperature of 50° C.

18. A steam-pipe, intended to convey steam at 110°, is formed of iron piping in lengths of 15 ft.; assuming that the temperature of the pipe, when it is not conveying steam, is 12°, find how much play must be allowed at each joint.

19. Two long slips of metal, one of iron, the other of brass, are firmly riveted together: describe what will happen when they are heated, and explain how the unequal expansion of metals has been applied:—

(1) In the determination of temperature.

(2) In the construction of delicate chronometers.

20. A rod of iron and a rod of brass are placed side by side, and firmly riveted together at one end. Their lengths at 0° are 1·5 and 2·5 metres respectively, so that the distance between their free ends at this temperature is exactly 1 metre. The compound bar is immersed in

a hot oil-bath at a temperature of 220°: what is now the distance between the free ends?

21. Show that the lengths of the metal bars of a compensation-pendulum should be inversely proportional to the coefficients of expansion of the metals. If the length of the iron bars be 87 centimetres, what should be the length of the zinc bars?

22. The time of vibration of a pendulum is proportional to the square root of its length, and a certain clock with an iron pendulum rod is made so as to keep correct time at 5°: how will its rate alter if the temperature rises to 30°?

When the pendulum keeps correct time (*i.e.* at 5°), it swings 86,400 times per day (supposing it to be a seconds pendulum; see p. 34). At 30° its length is increased in the ratio of 1·0003 : 1, and it now swings 86,400 $\sqrt{1/1{\cdot}0003}$ times per day. Referring to p. 21, it will be seen that $\sqrt{1/1{\cdot}0003} = 1/1{\cdot}00015 = 1 - 0{\cdot}00015$ approximately. Therefore the clock loses $86{,}400 \times 0{\cdot}00015 = 12{\cdot}96$ secs. per day.

23. A clock which keeps correct time at 25° has a pendulum rod made of brass: how many seconds a day will it gain if the temperature falls to the freezing point?

24. In order to measure the coefficient of expansion of a brass bar it was placed by the side of an iron bar, whose coefficient of expansion was known to be 0·0000118, and both were riveted together at one end. They were then placed in melting ice, and a fine mark was engraved across both at a distance of exactly one metre from the riveted end. On heating the bars and examining the marks under a microscope it could be seen that the brass bar expanded more than the iron, as was shown by the shifting forward of the mark upon it. At 100° the measured distance between the marks on the two bars was 0·74 mm. What was the coefficient of expansion of the brass?

25. A spherical iron ball, of 5·01 cm. diameter at 0°,

rests upon a copper ring, the internal diameter of which is exactly 5 cm. at the same temperature. To what temperature must both be heated in order that the ball may just pass through the ring?

Let t be the required temperature. The diameter of the iron ball at $t°$ is $5.01\ (1 + 0.000012t)$, and the internal diameter of the copper ring at the same temperature is $5(1 + 0.000017t)$. Assuming that the ball will just pass through the ring when their diameters are equal, we have

$$5.01(1 + 0.000012t) = 5(1 + 0.000017t),$$

$$\therefore\ 0.01 + (5.01 \times 0.000012t) = (5 \times 0.000017t),$$

and

$$t = 0.01/(5 \times 0.000017 - 5.01 \times 0.000012)$$
$$= 0.01/0.00002488,$$

or $$t = 402°.$$

26. A platinum wire and a strip of zinc are both measured at 0°, and their lengths are found to be 251 and 250 cm. respectively: at what temperature will their lengths be equal, and what will be their common length at this temperature?

27. Prove that the coefficient of contraction is, for most solids, approximately equal to the coefficient of expansion.

A glass tube which is 99·994 cm. long at 5°, and a brass rod of which the length at 22° is 100·019 cm., are found to be of exactly the same length at an intermediate temperature: what is this temperature?

28. Calculate the superficial expansion produced by a rise of 40° C. in a plate of sheet-iron 5 ft. long and 3 ft. broad.

29. A sheet of brass is 20 cm. long and 15 cm. broad at 0°: what is its superficial area at 80°?

30. Prove that the coefficient of cubical expansion of a substance is approximately three times its coefficient of linear expansion.

The coefficient of linear expansion of vulcanite is 0·00008; what change will be produced in the volume

of a slab of vulcanite on heating it to 90°, if the slab at 0° is 1 ft. long, 10 in. broad, and 1 in. thick?

31. The volume of a leaden bullet at 0° is 2·5 cubic centimetres, and its volume at 98° is found to be 2·521 cubic centimetres: prove that the coefficient of cubical expansion of lead is 0·0000857.

32. The density of standard silver at 0° is 10·31, and its coefficient of cubical expansion is 0·000058: find its density at 150°.

33. The coefficient of cubical expansion of sulphur is 0·000223, and a certain piece of sulphur is found to displace 48 c.c. of water at 0°: what volume of water will it displace at 35°?

2. Expansion of Liquids

34. Distinguish between the real and apparent expansion of a liquid contained in a glass vessel. If the apparent coefficient of expansion of mercury contained in a glass vessel is 1/6500, while its coefficient of absolute expansion is 1/5500, what is the coefficient of cubical expansion of the glass? [1]

[1] The absolute dilatation of a liquid is approximately equal to its apparent dilatation, together with the cubical expansion of the containing vessel. This may be proved as follows:—

Suppose the liquid to be contained in a vessel with a graduated stem, the graduation indicating the volume up to each mark on the stem, *and being correct at* 0°. At any other temperature the graduation will not be correct; for when the vessel is heated it expands, and its *real* volume up to any particular mark on the stem is greater than its *apparent* volume as indicated by the graduation.

Let V_0 be the common volume at 0° of the liquid, and of the portion of the vessel which it fills at that temperature. Also let V be the real volume of the liquid at any temperature $t°$, and V' its apparent volume at the same temperature.

If δ be the coefficient of real expansion of the liquid, its real volume at $t°$ is $V=V_0(1+\delta t)$; or, substituting Δ for δt,

$$V=V_0(1+\Delta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

The apparent volume of the liquid is

$$V'=V_0(1+D) \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

D being equal to dt, where d is the coefficient of apparent expansion,

35. Describe the behaviour of water when it is heated from 0° to 10° C. How would you show the existence of a point of maximum density between these temperatures?

36. The density of a liquid at 0° is D_0, and its coefficient of cubical expansion is k: show that its density at $t°$ is

$$D_t = D_0/(1 + kt).$$

37. Two thermometers, one filled with mercury, the other with water, are marked so as to agree at two fixed points, but their readings are found to differ at intermediate temperatures. Explain this fact, and discuss its bearing upon the general question of thermometric measurements.

38. Explain precisely what you understand by the symbol "200° C."

39. The volume of a gramme of water being 1 c.c. at 4°, and 1·0169 c.c. at 60°, what is the mean coefficient of expansion of water between these temperatures?

V′ is also the *apparent* volume of the portion of the vessel in which the liquid is contained at the temperature $t°$. If k be the coefficient of cubical expansion of the vessel, its real volume at this temperature is $V'(1+kt)$, and this is equal to V, the real volume of the liquid. Putting $kt=K$, we have

$$V=V'(1+K) \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

From equations (b) and (c), by multiplication, we have

$$V=V_0(1+D)(1+K).$$

Comparing this with equation (a) we see that

$$1+\Delta=(1+D)(1+K)$$
$$=1+D+K+DK.$$

Now both D and K are small quantities compared with unity, and hence their product may be neglected (Cp. § 13). Thus we have, approximately,

$$\Delta=D+K.$$

If we take $t=1°$, Δ, D and K become respectively δ, d and k; so that to the same degree of approximation

$$\delta=d+k,$$

or the coefficient of real expansion of the liquid is equal to its coefficient of apparent expansion, together with the coefficient of cubical expansion of the containing vessel.

40. The density of water at 4° is unity, and its density at 60° is 0·9834: find its mean coefficient of expansion between 4° and 60°.

41. Find the relation between the density d_t of water at $t°$ and its density d_0 at 0°, given that unit volume of water at 4° expands by an amount e from 4° to 0°, and by an amount e' from 4° to $t°$.

42. Describe Dulong and Petit's method of measuring the real expansion of mercury, giving a sketch of the apparatus employed. State the hydrostatic principle upon which the method is based, and show that if H_t and H_0 denote the height of the hot and cold columns respectively, the coefficient of expansion is

$$k = \frac{H_t - H_0}{H_0 . t}.$$

43. Point out the defects in the above method which render it difficult to make accurate measurements by means of it. Sketch and describe the modified form of apparatus introduced by Regnault.

44. In an experiment made by the above method, the heights of the mercury columns were 60 cm. and 61·09 cm., their temperatures being 0° and 100° respectively: what value does this give for the coefficient of expansion?

45. In an experiment made with paraffin oil by the same method the hot column was kept at 100° and the cold column at 14°. The height of the cold column was 60 cm., and the difference in height between the two columns was 5·1. Show that the coefficient of real expansion of the paraffin oil was 0·00099.

Note.—In Examples 46-55 the coefficient of real expansion of mercury is to be taken as 0·000182.

46. In an experiment made according to Dulong and Petit's method, the heights of the two columns of mercury were 90 cm. and 91·7 cm.: if the first column was at 0°, what was the temperature of the second?

47. Assuming that the density of mercury at 0° is 13·6, prove that its density at 120° is 13·3.

48. Show that the volume of a gramme of mercury at 110° is 0·075 c.c., and that the weight of 1 c.c. of mercury at 80° is 13·4 gm.

49. The height of the barometer is found to be 77·25 cm., the temperature of the air being 25°: prove that the corresponding barometric height reduced to zero would be 76·90 cm., *i.e.* that the barometric column would stand at this height if the mercury were at 0°.

50. A specific gravity bottle contains exactly 687 gm. of mercury at 70°: show that its internal volume at this temperature is 51·158 c.c. [Density of mercury at 0° = 13·6.]

51. Prove that the mean coefficient of real expansion of mercury between 0° and 300° is 0·0001864, having given that its density is 13·595 at 0°, and 12·875 at 300°.

52. A weight thermometer contains M grammes of a liquid at 0°; on heating it to a temperature t, m grammes of the liquid are expelled: show that the coefficient of apparent expansion of the liquid in its envelope is

$$k = m/(M - m)t.$$

53. A glass bulb, with bent capillary tube, was filled with mercury at the temperature of melting ice: it was then heated to 100°, and the expelled mercury carefully collected and weighed. Calculate the apparent coefficient of expansion of mercury between these temperatures from the data given below:—

Weight of mercury expelled = 10·2877 gm.
,, of bulb + mercury at 100° = 697·5 gm.
,, of bulb alone = 38·5 gm.

54. A weight thermometer weighs 40 gm. when empty, and 490 gm. when filled with mercury at 0°; on heating it to 100°, 6·85 gm. of mercury escapes. Cal-

culate the coefficient of expansion of the glass. (See notes on pp. 114 and 116.)

55. A glass flask contains 1·36 kilogramme of mercury at 0°: find the volume at 100° of the mercury which is expelled when the flask and its contents are heated to 100°. [Coefficient of cubical expansion of glass = 0·000025.]

Note.—In Examples 56-59 the coefficient of apparent expansion of mercury in glass is to be taken as 0·000155.

56. A weight thermometer which contains a kilogramme of mercury at 0° is placed in an oil-bath, and the mercury expelled is found to weigh 20 grammes. Find the temperature of the bath.

57. A glass vessel with a capillary stem weighs 104·53 gm. when empty, and holds 623·51 gm. of mercury at 0°. What is the temperature when the whole apparatus weighs 717·62 gm.?

58. How much mercury would have been expelled if the glass vessel in the preceding example had been heated to 100° exactly?

59. A glass flask holds 1360 grammes of mercury at 0°: how much of the mercury would overflow if the flask were immersed in boiling water? [Take the density of mercury as 13·6.]

60. The coefficient of linear expansion of glass is 0·000008, and a certain glass flask contains exactly 100 c.c. of water at 0°. What will be the volume of the water contained when the flask and its contents are heated to 100°?

61. A piece of glass, the weight of which in air was 46·76 gm., was found to weigh 31·29 gm. in water at its point of maximum density (4°), and 31·51 gm. in water at 60°. Find the coefficient of cubical expansion of water, taking that of glass as 0·000024.

The loss of weight of the glass when weighed in water at 4°

is $46{\cdot}76 - 31{\cdot}29 = 15{\cdot}47$ gm., so that the volume of the glass at 4° is $15{\cdot}47$ c.c. At 60° its volume is

$$15{\cdot}47 \times (1 + 0{\cdot}000024 \times 56) = 15{\cdot}47 \times 1{\cdot}001344$$
$$= 15{\cdot}4908 \text{ c.c.}$$

Now the loss of weight on weighing in water at 60° is $46{\cdot}76 - 31{\cdot}51 = 15{\cdot}25$ gm.; therefore $15{\cdot}4908$ c.c. is the volume occupied by $15{\cdot}25$ gm. of water at 60°. This quantity of water at 4° would occupy $15{\cdot}25$ c.c., so that if k be the coefficient of expansion of water,

$$15{\cdot}4908 = 15{\cdot}25(1 + 56k),$$

and $$\therefore k = \frac{0{\cdot}2408}{15{\cdot}25 \times 56} = 0{\cdot}000282.$$

62. The method indicated in the preceding example can also be employed for measuring the expansion of a solid when that of the liquid is known (Matthiesen's method). Explain fully how you would proceed to find *ab initio* the coefficients of cubical expansion of a solid and a liquid (say glass and water).

63. A glass rod which weighs 90 grammes in air is found to weigh 49·6 gm. in a certain liquid at 12°. At 97° its apparent weight in the same liquid is 51·9 gm.: find the coefficient of absolute expansion of the liquid.

The loss of weight at 12° is $90 - 49{\cdot}6 = 40{\cdot}4$ gm. If D be the density of the liquid at 12°, the volume of the glass at this temperature is

$$V = 40{\cdot}4/D.$$

The loss of weight at 97° is $90 - 51{\cdot}9 = 38{\cdot}1$ gm., and if D' be the density of the liquid at this temperature, the volume of the glass at the same temperature is

$$V' = 38{\cdot}1/D'.$$

But since the coefficient of cubical expansion of glass is $0{\cdot}000024$,

$$V' = V(1 + \overline{85 \times 0{\cdot}000024})$$
$$= V \times 1{\cdot}00204.$$

Therefore

$$38{\cdot}1/D' = 1{\cdot}00204 \times 40{\cdot}4/D.$$

Again, if k be the coefficient of expansion of the liquid between $12°$ and $97°$, $D' = D(1 + 85k)$. Thus

$$38{\cdot}1 \times (1 + 85k) = 1{\cdot}00204 \times 40{\cdot}4,$$
$$= 40{\cdot}4824;$$
$$\therefore\ 38{\cdot}1 \times 85k = 40{\cdot}4824 - 38{\cdot}1 = 2{\cdot}3824,$$
and $$k = 2{\cdot}3824/38{\cdot}1 \times 85 = 0{\cdot}0007356.$$

64. *A solid is found to weigh 29·9 gm. in a liquid of specific gravity 1·21 at 10°, its weight in air being 45·6 gm. It weighs 30·4 gm. in the same liquid at 95°, when its specific gravity is 1·17. Calculate the coefficient of cubical expansion of the solid.

3. Expansion of Gases

Charles's Law.—The volume of a given mass of gas, kept at a constant pressure, increases by a definite fraction of its amount at 0° for each degree rise of temperature.

For air, hydrogen, oxygen and nitrogen, the value of this fraction (which is the coefficient of cubical expansion) is 0·00366, or approximately 1/273: the latter value may be adopted in solving the examples in this chapter.

Thus if we take a quantity of gas whose volume at 0° is 1 (unity), its volume will become

$$1 + \frac{1}{273} \text{ at } 1°,$$
$$1 + \frac{2}{273} \text{ at } 2°,$$
$$1 + \frac{3}{273} \text{ at } 3°,$$
and $$1 + \frac{t}{273} \text{ at } t°.$$

Or if we denote by V_0 the volume at 0° and by V_t the volume at $t°$, then

$$V_t = V_0\left(1 + \frac{t}{273}\right) = V_0 \frac{273 + t}{273} \quad . \quad . \quad (1)$$

If we adopt the symbol a to denote the coefficient of expansion, we may put equation (1) into the algebraical form

$$V_t = V_0(1 + at) \quad . \quad . \quad . \quad (2)$$

from which we obtain

$$V_0 = \frac{V_t}{1 + at} \quad . \quad . \quad . \quad (3)$$

an equation by which we can find the volume at 0° when the volume at $t°$ is given.

65. A certain quantity of oxygen gas occupies a volume of 300 c.c. at 0°: find its volume at 91°.

The required volume is given by the equation

$$V_{91} = V_0\left(1 + \frac{91}{273}\right),$$

or
$$V_{91} = 300\left(1 + \frac{1}{3}\right) = 300 \times \frac{4}{3} = 400 \text{ c.c.}$$

66. The volume of a certain quantity of gas is 27·3 c.c. at 0° and 30 c.c. at 27°: what is its coefficient of expansion?

Let a denote the required coefficient; then

$$\begin{aligned} 30 &= 27{\cdot}3(1 + 27a) \\ &= 27{\cdot}3 + 27{\cdot}3 \times 27a, \\ \therefore a &= \frac{2{\cdot}7}{27{\cdot}3 \times 27} = \frac{1}{273}. \end{aligned}$$

67. What would be the volume at 0° of a mass of gas which at 78° occupies a volume of 9 litres?

According to equation (3) the required volume is

$$V_0 = \frac{9}{1+\frac{78}{273}} = \frac{273 \times 9}{273+78}$$

$$= \frac{9 \times 273}{351} = 7 \text{ litres.}$$

68. The weight of a litre of air at N.P.T. is 1·293 gm.: to what temperature must the air be heated so that it may weigh exactly 1 gm. per litre?

Let t° be the required temperature, then at t° 1·293 gm. of air occupies a volume of 1·293 litre, and the question is equivalent to the following: to what temperature must we heat a litre of air, taken at 0°, in order to increase its volume to 1·293 litre? The value of t is therefore given by the equation

$$1{\cdot}293 = 1 + t/273,$$

thus $$t = 0{\cdot}293 \times 273 = 79{\cdot}99,$$

and the required temperature is $79^\circ{\cdot}99$.

Notice carefully the following points in the statement of Charles's Law:—

(*a*) "Kept at a constant pressure."—These words are inserted because the law only holds good if the pressure remains constant: if it varies, the volume also varies according to Boyle's Law.

(*b*) "For each degree."—The increase of volume is proportional to the rise of temperature.

(*c*) The coefficient of expansion (which is *defined* by the law and the statement which follows it) is the *same* for all the gases named.

(*d*) "Of its amount at 0°."—Just as a certain pressure (76 cm. of mercury) is adopted as the standard atmospheric pressure, so this temperature (0°) is adopted as the standard or normal temperature for measuring the volume of a gas, and the value of the coefficient of expansion is stated with reference to this. If you are given the volume *at* 20° you are not at liberty to say that *at* 21° the volume will have increased by the *same* fraction

(1/273). You must either find by equation (3) its volume at 0°, and then by equation (2) its volume at 21°, or you may proceed as follows :—

Given the volume V_t *of a mass of gas at* $t°$, *to find its volume* $V_{t'}$ *at* $t'°$.

As in equation (2) we can refer the volumes at both temperatures to the volume V_0 at 0° thus

$$V_t = V_0(1 + at),$$

and

$$V_{t'} = V_0(1 + at').$$

It follows that

$$\frac{V_{t'}}{V_t} = \frac{1 + at'}{1 + at} \quad . \quad . \quad . \quad (4)$$

or

$$V_{t'} = V_t(1 + at')/(1 + at).$$

Since $a = \frac{1}{273}$ we may write equation (4) in the form

$$\frac{V_{t'}}{V_t} = \frac{1 + t'/273}{1 + t/273} = \frac{273 + t'}{273 + t} \quad . \quad . \quad (5)$$

which is convenient for calculation. (See what follows. Compare the above with the note on pp. 108-9 respecting the expansion of metal rods. Be careful not to adopt the method there given when dealing with expansion of gases. The coefficients of expansion of gases are much larger fractions than those which express the coefficients of linear expansion of solids, and hence the approximate calculation on p. 108 is not admissible here.)

69. 100 c.c. of air is measured off at 20°: if the temperature be raised to 50°, what will the volume now be, the pressure remaining constant?

By equation (5), if V_{60} be the required volume at 50°,

$$\frac{V_{60}}{100} = \frac{273 + 50}{273 + 20} = \frac{323}{293},$$

and

$$\therefore V_{60} = 32300/293 = 110{\cdot}24 \text{ c.c.}$$

Absolute Temperature.—Charles's Law is known to hold good (within certain limits) not only for expansion on heating, but also for contraction on cooling. Thus if the volume of a certain mass of gas at 0° be 1 (unity) its volume on cooling would become

$$1 - a \text{ at } -1^\circ,$$
$$1 - 2a \text{ at } -2^\circ,$$

and
$$1 - ta \text{ at } -t^\circ.$$

Now suppose the gas to be cooled down to a temperature $t = -\frac{1}{a}$: if this were possible, and *if* the law held good down to this temperature, the volume of the gas would be reduced to zero. The temperature at which this would occur is called the *absolute zero of temperature.* On the assumptions above made, and taking $a = 1/273$, the absolute zero would correspond to a temperature of -273° C. A scale of temperatures in which this is taken as the zero is called the absolute scale of temperature, and temperatures reckoned according to this scale are called *absolute temperatures.* Clearly the absolute temperature corresponding to any given temperature on the Centigrade scale will be found by adding 273 to it. Thus the following are corresponding temperatures:—

Centigrade Scale.	Absolute Scale.
-273°	0°
0°	273°
1°	274°
100°	373°
273°	546°
t°	$T^\circ = 273^\circ + t^\circ$

It will now be seen that the numerator and denomi nator on the right-hand side of equation (5) represent respectively the absolute temperatures corresponding to

t'° and t° on the Centigrade scale; denoting these by T′ and T we may write the equation in the form

$$\frac{V_{t'}}{V_t} = \frac{T'}{T} \quad . \quad . \quad . \quad . \quad (6)$$

or, *The volume of a given mass of gas is proportional to its absolute temperature.*

70. 15 litres of air are cooled from 27° to 7°: by how much will the volume diminish?

The absolute temperature corresponding to 27° C. is 273° + 27° = 300°; and the absolute temperature corresponding to 7° C. is 273° + 7° = 280°. Let V be the volume of the air at the lower temperature; since the volumes are proportional to the absolute temperatures,

$$\therefore \frac{V}{15} = \frac{280}{300} = \frac{14}{15},$$

or $$V = 14 \text{ litres.}$$

Thus the volume of the air becomes 1 litre less.

71. At what temperature will the volume of a given mass of gas be exactly double of what it is at 17°?

The absolute temperature corresponding to 17° is 273° + 17° = 290°. If T be the required temperature on the absolute scale, we must have

$$\frac{T}{290} = \frac{2}{1},$$

and, therefore, T = 580°. This corresponds to a Centigrade temperature of 580° − 273° = 307°.

Change of Pressure at Constant Volume.—If a gas be heated while its volume is kept constant, the pressure increases just as the volume increases when the pressure is kept constant; so that if P_0 denote the pressure at 0°, the pressure at any other temperature t° is

$$P_t = P_0(1 + at) \quad . \quad . \quad . \quad (7)$$

a denoting here the coefficient of increase of pressure at constant volume, which has the same value as the coefficient of increase of volume at constant pressure (1/273).

This last equation is similar to equation (2); and if we substitute P for V in the other equations (4 and 6), they will also hold good for change of pressure at constant volume. Thus

$$\frac{P_{t'}}{P_t} = \frac{1 + at'}{1 + at} \quad . \quad . \quad . \quad (8)$$

and

$$\frac{P_{t'}}{P_t} = \frac{T'}{T} \quad . \quad . \quad . \quad . \quad (9)$$

A gas is said to be at the normal pressure and temperature [N.P.T.] when its pressure is equal to that of a column of mercury 76 centimetres in height, and its temperature is 0° C.

72. The pressure inside a steel cylinder containing compressed oxygen is measured by means of a manometer, and is found to be 30 atmospheres at a temperature of 27°. The cylinder is now surrounded by a freezing mixture, which reduces the temperature to − 13°, and the pressure falls to 26 atmospheres. Find the coefficient of increase of pressure.

Let a denote the required coefficient. By equation (8) we have

$$\frac{1 + 27a}{1 - 13a} = \frac{P_{27}}{P_{-13}} = \frac{30}{26};$$

$$\therefore 26(1 + 27a) = 30(1 - 13a),$$

or
$$26 + 702a = 30 - 390a,$$

or
$$1092a = 4,$$

and
$$a = \frac{4}{1092} = \frac{1}{273}.$$

73. A closed flask containing air at 0° is connected with a mercury manometer which indicates that the pressure inside is less than that outside, the difference being equal to the pressure due to a column of mercury 15 cm. high. The flask is gradually heated: find the temperature at which the internal and external pressures will be equal, the barometric height being 75 cm. At what

temperature will the pressure inside the flask be two atmospheres?

The pressure inside the flask is equal to that due to a column of mercury the height of which is $75 - 15 = 60$ cm. Suppose that at $t°$ the pressure increases to 75 cm., then, by equation (8),

$$75 = 60(1 + t/273),$$
$$\therefore\ 60t/273 = 15,$$
and $$t = 273 \times 15/60 = 68°\cdot25.$$

Again, if $t'°$ be the temperature at which the pressure is 2 atmospheres ($= 150$ cm.),

$$150 = 60(1 + t'/273),$$
$$\therefore\ 60t'/273 = 90,$$
and $$t' = 273 \times 90/60 = 409°\cdot5.$$

Note.—Take $\alpha = 1/273$.

74. What change of volume would be produced by heating 26 litres of air from 0° to 21°?

75. A certain quantity of gas measures 260 c.c. at 0°: what would be its volume at 63°?

76. To what temperature must a gas be heated in order that its volume may become double of what it is at 0°?

77. A certain quantity of gas measures 90 c.c. at 0°: at what temperature will its volume become 120 c.c.?

78. The volume of a gramme of hydrogen at 0° is 11·16 litres: what is its volume (1) at 30°, (2) at 50°?

79. 200 cubic centimetres of air are heated from 0° to 30°, and at the latter temperature the volume is found to be 222 c.c.: what value does this give for the coefficient of expansion of air?

80. A closed glass tube filled with air at 0° and under atmospheric pressure is gradually heated. If the tube can safely stand a pressure of 3 atmospheres, to what temperature may it be heated?

81. A litre flask contains 1·293 gm. of air at 0°: how much will it contain at 100°?

82. A certain quantity of gas occupies a volume of

66 c.c. at 13°: what will be its volume at 52°? At what temperature will its volume be 63 c.c.?

83. At what temperature will the volume of a given mass of gas be exactly double of what it is at 30°?

84. What will be the volume at 75° of a quantity of air which occupies 2·5 litres at 0°. At what temperature will its volume be exactly 3 litres?

85. At 50° the volume of a gramme of hydrogen is 13·2 litres: what is its volume (1) at 0°, (2) at 30°?

86. What volume of gas measured at 30° will have a volume of 200 c.c. at 0°?

87. A litre of hydrogen weighs 0·0896 gm. at 0°: find the weight of 1602 c.c. of hydrogen measured at 110°.

88. The stopcock of a copper boiler containing air at the normal pressure is closed when the temperature is 25°: it is then immersed, first in melting ice, and secondly in boiling water. Calculate the pressure (in centimetres of mercury) inside the boiler in both cases, neglecting the expansion of the boiler.

89. A certain mass of gas has a volume of 1250 c.c. at 90°: find its volume at 363°.

90. Calculate the mean coefficient of expansion of air between 0° and 200°, having given that its density is 0·001293 at 0° and 0·0007457 at 200°.

91. If the volume of a quantity of air at 10° be 230 c.c., at what temperature will its volume have increased to 285 c.c.?

92. An iron cylinder at 13° contains oxygen gas at a pressure of 6 atmospheres: if the cylinder is made to stand a pressure of 21 atmospheres, show that it may be heated to 728° before bursting.

93. Describe carefully an experimental method of finding the relation (1) between the volume and temperature of a gas kept at constant pressure, (2) between the pressure and temperature of a gas kept at constant volume.

94. A glass flask containing air at 0° is heated to

100° (with the neck open), and during the process a gramme of air escapes. What weight of air did the flask originally contain? (You may neglect the expansion of the glass.)

95. A quantity of mercuric oxide is heated, and the oxygen given off is found to measure 300 c.c. On cooling to the temperature of the room (9°·5) the volume is reduced to 290·5 c.c.: what was the original temperature?

96. 6 litres of air at 10° are enclosed in the cylinder of an air engine, the cross-section of which is 200 sq. cm. The piston moves through a distance of 5 cm.: what elevation of temperature is required to keep the pressure constant?

97. Two spherical vessels contain equal quantities of air. The one is at a temperature of 70° and its internal radius is 7 cm.: the other is at a temperature of 239° and its internal radius is 8 cm. Prove that the pressure exerted by the air inside both is the same.

98. A litre flask is filled with air at N.P.T.; the temperature is then raised to 130°, the pressure remaining constant: find the volume that would be occupied by the air which escapes, if it were again cooled to 0°. (You may neglect the expansion of the flask.)

99. The density of carbon monoxide is to that of carbon dioxide as 28 is to 44: prove that the density of carbon dioxide at 156° is equal to that of carbon monoxide at 0°, the pressures being identical.

Simultaneous Changes of Pressure and Temperature.—It will perhaps be advisable for the student at first to calculate the effect of simultaneous changes of pressure and temperature on the volume of a gas as if each change occurred separately. An instance of this is given in Ex. 100; also in Ex. 101, which illustrates the process of "reducing the volume of a gas to the normal pressure and temperature." (See p. 131.)

Let v denote the volume of a given mass of gas, p its pressure, and T its absolute temperature. We know that according to Charles's Law the volume varies directly as the absolute temperature, or

$$v \propto T,$$

when p is constant.

Further, by Boyle's Law, the volume varies inversely as the pressure, or

$$v \propto \frac{1}{p},$$

when T is constant.

It follows that, when temperature and pressure change simultaneously, the volume varies directly as the absolute temperature and inversely as the pressure, or

$$v \propto \frac{T}{p}.$$

This may be written in the form

$$v = R \, . \frac{T}{p}, \text{ or } pv = RT \quad . \quad . \quad (10)$$

where R is a constant.

Thus *the product of the pressure and volume of a given mass of gas is always proportional to its absolute temperature.*

This may also be proved as follows. Let v_0 be the volume at 0° of a mass of gas, the pressure being p_0. If the pressure changes to p (the temperature remaining constant), then, according to Boyle's Law, the volume becomes $v_0 \times p_0/p$. Now let the gas be heated to any temperature $t°$; according to Charles's Law, the volume will now become

$$v = v_0 p_0(1 + at)/p,$$

and therefore

$$\frac{pv}{1 + at} = p_0 v_0$$

$$= \text{a constant} = k \text{ (say)}$$

Substituting for a its value $1/273$,

$$\frac{pv}{1+t/273}=k,$$

or
$$\frac{pv}{273+t}=\frac{k}{273}=\text{R (say)}.$$

Since $273+t=\text{T}$ (the absolute temperature corresponding to t° on the centigrade scale), we may write the last equation in the simple form

$$\frac{pv}{\text{T}}=\text{R, or } pv=\text{RT} \quad . \quad . \quad (11).$$

100. The volume of a certain mass of gas is 145 c.c. at 17° and under a pressure of 72·5 cm. of mercury. How will the volume be affected if the temperature falls to 7°? And if the pressure further falls to 70 cm. what will be the final volume?

Let V_7 denote the volume of the gas at 7° and 72·5 cm. pressure. By equation (5) we have

$$\frac{V_7}{145}=\frac{273+7}{273+17}=\frac{280}{290},$$

and
$$\therefore V_7=140 \text{ c.c.}$$

Again, let V denote the final volume after the pressure has fallen from 72·5 cm. to 70 cm. By Boyle's Law we have

$$\frac{V}{140}=\frac{72\cdot5}{70},$$

or
$$V=2\times72\cdot5=145 \text{ c.c.}$$

so that the final volume is the same as the original volume.

101. On heating a certain quantity of mercuric oxide it is found to give off 380 c.c. of oxygen gas, the temperature being 23° and the barometric height 74 cm.: what would be the volume of the oxygen measured at the normal pressure and temperature?

First allow for the change of pressure, according to Boyle's Law, assuming the temperature to remain constant. Since the volumes are inversely proportional to the pressures, the

gas would occupy under a pressure of 76 cm. a volume of $380 \times \frac{74}{76} = 370$ c.c.

Next allow for the change of temperature, the pressure remaining constant and equal to 76 cm. The absolute temperatures corresponding to 23° and 0° are 296° and 273° respectively. At the latter temperature the volume would be

$$370 \times \frac{273}{296} = \frac{2730}{8} = 341\tfrac{1}{4}.$$

Thus the volume at the normal pressure and temperature would be $341\frac{1}{4}$ c.c.

102. A litre of dry air weighs 1·293 gm. at N.P.T. At what temperature will a litre of air weigh a gramme, the pressure being 72 cm.?

Let t be the required temperature. A litre of air, taken at 0°, would at $t°$ have the volume $(1 + t/273)$ litres, if the pressure remained constant; but when it changes from the normal pressure (76 cm.) to 72 cm., the volume increases further, and by the preceding proposition becomes

$$(1 + t/273) \times 76/72.$$

This is the volume (in litres) of 1·293 gm. of air at $t°$; and, in order that 1 gm. of air at this temperature should occupy exactly a litre, we must have

$$(1 + t/273) \times 76/72 \times 1{\cdot}293 = 1.$$

Thus

$$(1 + t/273) \times 76 = 72 \times 1{\cdot}293 = 93{\cdot}1,$$

$$\therefore\ 76t/273 = 93{\cdot}1 - 76 = 17{\cdot}1,$$

and

$$t = 273 \times 17{\cdot}1/76 = 61°{\cdot}43.$$

103. A quantity of air at the atmospheric pressure and at a temperature of 7° is compressed until its volume is reduced to one-seventh, the temperature rising 20° during the process: find the pressure at the end of the operation.

By equation (10),

$$\frac{pv}{T} = \frac{p'v'}{T'},$$

the letters having the usual signification. The absolute temperatures corresponding to 7° and 27° are $T=273+7=280$, and $T'=273+27=300$. Taking the original volume as 7, and the final volume as 1, we have

$$1 \times 7/280 = p' \times 1/300$$
$$\therefore p' = 7 \times 30/28 = 7 \cdot 5,$$

or the pressure at the end of the operation is $7\frac{1}{2}$ atmospheres.

104. A quantity of air is contained in a straight vertical tube closed at the lower end, the air being shut off by a pellet of mercury, the weight of which may be neglected. When the temperature is 13°, the mercury is 66 cm. from the bottom of the tube. What will be its position when the temperature is 52°, and what is the temperature when the mercury stands at 63 cm.?

Since the tube is supposed to be uniform, the volume occupied by the air is proportional to the distance of the mercury from the bottom of the tube. Let d be the distance when the temperature is 52°: by equation (5)

$$\frac{d}{66} = \frac{273+52}{273+13} = \frac{25}{22},$$

and $$d = 3 \times 25 = 75 \text{ cm.}$$

Again, if t be the temperature when the mercury stands at 63 cm., by the same equation,

$$\frac{63}{66} = \frac{273+t}{273+13},$$
$$\therefore 273+t = 286 \times 63/66 = 273,$$

or the required temperature is 0°.

105. A quantity of a certain gas was collected and found to measure 54·02 c.c. at a temperature of 22°, the barometric height being 74 cm. On cooling down to 0° the volume became 49·3 c.c., the height of the barometer at the time being 75 cm.: calculate the coefficient of expansion of the gas.

Let α denote the coefficient of expansion; then, since the

volume of the gas at 0° and 75 cm. was 49·3 c.c., its volume at 0° and 74 cm. would, by Boyle's Law, be $49 \cdot 3 \times 75/74 = 49 \cdot 97$ c.c., and at 22° and 74 cm. the volume would become $49 \cdot 97 \times (1 + 22\alpha)$.

Thus

$$49 \cdot 97 \times (1 + 22\alpha) = 54 \cdot 02,$$

$$\therefore 49 \cdot 97 \times 22 \times \alpha = 54 \cdot 02 - 49 \cdot 97 = 4 \cdot 05,$$

and $$\alpha = 4 \cdot 05/49 \cdot 97 \times 22 = 0 \cdot 003685.$$

106. The pressure upon a gas is doubled, and at the same time the temperature is raised from 0° to 91°: how is the volume altered?

107. Find the volume at the normal pressure and temperature of a quantity of gas which measures 392 c.c. at 21°, and under a pressure of 80 cm. [See Ex. 101.]

108. The pressure upon a gas is doubled, and at the same time its temperature is raised from 13° to 299°: how does this affect its volume?

109. Compare the volumes of equal masses of air (1) at the normal pressure and temperature, and (2) at 10° and 85 cm. pressure.

110. 455 c.c. of air is measured off at 0°: it is then heated to 30°, and the pressure is reduced to one-half: what is now the volume?

111. A quantity of air at atmospheric pressure is compressed so that its volume is reduced to one-tenth, the temperature being raised from 23° to 37°: express the new pressure in atmospheres.

112. Compare the masses of equal volumes of air when measured (1) at 0° and 30 in. pressure, and (2) at 65° and 29 in. pressure.

113. Compare the mass of 100 cubic feet of air at 40° and under the atmospheric pressure, with the mass of 10 cubic feet of air at 0° and under a pressure of 20 atmospheres.

114. If the barometer falls from 75 cm. to 70 cm., what must be the alteration in the temperature of a

quantity of air originally at 17° in order that its volume may remain constant?

Note.—The density of dry air at N.P.T. is 0·001,293 gramme per cubic centimetre.

115. Show that the density of air at 76·8 cm. pressure and at 15° C. is 0·001239.

116. What is the mass of the air contained in a 500 c.c. flask at 10° and 73 cm. pressure?

117. The cubical content of a certain room is 750 cubic metres: calculate the mass of air contained in it at 17° and 77 cm. pressure.

118. The density of hydrogen is to that of air as 1 : 14·44: calculate the volume occupied by a gramme of hydrogen at 16° and 77 cm. pressure.

119. What is the weight of 10 litres of dry air at 14° and 74 cm. pressure?

120. The internal volume of a glass flask is 1 litre at 0°. It is filled with air at the normal pressure and temperature, and is then heated to 91° and opened under a pressure of 72 cm. Find the weight of the air that escapes.

121. Explain the construction of some practical form of air-thermometer, and the method of using it. What reasons have we for supposing that measurements of temperature made by such a thermometer are more reliable than those made by thermometers which depend upon liquid expansion?

122. If a mass of gas occupies a volume of 1750 c.c. at 8° and 79 cm. pressure, what will be its volume at 26° and 74 cm. pressure?

123. A flask containing air is corked up at 20°: find the pressure inside the flask after it has stood for some time in a steam-bath at 98°, the original pressure being the standard atmospheric pressure of 30 inches of mercury.

If the flask can stand a pressure of $2\frac{1}{2}$ atmospheres, at what temperature will it burst?

124. Compare the densities of the air at the bottom and top of a mine-shaft, when the temperatures and barometric pressures are 20° C. and 31 inches, and 5° C. and 30 inches respectively.

125. A quantity of air is collected at 0° and 76 cm. pressure. The pressure now increases to 78 cm.: what change of temperature will cause the volume to increase up to its original value?

126. Compare the masses of air contained in a room (1) when the temperature is 6° and the barometer stands at 78 cm., and (2) when the temperature is 20° and the pressure 73 cm.

127. If the volume of a gas at 13° be doubled, to what temperature must it be raised in order that the pressure may not be affected by the change of volume?

128. A cylindrical test-tube 10 inches in length and containing air at 0° is inverted over a mercury-bath and forced downwards until its upper (closed) end is level with the surface of the mercury in the bath, the barometric height at the time being 30 inches: to what temperature must the bath be raised in order that the air may fill the test-tube?

129. A mixture is made of 8 litres of hydrogen at 74 cm. pressure and 3 litres of oxygen at 76 cm., both gases being at a temperature of 14°. The volume of the mixture is reduced to 10 litres: show that the pressure is 82 cm., and prove that if the mixture is cooled to $-7°$ the pressure will be equal to the original pressure of the oxygen gas.

130. Equal quantities of air at temperatures t_1 and t_2 are contained in two hollow spheres whose radii are r_1 and r_2 respectively: prove that the pressures within the spheres are as $(1+at_1)/r_1{}^3 : (1+at_2)/r_2{}^3$, and that the whole pressures on the internal surfaces of the spheres are as $(1+at_1)/r_1 : (1+at_2)/r_2$.

131. A kilogramme weight is placed inside a bladder, which itself weighs 50 grammes. The bladder is then

partly filled with air at 0° and tied up, when its volume is exactly 1 litre: find the temperature at which it will just float in water, the expansion of the water itself and the weight of the air being neglected.

EXAMINATION QUESTIONS.

132. Define the coefficient of expansion of a substance with temperature. Calculate the extension, in inches, of a bar a mile long at 0° when heated from —40° to + 35°, the coefficient of expansion of its material being 0·000018. Matric. 1891.

133. Describe how to measure the coefficient of expansion of a solid by heat. A railway girder, 87 metres long, whose coefficient of expansion is 0·000018 per degree Centigrade, changes in temperature from − 10° C. to + 30° C. between winter and summer; calculate its change of length, and mark off, as well as you can, the actual extra length in your book.

Matric. 1892.

134. The coefficient of linear expansion of iron is 0·0000117. How much must the temperature of a block of iron be raised, in order that it may increase 1% in volume. Prel. Sc. 1890.

135. Define "coefficient of expansion." If the linear coefficient of expansion of iron is ·0000123, find the increase in the capacity (in litres) of a cylindrical steam engine boiler, which, at the freezing point is 7 metres in length and 3 in diameter, when heated from 15° to 150°. Matric. 1891.

136. A pendulum consists of a bob suspended by a steel wire, whose coefficient of expansion is $1 \cdot 24 \times 10^{-5}$. If the pendulum beat seconds at 0° C., find the number of seconds lost per day, supposing that the temperature is constant throughout the day and equal to 25° C.

Glasg. M.A. 1890.

137. Describe how to measure experimentally the

cubical expansion of a solid body, *e.g.* a crystal. If a crystal have a coefficient of expansion of ·0000013 in one direction, and of ·0000231 in every direction at 90° to the first, calculate its coefficient of cubical expansion. B.Sc. 1890.

138. How may the absolute expansion of any non-volatile liquid be directly determined? Explain why the balancing of a hot against a cold column eliminates the expansion of the vessel. If the cold column at 4° C. were 60 cms. high, and the hot column at 95° C. were $\frac{1}{3}$ cm. higher, what would be the absolute coefficient of cubical expansion of the liquid? S. K. 1891.

139. A thermometer is required to have a range of 130°; how much greater must the capacity of the bulb be than that of the stem? The apparent coefficient of expansion of mercury in glass is ·00015. What is meant by the apparent coefficient of expansion of mercury in glass? Prel. Sc. 1892.

140. If the coefficients of cubical expansion of glass and mercury are 0·000025 and 0·00018 respectively, what fraction of the whole volume of a glass vessel should be filled with mercury in order that the volume of the empty part should remain constant when the glass and mercury are heated to the same temperature?

S. K. 1892.

141. Describe the weight thermometer.

The weight of mercury required to fill a glass bulb when immersed in ice was 809·02 grammes, and when the bulb was placed in steam at 100° C. it was found that 12·012 grammes of mercury were expelled. Assuming the cubical coefficient of expansion of mercury to be ·0001813, find the linear coefficient of expansion of the glass. Camb. Schol. 1894.

142. The apparent mass of a piece of glass weighed in water at 4° is 25 grammes, its real mass being 37·5 grammes; its apparent mass when weighed in water at 100° is 25·486 grammes. The coefficient of cubical

expansion of glass per 1° C. is ·000026. Show that the volume of 1 gramme of water at 100° C. is 1·043 cubic centimetres. Ind. C. S. 1886.

143. Find the relation between the coefficients of linear and cubical expansion of a substance.

An iron bottle contains 20 lbs. of mercury at 0° C., but at 100° C. it only contains 19·72 lbs. The coefficient of linear expansion of iron is 0·000012. Find the coefficient of cubical expansion of mercury.

Camb. Schol. 1895.

144. * Experiments on the expansion of benzene gave the following results:—

Temperature.	Volume.
0°	1
20°	1·0241
40°	1·0500
60°	1·0776
80°	1·1070

Show that the coefficient of expansion can be represented by the formula $a+bt$, and determine the values of the constants a and b. B.Sc. 1884.

145. How is the apparent weight of a body affected by the air with which the body is surrounded?

For a hollow globe of glass with an external volume of two litres, find the apparent difference in the weight due to the fall of the barometer from 760 mm. to 750 mm., the temperature at the same time altering from 12° C. to 19° C. [The density of the air at 0° and 760 mm. may be taken at 0·00125 grammes per cubic centimetre.] The effect of the change of the pressure and temperature in the weight of the counterpoise may be neglected. Camb. Schol. 1894.

146. If 3000 cubic inches of air at 0° C. expand by 11 cubic inches for each degree rise of temperature, find the volume at 100° of a quantity of air which at 50° measures 100 cubic inches, the pressure being supposed to undergo no change. Matric. 1888.

147. Describe an experiment for showing that the

volume of a gas at constant pressure increases by approximately 1/273rd of its volume at 0° for each rise of 1° C. of temperature. The volume of a mass of gas at a pressure of half an atmosphere and temperature 15° C. is 150 c.c.; find the volume when the temperature is 303° C. and the pressure one atmosphere.

Matric. 1889.

148. What properties have liquids and gases in common, and what properties discriminate them from each other? What do you understand by the term "perfect gas"? A gramme of such a gas at 27° C. has the pressure on it halved, and is then cooled until it occupies the same volume as at first. What is its final temperature? Prel. Sc. 1892.

149. The mass of a cubic centimetre of hydrogen at 0° C. and atmospheric pressure is about 9×10^{-5} grammes. Find the pressure due to a gramme of hydrogen placed in a vessel having a capacity of 40 c.c. at 18° C.

Camb. B.A. 1888.

150. State the law connecting the pressure, volume, and absolute temperature of a gas.

A mass of air under a given pressure occupies 44 cubic inches at a temperature of 13° C. If the volume of the air be reduced to 24 cubic inches and the temperature raised to 39° C., show that the pressure will be doubled. Camb. B.A. 1889.

151. A mass of air under a given pressure occupies 24 cubic inches at the temperature of 39° C. If the pressure be diminished in the ratio of 4 : 3, and the temperature raised to 78° C., show that the volume of the air will be 36 cubic inches. Camb. B.A. 1889.

152. Determine the height of the barometer when a milligramme of air at 27° C. occupies a volume of 20 cub. cm. in a tube over mercury, the mercury standing 73 cm. higher inside the tube than outside. (1 gramme of air at 0° C. under a pressure of 76 cm. of mercury measures 773·4 cub. cm.) Int. Sc. 1885.

Let h denote the height of the barometer, then the pressure of the air in the tube over mercury is $h-73$. At this pressure, and at 27°, a milligramme of air measures 20 c.c., whereas at 0° and 76 cm. pressure it measures 0·7734 c.c. By equation (9), p. 126, we have

$$\frac{(h-73)\times 20}{273+27}=\frac{76\times 0{\cdot}7734}{273},$$

$$\therefore h-73=76\times 0{\cdot}7734\times 300/20\times 273=3{\cdot}23,$$

and the required barometric height is 76·23 cm.

153. At the sea-level the barometer stands at 750 mm., and the temperature is 7° C., while on the top of a mountain the barometer stands at 400 mm., and the temperature is – 13° C.; compare the weights of a cubic metre of air in the two places. Int. Sc. 1889.

154. Calculate the weight of air in a room 20 × 10 × 2 metres when the barometer stands at 77 centimetres and the centigrade thermometer at 15°.

If the room were at Cashmere, with the barometer at 42, and the thermometer at 25°, what weight of air would it contain?

(N.B.—·001293 gramme of air occupies 1 c.c. at 76 centim. and 0° C. The air may be taken in each case as dry, and no correction of the barometer for temperature need be made.) Prel. Sc. 1891.

155. * Distinguish carefully between a gas and a vapour. A mass of air is saturated with water vapour at a temperature of 100° C. On raising the temperature of the whole to 200° C. without change of volume, the pressure is found to be 2 atmospheres. Find the pressure at 0° C. of this volume of the dry air alone.

Int. Sc. (Hons.) 1889.

CHAPTER V

SPECIFIC AND LATENT HEAT

1. Specific Heat

1. A COIL of copper wire weighing 45·1 gm. was dropped into a calorimeter containing 52·5 gm. of water at 10°. The copper before immersion was at 99°·6, and the common temperature of copper and water after immersion was 16°·8. Find the specific heat of the copper wire.

The quantity of heat (Q) given out by a body of mass m and specific heat s in cooling through an interval of temperature θ is $Q = ms\theta$. Thus if s denote the specific heat of the copper, the amount of heat evolved by it in cooling from 99°·6 to 16°·8 is $45{\cdot}1 \times s \times (99{\cdot}6 - 16{\cdot}8)$.

Since the specific heat of the water is unity, the amount of heat required to raise its temperature from 10° to 16°·8 is $52{\cdot}5 \times 1 \times (16{\cdot}8 - 10)$, and as no heat is supposed to be gained or lost these two quantities are equal.

$$\therefore\ 45{\cdot}1 \times s \times 82{\cdot}8 = 52{\cdot}5 \times 6{\cdot}8,$$

and $$s = 357/3734{\cdot}3 = 0{\cdot}0956.$$

2. What is the temperature of an iron ball weighing 5 lbs., which, when immersed in 8 lbs. of water at 13°, raises the temperature to 48°? The specific heat of iron is 0·112.

If the temperature of the ball before immersion was $t°$, the

number of heat-units[1] which it gives out, in cooling to the final temperature of 48°, is $5 \times 0{\cdot}112 \times (t - 48)$.

Assuming that no heat is gained or lost during the experiment, this must be equal to the number of heat-units absorbed by the cold water, *i.e.* to $8 \times (48 - 13)$. Equating these two quantities, we have

$$5 \times 0{\cdot}112 \times (t - 48) = 8 \times (48 - 13),$$
$$\therefore\ 0{\cdot}56t = 8 \times 35 + (0{\cdot}56 \times 48)$$
$$= 280 + 26{\cdot}88 = 306{\cdot}88,$$

and $$t = 548^\circ.$$

3. In order to determine the specific heat of silver, a piece of the metal weighing 10·205 gm. was heated to 101°·9 and dropped into a calorimeter containing 81·34 gm. of water, the temperature of which was raised from 11°·09 to 11°·71. The water equivalent of the calorimeter, agitator, and thermometer employed was 2·91 gm.: find the specific heat of the silver.

The heat evolved by the hot body is $10{\cdot}205 \times s \times (101{\cdot}9 - 11{\cdot}71)$, where s is the specific heat of the silver. The heat is partly absorbed by the water and partly by the calorimeter, etc., and these are together equivalent to $(81{\cdot}34 + 2{\cdot}91)$ gm. $= 84.25$ gm. Since these are raised from 11°·09 to 11°·71, the heat absorbed is $84{\cdot}25\ (11{\cdot}71 - 11{\cdot}09)$.

Equating these quantities, we have

$$10{\cdot}205 \times s \times 90{\cdot}19 = 84{\cdot}25 \times 0{\cdot}62,$$

and $$\therefore\ s = 0{\cdot}05677.$$

[1] Since specific heat is merely a number, or a numerical ratio between quantities of heat, it is independent of the unit of mass (or weight) employed; and this is also true for latent heat. In the statement or solution of a problem it is a matter of indifference whether we take as our unit of mass the pound, the kilogramme, or the gramme, provided that we use this unit consistently, the corresponding units of heat in the three cases being the pound-degree (or amount of heat required to raise one pound of water through 1° C.), the kilogramme-degree, and the gramme-degree. The last, which is the C.G.S. unit, is sometimes called a "calorie," and Berthelot distinguishes the kilogramme-degree from this by calling it a "Calorie" (1 Calorie = 1000 calories).

4. The same piece of silver was heated to 102°·2 and immersed in 75·3 gm. of turpentine at 10°·98: the experiment was performed with the same apparatus as in Ex. 3, and the final temperature was 12°·47. Calculate the specific heat of the turpentine.

Taking the specific heat of the silver as 0·05677, the amount of heat which it gives out is

$$10{\cdot}205 \times 0{\cdot}05677 \times (102{\cdot}2 - 12{\cdot}47) = 51{\cdot}97.$$

Of this, $75{\cdot}3 \times s \times (12{\cdot}47 - 10{\cdot}98)$ is absorbed by the turpentine, s being its specific heat; and the calorimeter absorbs

$$2{\cdot}91 \times (12{\cdot}47 - 10{\cdot}98) = 4{\cdot}336.$$

Equating the amounts of heat absorbed and emitted,

$$51{\cdot}97 = (75{\cdot}3 \times s \times 1{\cdot}49) + 4{\cdot}336,$$

and
$$s = \frac{47{\cdot}634}{75{\cdot}3 \times 1{\cdot}49} = 0{\cdot}425.$$

5. A certain vessel holds 800 c.c. of water at its temperature of maximum density (4°). How much heat must be imparted to the water before it begins to boil?

6. Define specific heat. A body of mass M and specific heat S at a temperature T° is dropped into a mass m of a liquid of specific heat s at $t°$: prove that the final temperature is

$$\theta = \frac{\mathrm{MST} + mst}{\mathrm{MS} + ms},$$

and that, if the liquid is water,

$$\mathrm{S} = \frac{m(\theta - t)}{\mathrm{M}(\mathrm{T} - \theta)}.$$

7. How many units of heat are required to raise the temperature of 150 grammes of copper (of specific heat 0·095) from 10° to 150°?

8. How much heat is required to raise the temperature of a kilogramme of mercury (specific heat 0·033) from 20° to 170°?

9. If 342 units of heat are imparted to 150 grammes of iron (specific heat $= 0 \cdot 114$) originally at 10°, what will be the final temperature of the iron?

10. What amount of heat must be given to an iron armour-plate 2 metres long, 1 metre broad, and 20 cm. in thickness, in order to heat it from 10° to 140°? [Sp. gr. of iron, 7·7; sp. heat, 0·112.]

11. The thermal capacity (or water equivalent) of a body being defined as the product of its mass into its specific heat, calculate the capacity for heat of a copper calorimeter of 125 grammes. What special name is given to the thermal capacity of unit mass of a substance?

12. A body of mass M at a temperature T° is dropped into a mass m of a liquid of specific heat s contained in a calorimeter of mass m' made of a substance of specific heat s', both the calorimeter and the liquid being at t°: if the final temperature is θ, prove that the specific heat of the solid is

$$S = \frac{(ms + m's')(\theta - t)}{M(T - \theta)}.$$

[Expressions such as those developed in Examples 6 and 12 are convenient when a number of similar problems have to be solved, or in calculating the results of actual laboratory examples where corrections have to be applied; but the student will find that in general it is best to work out problems in specific and latent heat directly by equating the quantities of heat evolved by the hot body and absorbed by the cold body, as in the solved examples 1-4.]

13. Find the specific heat of a substance 100 grammes of which at 90°, when immersed in 250 grammes of water at 12°, gave a resulting temperature of 18°.

14. What is meant by the statement that the specific heat of water is thirty times the specific heat of mercury?

If a kilogramme of mercury at 120° is poured into a

vessel containing 200 gm. of ice-cold water, what will be the temperature after the whole is mixed? How would the weight and material of the vessel affect the result?

Note.—In Examples 15-18 the specific heat of mercury is to be taken as 1/30.

15. Two pounds of boiling water are poured upon ten pounds of mercury at 16°: what will be the common temperature after mixing?

16. Compare the thermal capacities of equal volumes of water and mercury, the density of mercury being 13·6.

17. A flask containing half a litre of mercury at 0° is immersed in boiling water, and allowed to remain there until the mercury has attained the temperature of the water: how many heat-units does it gain from the water?

18. Equal masses of boiling water and of mercury at $-5°$ are mixed together: prove that the resulting temperature is 96°·61.

19. 200 grammes of lead shot at 100° are dropped into 100 grammes of water at 9°·6. The temperature rises to 14°·9: what is the specific heat of the lead?

20. A kilogramme of mercury contained in a glass flask is heated by immersing the flask in a beaker of boiling water; it is then poured into a large flask containing 500 c.c. of water at 10°, and after shaking thoroughly the temperature is found to be 15°·6: find the specific heat of mercury.

What errors would probably occur in carrying out these operations, and how would they influence the result?

21. 105 grammes of copper are heated to 98°·5 and mixed with 90 grammes of water at 10°·3. The temperature after mixing is found to be 19°·2: what is the specific heat of copper?

22. $61\frac{1}{2}$ ounces of water are mixed with 100 ounces of alcohol, and the final temperature is found to be midway between the two initial temperatures: what is the specific heat of alcohol?

23. 60 grammes of iron nails at 100° are dropped into 120 grammes of water at 13°·2, and the final temperature is found to be 17°·8: what is the specific heat of the nails?

24. If you had at your command a supply of boiling water and of tap-water at 10°, what quantities of each would you take in order to prepare a bath containing 20 gallons of water at 35°?

25. A copper kettle weighing 2 lbs. and containing 6 lbs. of cold water is placed on a fire. Taking the specific heat of copper as 0·09, find in what proportion the heat absorbed from the fire is shared between the kettle and its contents.

26. A pound of boiling water is allowed to cool down to 10°: if all the heat given out were employed in warming 40 pounds of air, initially at 0°, to what temperature would it be raised? [Sp. heat of air = 0·237.]

27. Calculate the specific heat of silver from the following data:—

Weight of silver	10·2 gm.
Weight of water	84·0 ,,
Temperature of silver . . .	102°
Initial temperature of water . .	11°·08
Final temperature . . .	11°·69

28. A 7-lb. iron weight was taken out of an oil-bath and immediately immersed in 10 lbs. of water at 8°, whereupon the temperature rose to 20°. If the specific heat of iron is 0·112, what was the temperature of the oil-bath?

This suggests a method of measuring high temperatures, such as those of furnaces: how would you carry it out practically?

29. Equal volumes of turpentine at 70° and of alcohol at 10° are mixed together: find the resulting temperature.

[Sp. gr. of turpentine = 0·87, of alcohol = 0·80.
Sp. heat of turpentine = 0·47, of alcohol = 0·62.]

Let v be the volume taken, and θ the resulting temperature. The mass of the turpentine is $0{\cdot}87v$, and the amount of heat which it evolves in cooling from 70° to $\theta°$ is $v \times 0{\cdot}87 \times 0{\cdot}47(70-\theta)$. This is entirely spent in warming a mass $0{\cdot}8v$ of alcohol from 10° to $\theta°$, for which operation $v \times 0{\cdot}8 \times 0{\cdot}62 (\theta - 10)$ heat-units are required.

Equating these quantities, we have

$$0{\cdot}87 \times 0{\cdot}47(70-\theta) = 0{\cdot}8 \times 0{\cdot}62(\theta - 10).$$

Thus $$28{\cdot}623 + 4{\cdot}96 = \theta \times (0{\cdot}496 + 0{\cdot}4089),$$

and $$\theta = 33{\cdot}583/0{\cdot}9049, \quad = 37°{\cdot}11.$$

30. The densities of two substances are as 2 to 3, and their specific heats are 0·12 and 0·09 respectively: compare their thermal capacities per unit volume.

31. Assuming that the density of boiling water is 0·96, and that the density of mercury at 0° is 13·6, calculate the resulting temperature when equal volumes of boiling water and mercury at 0° are mixed.

32. The specific heat of air at constant pressure is 0·237, and a litre of air weighs 1·293 gramme: how much heat is given out by 50 litres of air in cooling from 25° to 5°?

33. Assuming the data given in the preceding question, find how many litres of air would be raised 1° in temperature if all the heat given out by a litre of water in cooling through 1° were imparted to the air. Mention any application of your results.

34. Hot air at 650° is used for superheating steam which is originally at 100°. The air and steam are kept at constant pressure during the operation, under which

circumstance their specific heats are 0·237 and 0·48 respectively, and they are introduced into the superheater in the proportion of 2 lbs. of air to 7 lbs. of steam. If the air is allowed to cool to 400°, to what temperature will the steam be raised?

35. What is meant by the statement that the specific heat of platinum is 0·03?

In order to ascertain the temperature of a furnace a platinum ball weighing 80 grammes is introduced into it; when this has attained the temperature of the furnace it is rapidly transferred to a copper vessel containing 400 grammes of water at 15°. The temperature of the water rises to 20°: what was the temperature of the furnace?

Discuss the errors that would arise through absorption of heat by the calorimeter, etc., in the above experiment, and the best methods of diminishing or allowing for them.

36. Three liquids, A, B, and C, are at temperatures of 30°, 20°, and 10° respectively. When equal parts (by weight) of A and B are mixed, the temperature of the mixture is 26°; and when equal parts by weight of A and C are mixed, the temperature is 25°. Prove that a mixture of equal parts of B and C will have a temperature of $16°\frac{2}{3}$.

Let S_a, S_b, and S_c denote the specific heats of the liquids A, B, and C respectively. The equation for the amounts of heat evolved and absorbed when equal parts of A and B are mixed reduces to

$$(30-26)S_a=(26-20)S_b,$$

and $$\therefore S_b=\frac{2}{3}S_a.$$

Similarly, when A and C are mixed,

$$(30-25)S_a=(25-10)S_b,$$

and $$\therefore S_c=\frac{1}{3}S_a.$$

If θ be the temperature of a mixture of equal parts of B and C,

$$(20-\theta)S_b = (\theta - 10)S_c,$$

$$i.e. \qquad (20-\theta)\cdot\frac{2}{3}\cdot S_a = (\theta-10)\cdot\frac{1}{3}\cdot S_a,$$

$$\therefore 40 - 2\theta = \theta - 10, \text{ and } \theta = \frac{50}{3} = 16^\circ\frac{2}{3}.$$

37. 10 grammes of a liquid at 90° were mixed with an unknown quantity of a second liquid of specific heat 0·25 and temperature 16°; the resulting temperature was 43°·75. If the specific heat of the first liquid was 0·45, what was the weight of the second?

38. A liquid of specific heat 0·54 and temperature 29° is mixed with another liquid of specific heat 0·36 and temperature 11°, and the final temperature was 17°. In what proportions were the liquids mixed?

39. Equal weights of three liquids, whose specific heats are s_1, s_2, s_3, and temperatures t_1°, t_2°, and t_3° respectively, are thoroughly mixed: find the temperature of the mixture.

40. C and C′ are two equal brass calorimeters, each weighing 40 grammes, and made of brass of specific heat 0·09. They contain two equal spirals S and S' of platinum wire, having the same resistance, so that when an electric current is passed through both equal quantities of heat are generated in each calorimeter. C contains 101·4 gm. of water, and C′ contains 86·4 gm. of turpentine. After a current has passed for quarter of an hour the temperature has risen 4°·35 inside C, and 11°·7 inside C′. What is the specific heat of the turpentine?

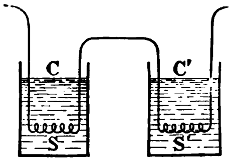

Fig. 5.

41. In order to find the specific heat of absolute alcohol, a quantity of it was boiled in a test-tube, and poured at

its boiling-point ($78^\circ\cdot5$) into a calorimeter containing 74 gm. of turpentine at $10^\circ\cdot6$. The calorimeter was weighed before and after the addition; the gain of weight was 14·7 gm., and the final temperature was $25^\circ\cdot2$. Find the specific heat of the alcohol, that of the turpentine being 0·466.

What advantage was there in using turpentine in this experiment?

42. 200 c.c. of water at 55° is poured into a copper calorimeter whose mass is 30 gm. and specific heat 0·095. Assuming that the calorimeter was previously at the temperature of the air, viz. 10°, and that the whole of the heat evolved by the water in cooling is absorbed by the copper, find the temperature to which the water is cooled.

43. 10 grammes of a metal were taken for a specific heat determination, and by a preliminary experiment it was found that the rise of temperature was insufficient, being only $3^\circ\cdot7$. The quantity of water in the calorimeter was reduced by one-third: what weight of the metal should now be taken to produce a rise of 12°, the other conditions remaining the same?

44. 40 gm. of water at 45° were poured into a leaden crucible weighing 300 grammes, which had previously been standing in a room the temperature of which was 16°. The water was cooled to $39^\circ\cdot46$: what is the specific heat of lead?

45. A calorimeter contains 20 grammes of water at 10°. 40 grammes of water at 40° are poured in, and the temperature after stirring is found to be 28°. What is the water-equivalent of the calorimeter?

46. In two experiments made to determine the specific heat of lead shot, the water equivalent of the calorimeter was 1·3, and that of the thermometer was 0·5. You are required to find the mean value from the results given, allowing for the heat absorbed by the calorimeter and thermometer.

	Exp. I.	Exp. II.
Weight of water . .	48·1 gm.	52·4 gm.
Weight of shot . .	60·9 ,,	90·0 ,,
Temperature of shot .	100°	100°
Initial temp. of water .	13°·0	14°·15
Final ,, ,, .	16°·2	18°·5

47. An audience of a thousand people is assembled in a concert-room on an evening when the external temperature is 2° C. In order to ventilate the room efficiently air has to be introduced at the rate of one-third of a cubic foot per second for each person present, and for the sake of comfort the air supplied has to be warmed to 15° C. Express in pound-degrees the amount of heat thus consumed in 2 hours, assuming that a cubic foot of air weighs 1·3 ounces, and that its specific heat is 0·24.

48. The specific heat of alcohol is 0·56, and that of mercury is 0·33: the density of alcohol is 0·82, and that of mercury is 13·6. Assuming these data, compare the rise of temperature produced by the application of a given amount of heat to a given volume of alcohol with that produced by the application of an equal amount of heat to the same volume of mercury. Consider also what further data you would require in order to institute a comparison between the suitabilities of the two liquids for thermometric purposes.

49. Describe the method employed by Regnault in determining the specific heats of gases at constant pressure, explaining in detail the construction of the heating apparatus and calorimeter, and the means adopted for obtaining a current of the gas at an uniform pressure.

50. If the quantity of heat required to raise unit mass of a substance from 0° to $t°$ be represented by

$$Q_t = at + bt^2 + ct^3,$$

show that the mean specific heat of the substance between $t°$ and $t'°$ is given by

$$S_t^{t'} = a + b(t+t') + c(t^2 + tt' + t'^2),$$

and that the true specific heat at t° is

$$S_t = a + 2bt + 3ct^2.$$

51. Regnault found that 100·5 units of heat were required to raise the temperature of unit mass of water from 0° to 100°, and 203·2 to raise its temperature to 200°; taking the specific heat of water at 0° as unity (*i.e.* putting $a = 1$ in the preceding equations), prove that the true specific heat of water at any temperature t°, between 0° and 200°, is given by the equation

$$S_t = 1 + 0{\cdot}00004t + 0{\cdot}0000009t^2.$$

Show also that the true specific heat of water at 150° is 1·02625.

2. Change of State and Latent Heat.

Note.—The latent heat of water is 80, and the latent heat of steam is 536.

52. How would you show that heat is absorbed when common salt is dissolved in water?

Anhydrous calcium chloride eagerly absorbs water, and dissolves in it with evolution of heat; whereas when crystallised chloride of calcium is dissolved in water, the temperature falls. How do you explain these facts?

53. A quantity of common salt is mixed with water, both being at 0° C. After solution the temperature is found to be below 0°, and if melting ice be employed instead of water, the fall in temperature is still more marked. Explain these results.

54. A glass flask is filled with a mixture of ice and water, and a narrow tube with an india-rubber stopper is fitted into the neck so as to force the water up to a certain height in the tube; the flask is then immersed in

lukewarm water. State exactly what will be observed in the tube.

55. Give a sketch of the reasoning by which it was predicted that the effect of a great increase of pressure would be to lower the melting-point of ice. Show how the prediction has been verified; and mention any important results arising from this fact.

56. If a fine wire with weights at its ends is hung over a block of ice at 0°, it is found that the wire cuts through the block, but that the ice reunites behind the wire, leaving the block still continuous. Explain this.

57. Define the latent heat of fusion of ice, and show that its numerical value may be deduced from the fact that when equal weights of boiling water and melting ice are intimately mixed, the ice all melts, and the resulting temperature is 10°.

58. How much ice at 0° will be melted by 500 grammes of boiling water?

59. How much hot water at 75° will just melt 10 lbs. of ice?

60. Equal weights of hot water and melting ice are mixed, and the result is water at 0°. What was the temperature of the hot water?

61. A quantity of ice is thrown into a basin containing 4 lbs. of water at 30°, and after all the ice has melted the temperature is found to have fallen to 8°: how much ice was thrown in?

62. How many grammes of ice must be dissolved in a litre of water at 20° in order to reduce its temperature to 5°?

63. 300 grammes of melting ice are mixed with 700 grammes of boiling water, and the resulting temperature is 46°: what is the latent heat of fusion?

64. Would a change in the thermometric scale affect the numerical value of the latent heat of a substance? The specific heat of lead is 0·0315 and its latent heat of fusion is 5·34 when the Centigrade scale is employed:

what are the corresponding numbers in the Fahrenheit scale?

65. A lump of iron of mass 750 grammes at a temperature of 400° is introduced into an ice calorimeter. 420 grammes of water are produced: what is the specific heat of the iron?

66. A 100-gramme weight of brass heated to 100° is dropped into an ice calorimeter, and 11·25 grammes of ice are melted: what is the specific heat of the brass?

67. In an experiment made to determine the latent heat of fusion of ice, 120 grammes of ice were dropped into a beaker containing 300 grammes of water at 50°. After all the ice was melted, the temperature of the water was found to be 13°: calculate the latent heat of fusion.

68. A ball of copper weighing 30 gm. was heated to 100° and placed in an ice calorimeter. In cooling down it evolved sufficient heat to melt 3·54 gm. of ice: what is the specific heat of copper?

69. 155 c.c. of water was obtained when a piece of iron at 100° was introduced into an ice calorimeter: if the mass of the iron was 1·08 kilogramme, what was its specific heat?

70. 100 c.c. of mercury are heated to 110°, and then poured into a cavity in a block of ice. It is found that 63 c.c. of water is obtained. Taking the density of mercury as 13·6, find its specific heat.

71. The bottom of a cylindrical vessel is covered by a layer of ice 1 decimetre thick. What must be the height of a column of boiling water which, when poured upon the ice, will just suffice to melt it? [Sp. gr. of ice = 0·917; sp. gr. of boiling water = 0·96.]

Let σ denote the sectional area (in sq. cm.) of the vessel, and h the height of the column of hot water. The volume of the water is $h\sigma$ c.c., its mass is $h\sigma \times 0{\cdot}96$ gm., and the number of heat-units which it evolves in cooling from 100° to 0° is $h\sigma \times 0{\cdot}96 \times 100$.

Again, the mass of the layer of ice is $10\sigma \times 0{\cdot}917$, and the number of heat-units required to melt it is $10\sigma \times 0{\cdot}917 \times 80$. Equating, we have

$$96h\sigma = 800 \times 0{\cdot}917\sigma,$$

and $$h = 733{\cdot}6/96 = 7{\cdot}64.$$

Thus a column of water 7·64 cm. high will give out just enough heat to melt the ice.

72. Assuming that the specific heat of ice is 0·5, its specific gravity 0·92, and the weight of a cubic foot of water 62·5 lbs., find how many pound-degree units of heat are required to convert a block of ice 1 ft. long, 6 in. thick, and 9 in. broad, at -10° into steam at 100°.

73. 1 cwt. of ice at 0° was taken into a warm room: some time afterwards it was found to have melted completely, and the water produced by the fusion was at 21°. Express the cooling effect of the ice in pound-degrees of heat.

If the air in the room mentioned was originally at 29° and finally at the temperature of the water, find how many pounds of air were cooled from 29° to 21° by the changes mentioned, taking the specific heat of air as 0·237.

74. Find the result of mixing 1 lb. of snow at 0° with 4 lbs. of water at 30°.

When snow (or ice) is mixed with water, one of two things must happen: either a portion only of the snow will be melted, in which case the mixture of snow and ice will have a temperature of 0°; or the whole of the snow will be melted, in which case the final temperature can be found as below. In solving such questions the student should first find by inspection whether the whole or part only is melted; otherwise he will only be able to form from the given data a single equation to find two unknown quantities.

The amount of heat required to melt 1 lb. of snow at 0° is 80 pound-degree units of heat, and the number of these heat-units which 4 lbs. of water would evolve, in cooling from 30° to 0°, is $4 \times 30 = 120$; since $120 > 80$, it is clear

that the whole of the snow will be melted. Let θ be the final temperature of the mixture: the heat evolved by the water is $4 \times (30 - \theta)$, and the heat absorbed by the snow (first in melting, and then in being raised θ°) is $80 + \theta$. Equating, we have

$$120 - 4\theta = 80 + \theta,$$

and $$\theta = 40/5 = 8.$$

Thus the result is 5 lbs. of water at 8°.

75. Find the result of mixing 2 lbs. of ice at 0° with 3 lbs. of water at 45°.

Only a portion of the ice will be melted, for the amount of heat which the water can give out in cooling to 0° is only $3 \times 45 = 135$; whereas $2 \times 80 = 160$ heat-units are required to melt all the ice. Thus the final temperature will be that of a mixture of ice and water, viz. 0°. If x denote the amount of ice melted,

$$80x = 3 \times 45, \text{ and } \therefore x = 1{\cdot}69.$$

$$\text{Result} \left\{ \begin{array}{lll} 4{\cdot}69 \text{ lbs. of water} \\ 0{\cdot}31 \quad ,, \quad \text{ice} \end{array} \right\} \text{all at } 0^\circ.$$

76. If 5 lbs. of snow be mixed with 2 lbs. of water at 60°, how much snow will be melted?

77. Explain clearly what is meant by the statements: "the specific heat of ice is 0·5," "the latent heat of water is 80." What is the result of mixing a pound of ice at -10° with a pound of water at 50°?

78. If a quantity of snow be dissolved in five times as much water at 25°, by how much will the temperature of the water be lowered?

79. Find the result of mixing 3 lbs. of melting ice with 7 lbs. of water at 60°.

80. What will happen if two lbs. of boiling water are poured upon two lbs. of snow at 0°?

81. Phosphorus melts at 44°·2, but can be cooled considerably below this temperature before solidification sets in. Show that if solidification commences at a temperature of 30°·7, just one-half of the mass will remain in the molten state. The latent heat of fusion

of phosphorus is 5·4, and its specific heat in the molten state is 0·2.

Let the quantity of melted phosphorus be denoted by unity, and let x be the amount which solidifies: on solidification the temperature of the whole rises suddenly, the result being a mass x of solid phosphorus, and a mass $(1 - x)$ of molten phosphorus, both at $44^\circ{\cdot}2$. The amount of heat evolved in the solidification is $5{\cdot}4\,x$, and the amount required to raise the whole mass to $44^\circ{\cdot}2$ is $(44{\cdot}2 - 30{\cdot}7) \times 0{\cdot}2$. Equating these quantities, we have $5{\cdot}4\,x = 13{\cdot}5 \times 0{\cdot}2$, and $x = 2{\cdot}7/5{\cdot}4 = 0{\cdot}5$, so that exactly one-half of the phosphorus remains in the molten state.

82. 10 gm. of phosphorus is melted and then gradually cooled to 26°, at which temperature solidification commences: find how much will remain in the liquid state.

83. A quantity of water is cooled down gradually to − 15°, and it then commences to freeze: assuming that the specific heat of water below the freezing point is unity, find how much ice will be produced.

84. A lump of ice weighing 80 grammes and at a temperature of − 10° is dropped into water at 0°. 5 grammes of water freeze on to the lump and the temperature of the ice rises to 0°. Calculate from the above the specific heat of ice.

85. Supposing the ground to be covered with snow of average density 0·2 to a depth of 10 cm., how much heat would have to be absorbed from the sun per square metre of surface in order to melt it?

86. Express (in centimetres) the minimum depth of rainfall which would be required to melt the snow in the preceding question, it being assumed that the rain has originally the temperature 10° and that it is cooled to 0°.

87. Discuss fully the relative advantages and disadvantages of the various methods of determining the specific heat of a substance, stating which you would

adopt when the amount of the substance at your disposal is very small. How would you proceed to determine the specific heat of a substance by means of Bunsen's ice calorimeter?

Theory of the Bunsen Ice Calorimeter.—The latent heat of fusion of ice is 80, *i.e.* 80 heat-units are required to melt 1 gramme of ice. Now since the density of ice is 0·91674, 1 c.c. of ice weighs 0·91674 gramme; hence the volume of a gramme of ice is $1/0{\cdot}91674 = 1{\cdot}0908$ c.c., and a mixture of ice and water will diminish in volume by 0·0908 c.c. for each gramme of ice melted. The application of a single unit of heat to such a mixture will cause a diminution in volume of $0{\cdot}0908/80 = 1/881$ c.c.; in other words, 881 heat-units are required to cause a contraction of 1 c.c.

Let a body of mass m, and specific heat s, be heated to t° and dropped into the calorimeter; in cooling down to 0° it will give out mst units of heat. If the observed diminution in volume of the mixture of ice and water be denoted by v, then $881v$ heat-units must have been evolved. Equating these two quantities, we have,

$$ms\theta = 881v,$$

an equation to find s.

[The value given above for the density of ice,—0·91674,—is that obtained by Bunsen. Bunsen also made a special determination of the latent heat of fusion of ice, and found it to be 80·025; other important determinations have given 79·24 (Regnault), and 79·25 (Person, De la Provostaye and Desains). The calculation is not quite correct, because we have assumed that a gramme of water at 0° has unit volume, whereas its volume is really somewhat greater, viz. 1·00012 c.c.]

88. 0·484 gm. of a metal at 100° is dropped into a Bunsen's calorimeter, and the thread of mercury moves backwards through 1·21 cm. in the capillary tube, the diameter of which is 0·6 mm.: assuming that 881 heat-

units are required to cause a contraction of 1 c.c., show that the specific heat of the metal is 0·06227.

Note.—In solving Examples 89-94 the value of the constant above given (881) should not be assumed, but each example should be worked out from the given data. The volume of a gramme of ice is to be taken as 1·09 c.c., and the latent heat of water as 80.

89. What change will be produced in the volume of a mixture of ice and water when 150 heat-units are imparted to it?

90. A mixture of ice and water was placed in a test-tube and occupied a volume of 30 c.c.; the test-tube was held in hot water until the volume had diminished to 29 c.c.: how much heat had been absorbed in the meantime?

91. A 10-gramme weight of brass heated to 100° and then dropped into a Bunsen ice calorimeter produces a contraction of 0·1021 c.c. What is the specific heat of the brass?

92. Find the specific heat of mercury, having given that 15 grammes of mercury at 100° produce a contraction of 0·0567 c.c. when introduced into an ice calorimeter.

93. What change of volume would be produced in an ice calorimeter by placing in it 2·5 gm. of a substance of specific heat 0·076 at a temperature of 100°?

94. Calculate the specific heat of a metal which gave the following results according to Bunsen's method:—0·96 gm. was heated to 99°·5, and dropped into the calorimeter; the thread of mercury retreated through a distance of 8·3 mm. in the capillary tube, whose bore had a cross-section of 1 sq. mm.

95. 10 grammes of water at 88° are placed in the inner tube of a Bunsen's calorimeter, and it is found that the volume of the contents of the outer portion decreases by exactly 1 c.c.; taking the latent heat of

water as 80, what value does this give for the specific gravity of ice?

96. 0·87 gramme of a substance was heated to 98°·6, and then dropped into a Bunsen's ice calorimeter; the movement of the mercury column showed that the volume of the mixture of ice and water surrounding the body had diminished by 7·9 cub. mm.: find the quantity of ice melted, and the specific heat of the substance from the data given. [Latent heat of fusion of ice = 80. Sp. gr. of ice = 0·917.]

97. It is said that the vapour of any liquid exerts a definite pressure which depends upon (1) the nature of the liquid, and (2) the temperature: how would you proceed to show that this is the case?

The surface of a liquid is exposed to the atmosphere, and heat is gradually applied until the pressure of the vapour of the liquid becomes equal to the atmospheric pressure: explain fully what occurs.

98. What do you understand by the expression "maximum tension of water-vapour" at a given temperature? How do you explain the existence of a maximum vapour-pressure at every temperature, and why does it increase with the temperature?

99. Describe briefly the methods which you would employ for measuring the maximum pressure of water-vapour (1) below 60°, (2) between 60° and 100°, and (3) above 100°.

100. How would you find by experiment (1) the minimum pressure at which a very volatile liquid such as liquefied sulphur dioxide can be kept exposed without boiling, and (2) the maximum pressure at which common ether will begin to boil, both substances being supposed to be kept at the atmospheric temperature?

101. Define the boiling point of a liquid, and distinguish between ebullition and evaporation. What condition determines whether a liquid will boil or evaporate?

102. State the laws which govern the phenomenon of boiling, and mention any abnormal cases in which these laws are not followed.

What precautions would you take (1) in order to make a liquid boil very regularly, (2) to make it boil at as low a temperature as possible?

103. Water at 15° is sprinkled upon the floor of a chamber which contains dry air at 15° and 76 cm. pressure: how will the pressure be affected (1) if the chamber is air-tight, (2) if there is free communication with the atmosphere? Will the temperature be affected at all? [The vapour pressure of water at 15° is 1·27 cm.]

104. The table below gives the maximum pressure of water-vapour in millimetres of mercury:—

4°	6·1	8°	8·0	12°	10·4
6°	7·0	10°	9·1	14°	11·9

State briefly how these numbers have been obtained; and find the actual pressure of the water-vapour present in a room at 14° when the dew-point is found to be 5°. What is the relative humidity of the air in the room?

105. 40 litres of air at 18° are passed through drying-tubes, and the increase of weight produced (by absorption of aqueous vapour) is 0·396 gm. Having given that 15·2 gm. of water-vapour is required to saturate a cubic metre of air at 18°, show that the relative humidity of the air in question is 0·65 (65 %).

106. It frequently happens that a cloud is precipitated when two currents of air at different temperatures mingle, although neither current is beforehand quite saturated with aqueous vapour. How do you explain this?

107. Distinguish carefully between saturated and unsaturated vapours. Into a vacuous cylinder provided with a piston there is introduced just enough water to saturate the space with vapour at 20°. Describe fully

what happens (especially with regard to pressure) under the following conditions. (1) The volume of the space is increased by pulling out the piston. (2) The volume is diminished by forcing the piston down. (3) The volume remaining as at first, the temperature increases to 30°. (4) The temperature falls to 10°.

108. Calculate the latent heat of steam from the results of the following experiment, allowing for the heat absorbed by the brass calorimeter:—

Weight of calorimeter	326·3 gm.
,, ,, + water . .	757·7 ,,
,, steam condensed . . .	46·35 ,,
Temperature of steam	100°
,, water before experiment	7°·5
,, ,, after experiment .	65°·2

Taking the specific heat of brass as 0·09, the water-equivalent of the calorimeter is $326·3 \times 0·09 = 29·4$ (*q.p.*)

The steam in condensing gives out $46·35x$ units of heat, x denoting the required value of the latent heat of steam. Further, in cooling from 100° to 65°·2 it evolves $46·35 \times 34·8 = 1612·9$ heat-units. These are together equal to the amount of heat required to raise the water and calorimeter from 7°·5 to 65·2, *i.e.* to

$$(431·4 + 29·4)(65·2 - 7·5) = 460·8 \times 57·7 = 26588·2.$$

Thus $\qquad 46·35x + 1612·9 = 26588·2,$

and $\qquad x = 24975·3/46·35 = 538·8.$

109. The condenser of a steam-engine is supplied with injection water at a temperature $t°$, and the steam enters the condenser at 100°. How many pounds of injection water must be supplied for every pound of steam condensed, in order that the water may leave the condenser at a temperature T° ?

1 lb. of steam in condensing to water at 100° sets free 536 units (pound-degrees) of heat, and in cooling from 100° to T° it further gives out $(100 - \text{T})$ heat-units. These are together equal to $x(\text{T} - t)$, the amount of heat absorbed by

the injection water, x being the required weight of water, which is heated from $t°$ to T°.

Thus $$x(T - t) = 536 + (100 - T),$$
and $$x = (636 - T)/(T - t).$$

110. What is meant by saying that the latent heat of steam is 536? How many heat-units are required to convert 50 grammes of water at 12° into steam at 100°?

111. A vessel containing 30 gm. of ice is placed over a spirit-lamp: how much heat will be required to melt it and vaporise the water completely?

112. How many pounds of steam at 100° will just melt 50 pounds of ice at 0°?

113. 10 gm. of steam at 100° is condensed in a kilogramme of water at 0°, and the temperature of the water is thereby raised to 6°·3: what value does this give for the latent heat of steam?

114. How many grammes of steam at 100° must be passed into 200 gm. of ice-cold water in order to raise it to the boiling point? What will happen if more steam than this is passed in?

115. m grammes of steam at 100° are passed into M grammes of water at $t°$, thereby raising its temperature to $t'°$. Show that the value of the latent heat (x) of steam is given by the equation

$$x = \frac{M}{m}(t' - t) - 100 + t'.$$

116. Into a calorimeter containing 120 gm. of water at 10° steam at 100° is passed. The total weight of steam condensed is 5 gm., and the final temperature of the water in the calorimeter is 35°. What value does this give for the latent heat of steam?

117. The calorific power of Welsh steam coal is 8240. Show that the heat evolved in the combustion of 1 lb. of this coal is sufficient to convert 15·4 lbs. of water at 100° into steam at the same temperature.

118. 1 lb. of a certain sample of coal is found to be

sufficient to evaporate 15 lbs. of water at 100°: how much heat does it give out in burning?

119. Compare the amount of heat required to convert a given mass of ice at $-3^\circ\cdot2$ into water at 38°, with that required to convert the same mass of water at 38° into steam at 100°. [The specific heat of ice is 0·5.]

120. A kilogramme of cold ice is taken at -10°, and heat is continually applied to it until a temperature of 1000° is attained: trace the successive effects produced, stating the amount of heat required for each of these effects.

121. Calculate the latent heat of steam from the results of the following experiment, made with the same apparatus as that of Ex. 108:—

Weight of calorimeter	326·3 gm.
,, ,, + water . .	915·3 ,,
,, steam condensed . . .	28·38 ,,
Temperature of steam	100°
,, water before experiment	2°·8
,, ,, after experiment .	30°·8

122. A calorimeter weighing 150 gm., and made of silver of specific heat 0·056, contains 350 gm. of water at 8°. If 10 gm. of steam at 100° is passed into the water, what will be the temperature after equilibrium has been attained, supposing no heat to be lost or gained?

123. It has been proposed that the condensation of steam on a cold body should be used as a means of measuring its specific heat. In an experiment made according to this method, a piece of cast zinc weighing 48·3 grammes, at a temperature of 10°·7, was immersed in a current of steam at 100°, and was found to condense 0·762 gramme of steam. Calculate the specific heat of zinc, taking the latent heat of steam as 536?

EXAMINATION QUESTIONS

124. Define "specific heat," and state how it is measured for any substance by the method of mixtures,

explaining carefully how the quantity calculated from the observation is really the specific heat as defined.

Calculate the specific heat of a solid from the following data :—A fragment of it, weighing 150 grammes, is heated to 100° C., and put into 500 grammes of water at 12° C., contained in a vessel whose heat-capacity is equivalent to 40 grammes of water; the temperature of the whole thereby becomes 15° C. Prel. Sc. 1890.

125. Explain fully how the specific heat of a substance is obtained by the method of mixtures, pointing out what is meant by "the water equivalent of the calorimeter."

Into a calorimeter at a temperature of 16° C. are thrown 8·5 grammes of water at a temperature of 84° C., and the temperature of the two becomes 18°·5 C.; find the water equivalent of the calorimeter.

Camb. Schol. 1894.

126. Define the specific heat of a substance, and the thermal capacity of a body. A certain stone, heated to 100° C., and dropped into 10 lbs. of water at 0°, raises the temperature of the water by 5°. What effect would it have produced had there been 15 lbs. of water at a temperature of 30°? Edinb. M.A. 1890.

127. What will be the result of placing (*a*) 5 lbs. of copper at 100° C., (*b*) 30 lbs. of copper at 80° C., in contact with 1·5 lbs. of ice at 0° C.? [Specific heat of copper = 0·1.] Camb. B.A. 1891.

128. A mass of 200 grammes of copper, whose specific heat is ·095, is heated to 100° C. and placed in 100 grammes of alcohol at 8° C., contained in a copper calorimeter, whose mass is 25 grammes, and the temperature rises to 28°·5 C. Find the specific heat of alcohol. Int. Sc. 1889.

129. A mass of 700 grammes of copper at 98° C., put into 800 grammes of water at 15° C., contained in a copper vessel weighing 200 grammes, raises the temperature of the water to 21° C. Find the specific heat of copper. S. K. 1889.

130. A concave mirror whose area is 1 sq. ft. is placed in the sunshine and the sun's rays are brought to a focus by it upon a copper calorimeter containing water. The mass of the copper is 1 oz. and it contains 2 oz. of water. The temperature of the water is found to rise 30° F. in 5 minutes. Find the amount of heat in thermal units received by the earth per square yard per minute and find the equivalent in H.P. Specific heat of copper $= \cdot 1$. Camb. B.A. 1893.

131. Calculate the specific heat of a liquid from the following data:—A piece of copper is heated to 100° C., and quickly immersed in 150 gms. of water in a calorimeter (water-equivalent, 10 gms.), whereby the temperature is raised from 14°·3 C. to 17°·8 C. A similar experiment is made with the same piece of copper when the water in the calorimeter is replaced by 180 gms. of the liquid, in which case the alteration in temperature is from 15°·8 to 18°·7. Vict. Int. Sc. 1890.

132. *A copper globe 4 centimetres in diameter is hung up in the air to cool. It loses heat at the rate of $\frac{1}{4000}$ of a gramme water degree Centigrade unit per square centimetre per second per degree difference of temperature of cooling body and surroundings. Its initial temperature is 75° C., and that of the surrounding enclosure is 15°, and remains constant. Find the temperature of the globe after 20 minutes. The capacity for heat of the globe is 28·3 gramme water units.

Glasg. M.A. 1890.

133. A piece of iron weighing 16 grammes is dropped at a temperature of 112°·5 C. into a cavity in a block of ice, of which it melts 2·5 grammes. If the latent heat of ice is 80, find the specific heat of iron. S. K. 1892.

134. Twenty-five grammes of water at 15° C. are put into the tube of a Bunsen ice calorimeter, and it is observed that the mercury moves through 29 centimetres. Fifteen grammes of a metal at 100° C. are then placed

in the water and the mercury moves through 12 centimetres. Find the specific heat of the metal.

S. K. 1893.

135. A lump of lead weighing two pounds, and heated to boiling water temperature, is dropped into a hole cut in a block of ice. How much ice is melted? [Specific heat of lead = 0·032.] Glasg. M.A. 1887.

136. Explain the statement that the latent heat of water is 80. To a pound of ice at 0° are communicated 100 units of heat (pound-degrees Centigrade). What change of temperature does the ice undergo, and in what way is its volume altered? Matric. 1888.

137. Describe an experiment by which you would show that ice contracts when it melts, and that the resultant water goes on contracting if it be warmed.

Matric. 1889.

138. If dry ice, at its melting point, be thrown into 20 lbs. of water at 60° C. until the whole weighs 23 lbs., what fall of temperature will have been thereby produced? How much mechanical work must be done on the water in order to restore its original temperature?

Matric. 1891.

139. What do you understand by the expressions: thermal unit, specific heat, temperature?

It is found that 63·2 grammes of copper at 50° C. will just melt 3·8 grammes of ice. Given that the latent heat of fusion of ice is 79, find the specific heat of copper. Camb. B.A. 1888.

140. Explain carefully the statement that the latent heat of fusion of water is 80. What is the unit in terms of which latent heat is measured? Trace the changes in the temperature and volume of a kilogramme of ice at − 5° C., to which heat is applied until it is converted into steam. Int. Sc. 1888.

141. A gramme of ice at 0° contracts 0·091 c.c. in becoming water at 0°. A piece of metal weighing 10 grammes is heated to 50°, and then dropped into the

calorimeter. The total contraction is ·063 c.c.: find the specific heat of the metal, taking the latent heat of ice as 80. Vict. B.Sc. 1886.

142. What experiments are required to prove Dalton's law as to the pressure of vapours and gases? Enough water is placed in a closed vessel to saturate the air it contains at 15°, and the pressure of the dry air in the vessel at 0° is 760 mm. Find the pressure at 25°. The saturating pressure of vapour at 15° is 16·1 mm. Matric. 1890.

143. Define latent heat.

How much ice at 0° C. would a kilogramme of steam at 100° C. melt if all the resulting water was at 0°? S. K. 1894.

144. How are units of heat measured? When are two bodies said to be in thermal equilibrium?

It is found that one pound of steam at 100° C., when passed into 15 lbs. of water at 0° C., raised the temperature of the water to 40° C. From these data calculate the latent heat of steam. Camb. B.A. 1888.

145. Distinguish between calorimetry and thermometry.

20 grammes of steam at 100° C. are condensed in a metal worm surrounded by 200 grammes of water at 10° C. If the water equivalent of the worm be 10 grammes, and the latent heat of steam be 536, determine the temperature to which the water is raised. Matric. 1886.

146. Explain the term *latent heat.* If 25 grammes of steam at 100° C. be passed into 300 grammes of ice-cold water, what will be the temperature of the mixture, the latent heat of steam being taken as 536. Matric. 1887.

147. What is meant by the statement that the "latent heat of steam is 536"?

Steam is passed into 100 grammes of water at 15° in a calorimeter If the mass of the water in the cal-

orimeter be by this means increased by 10 grammes, find the final temperature, supposing no heat to have been lost, and that the heat taken by the calorimeter may be neglected. Prel. Sc. 1889.

148. Distinguish carefully between conduction and convection of heat, and explain how each is involved in heating a house by means of steam pipes. If the latent heat of vaporisation of water at 100° C. be 537, how much heat per hour would be given out by a stack of pipes in which 0·12 kilogrammes of steam, at atmospheric pressure, was condensed per minute?

Matric. 1892.

149. A calorimeter whose capacity for heat is 48 water-grammes has 352 c.c. of water in it, and the whole weighs 882 grammes. Into this steam at atmospheric pressure is condensed till its temperature rises from 12·2° C. to 18·7° C., and on weighing again the calorimeter weighs 886·2 grammes. Calculate the latent heat of vaporisation of water. Int. Sc. 1890.

150. Alcohol boils at 78° C., its latent heat of evaporation is 202, and its mean specific heat, when liquid, is ·65; calculate the least quantity of water at 10° C. needed to condense 100 gr. of alcohol from vapour at 78° C. into liquid at 15° C. Matric. 1891.

151. State Dalton's law of the pressure of mixed gases and vapours, and explain how it is proved.

Examine the accuracy of the following statement: "The numerical value in millimetres of the pressure of aqueous vapour in air is the same as the mass in grammes of the aqueous vapour in one cubic metre of the air." (Density of dry air at 0° and 760 mm. = 0·001293 gm. per c.c. Specific gravity of aqueous vapour referred to air at same temperature and pressure = ·622.)

Nat. Sc. Tripos 1889.

152. The latent heat of steam is 537 when the Centigrade scale is used. A certain vessel, suspended by strings and coated with lampblack, is filled with

water at 90° C. It is found that, if steam is passed in and condensed at the rate of 1 gramme per hour, the temperature of the water can be kept constant at 80° C. What is the loss of heat per hour, in what way does it take place, and how can the rate of loss be diminished?

Ind. C. S. 1889.

153. What is known about the latent heat of vaporisation of water at different temperatures?

If a boiler receives 30,000 units of heat per minute through every square metre of its fire-box surface, the total surface being, say, 5 square metres; and if its temperature be 140° C., while it is fed with condensed water at 45°; what weight of steam would you expect to be able to regularly draw off per hour? The latent heat of vaporisation of water at 140° C. is 509.

Int. Sc. 1891.

154. Discuss the conditions that determine the temperature of the wet bulb of a wet-and-dry bulb hygrometer. Calculate the hygrometric state of the air from the following data:—

Actual temperature	. . .	14°·5
Temperature at which dew appears		9°·2
Vapour Tension of water at 5°	.	·0087 atm.
,, ,, at 10°	.	·0122 ,,
,, ,, at 15°	.	·0169 ,,

Explain exactly what you mean by a "hygrometric state," and why the number which you give expresses it.

Int. Sc. (Hons.) 1891.

CHAPTER VI

CONDUCTIVITY AND THERMODYNAMICS

Conductivity.—The thermal conductivity (or coefficient of conductivity) of a substance is measured by the number of units of heat which pass in unit time across unit area of a plate whose thickness is unity, when its opposite faces are kept at temperatures differing by one degree. In the statement of this definition it is supposed that the flow of heat has become *steady*, and that the lines of flow of heat are perpendicular to the surfaces of the plate.

The thermal conductivity of a substance, in the C.G.S. system, is measured by the quantity of heat which flows per second, under these circumstances, across one square centimetre of a plate one centimetre in thickness, the opposite faces of the plate being kept at temperatures differing by 1° C.

The quantity of heat (H) which flows in a given time across a plate of given dimensions is inversely proportional to the thickness (d) of the plate, and is directly proportional to the area of its surface (s), to the difference of temperature (θ) between its opposite faces, and to the time t. If the thermal conductivity of the substance be denoted by k, then

$$H = k \cdot \frac{s\,\theta\,t}{d} \quad . \quad . \quad . \quad . \quad (1)$$

1. A large tank is covered with a layer of ice 6 cm.

thick and 24 square metres in area: assuming that the coefficient of conduction of ice in C.G.S. units is 0·003, determine the amount of heat transmitted per hour from the water up through the ice, the upper surface of which is at the temperature of the air, viz. -10° C.

In order that the answer may be given in gramme-degrees of heat, we must express all quantities in terms of the corresponding C.G.S. units. Thus $s=24$ sq. metres $=$ 240,000 sq. cm., and $t=60$ minutes $=3600$ seconds. Substituting these values in equation (1), we have

$$\begin{aligned} H &= 0\cdot003 \times 240{,}000 \times 10 \times 3600/6 \\ &= 4{,}320{,}000 \text{ units.} \end{aligned}$$

2. The top of a steam chamber is formed of a stone slab 6 decimetres long, 5 decimetres broad, and 1 decimetre thick. Ice is piled upon the slab, and it is found that 5 kilogrammes of ice are melted in half an hour: what is the thermal conductivity of the stone?

Since the latent heat of fusion of ice is 80, the amount of heat required to melt 5 kilogrammes of ice is

$$H = 5 \times 1000 \times 80 = 400{,}000 \text{ heat-units.}$$

This amount of heat is transmitted in 1800 seconds through a slab of $60 \times 50 = 3000$ sq. cm. area and 10 cm. thickness, its opposite faces being kept at 0° and 100° degrees respectively. Thus

$$400{,}000 = k \times 3000 \times 100 \times 1800/10,$$

and

$$k = 4/540 = 0\cdot00741.$$

3. The coefficient of conduction of copper is 0·96: how many heat-units will pass per minute across a plate of copper 1 metre long, 1 metre broad, and 1 cm. thick, when its opposite faces are kept at temperatures differing by 10°?

4. The wall of a room is 20 cm. thick, and is built of stone whose thermal conductivity is 0·008; the temperature inside the room is 18°, while the outside

temperature is 2°: how much heat is lost by transmission per hour across each square metre of the wall?

5. A metal plate 2 feet long, 1 foot broad, and three-tenths of an inch thick has one face kept at a temperature of 40°, while the other face is kept at 10°. It is found that under these circumstances the heat transmitted across the plate in 10 minutes would suffice to raise the temperature of 5184 pounds of water through one degree. Taking as units the inch, pound, and second, express the conductivity of the metal.

6. It is found that $1 \cdot 44 \times 10^7$ heat-units are transmitted per hour across an iron plate 2 cm. thick and 500 sq. cm. in area, when its opposite sides are kept at 0° and 100° respectively: what is its coefficient of conductivity?

7. The walls of a cottage are 3 decimetres thick, and are built of materials having a thermal conductivity of 0.0035. The temperature inside the cottage is kept at 15°, while the outside temperature is 5°. The area of the walls is 1000 square metres: find how much heat is lost by conduction per hour, and what is the minimum quantity of coal of calorific power 8400 that must be burned in order to keep the temperature constant.

8. An iron boiler is made of plate 0·8 cm. thick, and its total exposed surface is 8 square metres: the water inside is at a temperature of 120°, and the exposed surface of the boiler is at 95°. Assuming that the thermal conductivity of the iron is 0·164, find how much heat is lost by conduction per hour.

9. A metal plate, 1 sq. decimetre in area and 0·5 cm. thick, has the whole of one face covered with melting ice, while the other face is in contact with boiling water. The coefficient of conductivity of the metal is 0·14: how many kilogrammes of ice will be melted in an hour?

10. "In a solid, heat may be transmitted from point to point in two ways, and in a fluid in three ways." Discuss this statement, and comment upon the following

facts : When a sheet of glass is held in front of a hot stove it appears to cut off the heat given out by the stove ; but when the sun shines upon the glass windows of a greenhouse the heat passes readily through without producing any considerable change in the temperature of the glass itself. Explain carefully how it is that the air inside the greenhouse may in this way become hotter than the outside air.

11. An iron vessel containing a kilogramme of ice is partially immersed in a tank of water at 15°, so that the total area of the immersed surface is 400 sq. cm. The mean thickness of the wall of the vessel is 0·8 cm., and exactly one minute after immersion all the ice is found to have melted. Calculate from this the thermal conductivity of iron, and discuss the validity of any assumptions made in your solution.

12. Péclet states that the quantity of heat which passes in an hour through a plate of lead 1 square metre in area and 1 metre thick, with a difference of 1° between the temperatures of its surfaces, is 13·83 kilogramme-degrees : what value does this give for the C.G.S. coefficient of conductivity for lead ?

13. An iron boiler is 1 cm. thick, and has a heating surface of 2 square metres. The water in the boiler is at 100°, and the heating surface is kept at 280°. Taking the latent heat of steam as 536, and the conductivity of iron as 0·16 : find how much water can be evaporated per hour.

14. Point out the experimental difficulties which would be met if you attempted to carry out practically the idea contained in the definition of conductivity given on p. 172. Describe the principle and the general results of Forbes's experiments, indicating how the calculations were made ; and find the value of the multiplier for reducing to the C.G.S. system coefficients of conductivity expressed in kilogramme-degrees per square metre, per millimetre, per second.

Mechanical Equivalent of Heat.—The amount of work which is equivalent to one heat-unit is called the mechanical equivalent of heat. Its value was first accurately determined by Joule (whence it is sometimes called "Joule's equivalent"), and is denoted by the letter J.

Joule found that 772·5 foot-pounds of work were required to raise the temperature of a pound of water through one degree Fahrenheit; the corresponding number in terms of the degree Centigrade is $772 \cdot 5 \times 9/5$, or 1390·5. More recent experiments by Rowland, Miculescu, Griffiths, Schuster and Gannon indicate that the numbers 778 (Fahr.) and 1400 (Cent.) are nearer the truth. Since 1 foot = 30·48 cm., the mechanical equivalent is $1400 \times 30 \cdot 48 = 42672$ gramme-centimetres per calorie or gramme-degree, or about 427 kilogramme-metres per kilogramme-degree of heat.[1] To convert this into absolute measure we have to multiply by g ($=981$), which gives 42672×981, or $4 \cdot 196 \times 10^7$ ergs per gramme-degree. We shall adopt in our calculations the approximate value $J = 4 \cdot 2 \times 10^7$.

It should be noticed that all the above numbers—778, 1400, 427, etc.—with the exception of the last ($4 \cdot 2 \times 10^7$), give the value of J *in gravitation measure*, and care should be taken to make the required conversion into absolute measure (or *vice versa*) where it is necessary. In working problems relating to potential energy, or the energy of bodies falling from a height, it is convenient to use gravitation measure; but it is preferable to work in absolute measure throughout.

[1] Observe that the value of J is not affected by a change in the unit of *mass* employed, for it is a ratio between an amount of work and a corresponding amount of heat, and the unit of mass is involved in the same degree (*i.e.* to the first power) in the units of work and heat. On this account the usual statement that the "mechanical equivalent of heat is 1390 foot-*pounds*" is somewhat misleading.

The energy of a body of mass m moving with velocity v is $mv^2/2$ in dynamical measure (pp. 40-43). The thermal equivalent of this is $mv^2/2J$. Suppose the body to meet an obstacle and to fall dead. Also let s denote its specific heat, and let us assume that all the heat developed by its impact goes to raise its temperature through (say) $\theta°$. The amount of heat required to produce this rise of temperature is $ms\theta$. Thus

$$mv^2/2J = ms\theta,$$

and $$\theta = v^2/2Js.$$

Similarly, if the body be raised to a height h above the ground, its potential energy is mgh ergs, and the thermal equivalent of this is mgh/J. If the body falls to the ground, and if we suppose that all the heat produced by the arrest of its motion is spent in warming it, the rise of temperature produced will be

$$\theta = gh/Js.$$

In both cases the elevation of temperature is independent of the mass of the body.

15. A leaden bullet of specific heat 0·032 strikes against an iron target with a velocity of 400 metres per second. If the bullet falls dead, and the heat produced is equally divided between it and the target, find its temperature, supposing it originally at 10°.

If the mass of the bullet be m, its kinetic energy is $(40{,}000)^2 \times m/2$, and the equivalent of this in heat-units is

$$(40{,}000)^2 \times m/2 \times 4{\cdot}2 \times 10^7 = 80m/4{\cdot}2.$$

Half of this goes to heat the bullet. Suppose the rise of temperature produced is $\theta°$, then

$$40m/4{\cdot}2 = m \times 0{\cdot}032\,\theta,$$

and $$\theta = 40/4{\cdot}2 \times 0{\cdot}032 = 297{\cdot}7.$$

Since the bullet was originally at 10°, its temperature after striking the target is 307°·7.

16. How much heat is set free when a body of mass 100 grammes, moving at the rate of 25 metres per second, is suddenly brought to rest?

17. From what height must a raindrop fall to the ground in order that its temperature may be raised 5° C.? [Take $J = 1400$.]

18. Find the equivalent in ergs of the amount of heat required to raise 1 lb. of water through 1° C.

19. A block of ice is dropped into a well of water, both ice and water being at 0°. From what height must the ice fall in order that one-fiftieth of it may be melted? [Take $J = 427$.]

20. Mercury falls from a height of 10 metres upon a perfectly non-conducting surface. How much warmer will it be after the fall? [Sp. heat = 0·033.]

21. A lead bullet falls from a height of 50 metres to the ground. Taking the specific heat of lead as 0·031, and assuming that half the heat generated goes to warm the bullet while the rest is lost, find the rise of temperature.

22. How many heat-units are required to raise the temperature of a kilogramme of iron [specific heat = 0·112] through 40°? If the equivalent amount of kinetic energy were imparted to it, what would be its velocity?

23. The heat of combustion of hydrogen is 34,460 calories, *i.e.* 1 gramme of hydrogen in burning gives out enough heat to raise the temperature of 34,460 grammes of water through 1°. Express in watts (p. 4) the power which would be obtained if the heat produced by the combustion of 50 grammes of hydrogen per hour were completely converted into work.

24. Power equal to 100 watts is entirely converted by friction into heat: how much water can be heated in this manner through 1° in an hour?

25. In one of Rumford's experiments on the boring of brass cannon, the heat developed by a horse working for 2½ hours was found to be sufficient to raise the tem-

perature of 26·5 lbs. of water from 0° to 100°. Calculate the number of foot-pounds of work done, and the rate at which the horse worked.

26. A cannon-ball moving at the rate of 800 feet per second strikes against a target, and the heat produced is equally divided between the target and the ball: supposing the latter to be made of iron of specific heat 0·112, prove that its temperature will be raised by 32°.

27. Lead melts at 335°, and its latent heat of fusion is 5·4. Taking its mean specific heat as 0·032, calculate the equivalent (in ergs) of the amount of heat required to raise the temperature of 10 grammes of lead from 0° to its melting point, and to melt it.

28. With what velocity must a leaden bullet, at a temperature of 15°, strike a target in order that the heat produced may be just sufficient to melt it? (Assume that all the energy goes to heat the bullet.)

29. In obtaining work from an engine at the rate of 20 H.-P., 56 lbs. of coal are consumed per hour: find the efficiency of the engine, assuming that the heat produced by the combustion of 1 lb. of coal is sufficient to convert 15 lbs. of water at 100° into steam at the same temperature.

The *efficiency* of a steam engine is the ratio between the useful work performed and the work represented by the heat of combustion of the fuel.

The work done per minute by the engine is

$$20 \times 33{,}000 = 660{,}000 \text{ ft.-lbs.}$$

The combustion of 1 lb. of coal produces

$$15 \times 536 = 8040 \text{ thermal units.}$$

Since 56 lbs. of coal are consumed per hour, the amount of heat evolved per minute is

$$56 \times 8040/60 = 7504,$$

which is equivalent to $7504 \times 1400 = 10{,}505{,}600$ ft.-lbs. Thus the efficiency of the engine is

$$E = 660000/10505600 = 0{\cdot}06279$$

(or 6·279 per cent).

30. The calorific power of a certain kind of coal is 7786: what is the mechanical equivalent of this in ergs per kilogramme of coal burned?

An engine burning such coal draws a train of total weight 100,000 kilogrammes slowly along a level line on which the resistances amount to 1/200th of the weight. If the efficiency of the engine is 0·05 (5 per cent), how far will the combustion of 9 kilogrammes of coal suffice to draw the train?

The combustion of 1 gramme of the coal produces 7786 gramme-degrees of heat: this is equivalent to $7786 \times 4 \cdot 2 \times 10^7 = 3 \cdot 27 \times 10^{11}$ ergs. Hence the mechanical equivalent in ergs per kilogramme of coal burned is $3 \cdot 27 \times 10^{14}$.

The resistance to be overcome in the motion of the train is the weight of $\frac{100{,}000}{200} = 500$ kilogrammes. If the train is drawn x metres, the work done is $500x$ kilogramme-metres $= 500x \times 10^5$ gramme-centimetres $= 5x \times 10^7 \times 981$ ergs.

Now the useful work done per kilogramme of coal consumed is

$$\frac{1}{20} \text{ of } 3 \cdot 27 \times 10^{14}.$$

Hence the distance x through which the train is drawn during the consumption of 9 kilogrammes of coal is given by the equation

$$5x \times 10^7 \times 981 = 9 \times \frac{1}{20} \times 3 \cdot 27 \times 10^{14}.$$

Thus $$100x \times 981 = 9 \times 3 \cdot 27 \times 10^7,$$

and $$x = \frac{9 \times 3 \cdot 27 \times 10^5}{981} = \frac{9 \times 10^5}{300} = 3000,$$

or the required distance is 3000 metres.

31. Calculate the efficiency of a steam engine from the following details :—

Diameter of cylinder . . .	16 in.
Length of stroke	2 ft. 6 in.
Number of revolutions per minute .	100
Mean pressure on piston . . .	22 lbs. per sq. in.
Consumption of coal . . .	$1\frac{1}{4}$ cwt. per hour.

[See Chap. I., Ex. 166. In this and the four following Examples assume the data given in Ex. 29.]

32. What is the efficiency of an engine which does 300 foot-tons of work for every pound of coal consumed?

33. An engine consumes per hour 10 lbs. of coal, and in three hours pumps 6000 gallons of water from a depth of 200 feet. Find the percentage of the heat evolved which is employed in doing useful work.

34. A train of 400 tons is drawn by an engine of which the efficiency is 0·05 (or 5 per cent). How far will the consumption of half a hundredweight of coal suffice to carry the train on a level line if the resistances to motion amount to 1/200th of the weight?

35. A horse working at the rate of 28,000 ft.-lbs. per minute is to be replaced by a small pumping-engine of efficiency 0·04. How much coal must be supplied to the engine per hour so as to pump at the same rate?

Work done in Expansion.—Let a cylinder containing air be closed by a weightless movable piston, and let a be the area of the piston, and p the pressure upon it per unit of area: the whole pressure upon the piston is $P = pa$. Now let the air expand and force the piston through a distance d against this constant pressure. The work done is the product of the force into the distance through which it is overcome, or

$$W = P \cdot d = pa \cdot d.$$

But $a\,d$ is the volume of the space through which the piston has moved, *i.e.* the change of volume produced by expansion of the air. Denoting this by dv, we have

$$W = p \cdot dv,$$

or the work done in expansion is equal to the pressure per unit area multiplied by the increment of volume.

If the work done is to be expressed in ergs, the pressure (p) must be measured in dynes per square centimetre, and the change of volume (dv) in cubic centimetres. When the expansion is caused by heat, and the original volume and initial and final temperatures are known, the change of volume can be calculated by applying Charles's law. If the barometric height (h) be given, the atmospheric pressure in terms of the above units (see p. 93) is

$$\Pi = h\rho g,$$

or, taking the normal barometric height of 76 cm.,

$$\Pi = 76 \times 13{\cdot}596 \times 981 = 1{,}013{,}663.$$

To find the specific heat of air at constant volume when its specific heat at constant pressure and the mechanical equivalent of heat are known.

Let m denote the density of air (or its mass per unit volume), C its specific heat at constant pressure, and c its specific heat at constant volume.

Suppose 1 c.c. of air taken at 0° to be heated to 273° under the constant atmospheric pressure Π. The amount of heat absorbed is

$$H = 273\,mC.$$

Part of this is spent in doing external work (p. 181), for during the process the volume of the air is doubled; the change of volume (dv) is 1 c.c., and the work done during the expansion is Π ergs. The thermal equivalent of this is

$$h = \Pi/J.$$

Now if the temperature of the air had been raised to 273° without allowing it to do any external work (*i.e.* if it had been heated at constant volume), the amount of heat absorbed would have been

$$H' = 273\,mc,$$

and it is clear that

$$H = H' + h,$$

or

$$H' = H - h.$$

Thus $$273mc = 273mC - \Pi/J,$$

and the specific heat of air at constant volume is

$$c = C - \Pi/273mJ.$$

Now we have seen that $\Pi = 1{,}013{,}663$ (p. 182), and $J = 4{\cdot}2 \times 10^7$ (pp. 176, 177). Also $C = 0{\cdot}237$, and $m = 0{\cdot}001293$, so that the numerical value of c is

$$0{\cdot}237 - 1013663/273 \times 0{\cdot}001293 \times 4{\cdot}2 \times 10^7 = 0{\cdot}237 - 0{\cdot}0684$$
$$= 0{\cdot}1686.$$

The ratio between the two specific heats is

$$\gamma = 0{\cdot}237/0{\cdot}1686 = 1{\cdot}4.$$

36. A gramme of air is heated from 0° to 100° under a pressure of 76 cm.: how much external work is done in the expansion?

The pressure is (p. 182) equal to 1,013,663 dynes per sq. cm., and since the density of air is 0·001293, the volume of a gramme of air is $1/0{\cdot}001293 = 773{\cdot}4$ c.c. at 0°.

On heating from 0° to 100° there is an increase of volume $dv = (773{\cdot}4 \times 100/273)$ c.c., and the external work done during the expansion is

$$p.dv = 1{,}013{,}663 \times 773{\cdot}4 \times 100/273 = 2{\cdot}872 \times 10^8 \text{ ergs}.$$

37. In one of Joule's earlier experiments air was compressed into a copper receiver standing in a calorimeter, the water equivalent of the calorimeter and its contents being 10,682 gm.; the air was then allowed to escape slowly, and was found to measure 44·6 litres, the atmospheric pressure being 76·5 cm. The cooling effect in the calorimeter was 0°·097: what value does this give for the mechanical equivalent?

The work done (in ergs) is

$$W = \Pi \cdot dv = h\rho g \cdot dv$$
$$= 76{\cdot}5 \times 13{\cdot}596 \times 981 \times 44600 = 4{\cdot}55 \times 10^{10}.$$

The corresponding amount of heat absorbed is

$$H = 10682 \times 0{\cdot}097 = 1{\cdot}036 \times 10^3 \text{ heat-units.}$$

This gives for the mechanical equivalent of heat

$$J = W/H = 4{\cdot}55 \times 10^7/1{\cdot}036 = 4{\cdot}39 \times 10^7.$$

38. A substance of density 1·11 and latent heat 50 melts at 60° C. at the atmospheric pressure, and in so doing expands 10 per cent in volume. What change in its melting-point will be produced by an additional 10 atmospheres of pressure?

By supposing the substance to go through the operations of a Carnot's cycle it can be shown that

$$T = \frac{(Tv_l - v_s)}{L}\delta p,$$

where L is the mechanical equivalent of the latent heat of fusion per unit mass, v_s the volume of unit mass of the solid, and v_l the volume of unit mass of the liquid at the absolute temperature T; and where δT is the change in the melting-point produced by the change of pressure δp.

Here $T = 273 + 60 = 333$. The change of volume $v_l - v_s$ is positive and equal to $(1/1{\cdot}11) \times 0{\cdot}1 = 0{\cdot}09$ c.c. The latent heat (expressed in ergs) is $50 \times 4{\cdot}2 \times 10^7$, and the change of pressure (expressed in dynes,—see p. 93) is $10 \times 1{\cdot}014 \times 10^6$.

Thus $$\delta T = \frac{333 \times 0{\cdot}09 \times 10 \times 1{\cdot}014 \times 10^6}{50 \times 4{\cdot}2 \times 10^7}$$

$$= 0{\cdot}182.$$

Hence the melting-point is raised 0°·182 C. If the change of volume in melting were negative (as in the case of ice and water), δT would be negative and the melting-point would be lowered by increase of pressure.

39. A cubic metre of air is heated under the normal pressure until its volume is doubled: how much of the heat supplied to it is converted into external work?

40. In the transmission of energy by means of compressed air, the air is compressed by means of a condensing pump until its pressure is 8 atmospheres. Find the

percentage loss of energy which should theoretically occur owing to the cooling of the air to its initial temperature. Would you expect the actual loss in the whole process of transmission to be greater or less, and why?

41. Explain what is meant by an indicator diagram, and draw (to any scale) the isothermals for a perfect gas at 0°, 10°, and 20°. Show in your figure the lines that would be traced if the gas were heated (*a*) at constant pressure from 10° to 20°, (*b*) at constant volume through the same range of temperature. How would you represent on the diagram the mechanical equivalents of the quantities of heat required for the two operations?

42. Define the critical temperature of a fluid. Give rough sketches showing the probable forms of the isothermals for a body both above and below its critical point; and specify the most important distinction between the two classes of isothermals.

43. The density of air at 0° and under a pressure of 10^6 dynes per square centimetre is 0·001277, and its specific heat is 0·237. One unit of heat is applied to a cubic centimetre of the air: express in ergs the external work done by expansion.

44. A cubic foot of air is heated from 0° to 273°, the barometric height at the time being 30 inches (p. 93): how many foot-pounds of work are done during the expansion? and what is the heat equivalent of this work?

45. Explain precisely what is meant by the *efficiency* of an engine. What do you understand by a *reversible* engine, and what are the conditions of reversibility in a heat engine?

Would you expect to get more work from 1 lb. of water in cooling from 100° to 0°, or from 100 lbs. of water in cooling from 1° to 0°, and why?

46. The density of air at 0° and 76 cm. is 0·001293; its specific heat at constant pressure is 0·237, and its

specific heat at constant volume is 0·168. Calculate from these data the value of the mechanical equivalent of heat.

47. Write a short essay on temperature, starting from the elementary ideas of "hot" and "cold," and criticising the methods commonly adopted for the measurement of temperature. Explain what is generally meant by *absolute temperature*, and indicate the process by which it has been rendered possible (in a stricter sense) to measure temperatures absolutely.

48. *The latent heat of fusion of ice is 80 and the volume of 1 gm. of ice at 0° is 1·0908 c.c. Prove that the effect of an additional atmosphere pressure will be to lower the melting-point of ice by 0°·0074 C. [Take the pressure of an atmosphere as $1{\cdot}014 \times 10^6$ dynes, and remember to express the latent heat by its mechanical equivalent.]

EXAMINATION QUESTIONS

49. Define thermal conductivity. A metal vessel, 1 square metre in area, and whose sides are 0·5 cm. thick, is filled with melting ice, and is kept surrounded by water at 100° C. How much ice will be melted in an hour? The conductivity of the metal is 0·02, and the latent heat of fusion of ice 80. Int. Sc. 1895.

50. A pond, 400 square metres in area, is covered with a sheet of ice 5 cm. thick; the temperature of the air is $-5°$ C.: how much heat will pass in an hour from the water through the ice? The thermal conductivity of ice is 0·00568. Prel. Sc. 1894.

51. Distinguish between conduction and convection of heat, giving illustrations of both processes.

The thermal conductivity of india-rubber is ·0041

C.G.S. units. How much heat would be conducted per hour across a square metre of india-rubber 3 centimetres thick if the difference of temperature of the faces were 12° C.? Vict. Int. Sc. 1890.

52. Define conductivity for heat, and show how the fundamental units of length, mass, and time enter into its numerical specification. Taking the conductivity of iron as ·17 in C.G.S. units, what difference of temperature would exist between the surfaces of an iron wall, 3 centimetres thick, through every square metre of which heat is streaming, from a furnace on one side into boiling water on the other, at the rate of 30,000 C.G.S. units per minute? Int. Sc. 1891.

53. How may the conductivity of a substance for heat be measured (*a*) for a metal, (*b*) for a poor conductor? If the conductivity of sandstone is ·0027 C.G.S. units, and if the underground temperature in a sandstone district increases 1° C. for 27 metres descent, calculate the heat lost per hour by a square kilometre of the earth's surface in that district. S. K. (Hons.) 1891.

54. Define conducting power for heat. If the conducting power of a woollen fabric be ·000122 C.G.S. units, calculate the heat lost per minute per square metre of the surface of the body of a man, whose surface temperature is 30° C., when the external air is at 5° C., and when the thickness of clothing is 3 millimetres. What number of kilogram-metres of energy does this loss represent? Prel. Sc. 1892.

55. Define thermal conductivity. Radiation has long been falling on a slab with blackened surface, each square decimetre of which absorbs 10,000 ergs per second; and the energy is transmitted to a back surface, half a centimetre distant, where it is removed by water. What steady difference of temperature must exist between the two surfaces of the slab if its conductivity is 0·02 C.G.S. units? Int. Sc. 1892.

56. A pound of coal in burning can raise 8000 lbs.

of water 1° C. Used in an engine the coal supplies 1,400,000 foot-pounds of work per pound burnt. What fraction of the heat is transformed to work? [Mechanical Equivalent = 1400, using feet, pounds, and degrees Centigrade.] Int. Sc. 1894.

57. What do you understand by the mechanical equivalent of heat? Taking the mechanical equivalent as 1400 foot-pounds per degree Centigrade, determine the heat produced in stopping by friction a fly-wheel 112 lbs. in mass, and 2 feet in radius, rotating at the rate of 1 turn per second, assuming the whole mass concentrated in the rim. S. K. 1894.

58. *Two heavy blocks of metal (each weighing 200 lbs.) are arranged as bobs of pendulums 6 feet long: they are suspended side by side, so that they can swing in the same plane, and so that when at rest the bobs just touch at A. The blocks are then separated as far as possible, the pendulum rods being horizontal. A lead bullet (1 ounce) is placed at A, and the blocks are simultaneously freed. They collide, crush the bullet, and rebound through a third of a quadrant. The bullet falls into a vessel of water containing $\frac{1}{2}$ lb. of water, and there is a rise of temperature in the water of 5 degrees Fahrenheit. Calculate the mechanical equivalent of heat; neglect the heating effects in the bobs, and the energy of the sound produced. (Specific heat of lead = ·03.) Camb. B.A. 1888.

59. Suppose a leaden bullet weighing fifty grammes to strike a target with a velocity of fifty metres per second, and all the heat generated by the stoppage of the motion to be expended in raising the temperature of the bullet. Find the increase in temperature of the bullet, the specific heat of lead being 0·0314. Camb. Schol. 1891.

60. If a horse does 60 kilogramme-metres of work per second for 5 hours each day, how much, at least, of oats per week must he eat to supply energy for

this work, if the combustion of 1 gramme of oats would warm 10 kilogrammes of water 1° C.?

Prel. Sc. 1891.

61. Describe carefully how to make experiments on the mechanical equivalent of heat by observing the heat produced when a substance is deformed beyond its limits of elasticity. How much heat would you expect to get out of the energy stored in a three-hundredweight block hung by two parallel strings, each 20 feet long, and slanted at 30° with the vertical?

Int. Sc. (Hons.) 1891.

62. A ton weight sliding down an inclined plane, 9 feet high and 21 feet long, reaches the bottom with a velocity of 5 feet per second. How much energy has been rubbed out of it during the descent, and how much heat has been developed. Prel. Sc. 1892.

63. A pound of coal in burning can raise 8000 lbs. of water 1° C. Used in an engine the coal supplies 1,400,000 foot-pounds of work per pound burnt. What fraction of the heat is transformed to work? [Mechanical Equivalent = 1400, using feet, pounds and degrees Centigrade.] Int. Sc. 1894.

64. The latent heat of steam at 100° C. is 536. If a kilogramme of water, when converted into saturated steam at atmospheric pressure, occupies 1·651 cubic metres, calculate the amount of heat spent in internal work during the conversion of water at 100° C. into steam at the same temperature. S. K. 1893.

65. What is meant by an isothermal line on the indicator diagram for a substance? Draw that for a given mass of air at a given temperature as the pressure varies from $\frac{1}{4}$ to 4 atmospheres, representing volumes and pressure by equal lengths at 1 atmosphere.

Int. Sc. (Hons.) 1893.

66. How would you determine accurately the coefficient of expansion of a gas under constant pressure? A gramme of air is heated from 15° to 60° C. under a

pressure of 75 cm. of mercury; how much external work is done in the expansion? [Density of mercury = 13·6; g = 981.] S. K. 1894.

67. An engine consuming 60 lbs. of coal per hour develops 30 horse-power. The heat given out by the combustion of 1 lb. of the coal is capable of converting 14 lbs. of water at 100° C. into steam at the same temperature. What fraction of the chemical energy of the coal is made available for mechanical work by the engine? [Mechanical equivalent of heat = 1390. Latent heat of steam = 536. 1 horse-power = 33,000 foot-pounds per minute.] Vict. B.Sc. 1890.

68. With what velocity must a leaden bullet at 50° C. strike against an obstacle in order that the heat produced by the arrest of its motion if all produced within the bullet might be just sufficient to melt it?

Take specific heat of lead = 0·031.
,, melting-point . = 335° C.
,, latent heat of fusion = 5·37. S. K. 1895.

69. What is meant by the Mechanical Equivalent of Heat?

A pound of coal in burning can raise 8000 lbs. of water 1° C. Used in an engine the coal supplies 1,400,000 foot-pounds of work per pound burnt. What fraction of the heat is transformed to work? [Mechanical Equivalent = 1400, using feet, pounds, and degrees Centigrade.] Int. Sc. 1894.

70. A heat engine works between the temperatures 127° and 52° C. It is found that only one-third of the largest amount of heat that could possibly be utilised is actually converted into useful work. What fraction of the total amount of heat supplied is usefully employed? S. K. 1895.

71. *A 20-H.P. non-condensing engine is supplied with steam at a temperature of 150° C., and requires 120 lbs. of coal per hour; a 10-H.P. engine is supplied with steam at a temperature of 140° C., and requires

61 lbs. of coal per hour. Taking into consideration the greatest attainable efficiency of each engine, determine which is the more nearly perfect. B.Sc. (Hons.) 1889.

72. *If a quantity of air at 15° C. is suddenly compressed to half its volume, show how to calculate the temperature it will momentarily attain. Explain the importance of this knowledge in the theory of sound-propagation.

Show how a numerical answer can be given with the help of the following data :—

$$\log 2 \quad = 0 \cdot 3010,$$
$$\log 1 \cdot 32 = 0 \cdot 1204.$$

S. K. (Hons.) 1889.

73. *State the second law of thermodynamics, and apply it to the determination of the effect of pressure on the melting point of a solid.

If a substance of latent heat 50 melts at 127° C. *in vacuo*, and contracts one-tenth of its bulk in so doing, at what temperature will it melt under a pressure of 100 atmospheres? S. K. (Hons.) 1891.

74. *Give the theory of the change in the melting-point of a solid by change of pressure, and describe experiments which have been made to test the theory.

If sulphur has density 2·05 just before, and 1·95 just after melting, the melting-point being 115°, and the latent heat 9·3, find the alteration in melting-point per atmosphere change of pressure. B. Sc. (Hons.) 1893.

CHAPTER VII

LIGHT

1. A CANDLE is placed at a distance of 1 ft. from a card-board screen, and a lamp of 9 candle-power is placed at a distance of 12 ft. on the other side. Compare the illumination on the two sides of the screen.

The intensity of the illumination is directly proportional to the illuminating power of the source, and inversely proportional to the square of its distance from the screen. Thus the intensity of the illumination on the side facing the candle is to that on the side facing the lamp as $\frac{1}{(1)^2}$ to $\frac{9}{(12)^2}$, or as 1 to $\frac{9}{144}$; *i.e.* as 144 to 9, or as 16 to 1.

2. How near to the screen would the lamp in Ex. 1 have to be moved in order to reverse the proportion, *i.e.* to make the intensities of illumination as 1 to 16?

3. If the light of the full moon is found to produce the same degree of illumination as a standard candle does at a distance of 4 ft., what is the equivalent in candle-power of the moon's light? (Take the distance of the moon as 240,000 miles.)

Let E denote the equivalent in candle-power: then, expressing distances in feet, we have

$$E/(240{,}000 \times 1760 \times 3)^2 = 1/4^2,$$
$$\therefore E = (240{,}000 \times 1760 \times 3/4)^2,$$
$$= 10{,}036{,}224 \times 10^{10}.$$

4. A standard candle and a gas-flame are placed 6 ft. apart, the gas-flame being of 4 candle-power: where must a screen be placed, on the line joining the candle and gas-flame, so that it may be equally illuminated by each of them?

Let x denote the distance of the screen from the candle; its distance from the gas-flame will then be $6-x$. If the screen is to receive equal illumination from each of the sources of light we must have

$$\frac{1}{x^2}=\frac{4}{(6-x)^2},$$

$$i.e.\ (6-x)^2=4x^2;$$

or, taking square roots, $6-x=\pm 2x$,

and $\therefore x=+2$ or -6.

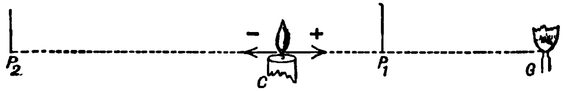

Fig. 6.

Corresponding to these two values of x there are two positions (P_1 and P_2 in Fig. 6) of the disc which satisfy the conditions of the problem. The first, P_1, is 2 ft. to the *right* of the candle, *i.e.* between the candle and the gas-flame; the second, P_2, is 6 ft. to the *left* of the candle.

Both solutions can easily be verified.

5. Distinguish between *illuminating power* and *intensity of illumination.* Two equal sources of light are placed on opposite sides of a screen, one being 20 cm. from it and the other 30 cm. Compare the intensity of illumination on the two sides of the screen.

6. In testing a night-light against a candle by means of a Rumford photometer, it is found that the shadows are of equal depth when the night-light is 2 ft. from the screen, and the candle 3 ft. Compare their illuminating powers.

7. In measuring the illuminating power of a gas-flame

by a Bunsen photometer, the distance from the gas-flame to the grease spot was 96 cm., and from this to the standard candle 30 cm. What was the candle-power of the gas-flame?

8. Compare the intensities of the illumination produced on the floor of a hall (1) by a Sunbeam lamp of 400 candle-power at a height of 16 ft., and (2) by an arc lamp of 1000 candle-power at a height of 40 ft.

9. In determining the illuminating power of a gas-flame by Bunsen's photometer, the distance from the gas-flame to the grease spot was 84 cm., and from this to the standard candle 40 cm.: what was the candle-power of the gas-flame?

10. A Carcel lamp of 9 candle-power is placed at a distance of 4 yards from a standard candle: determine, as in Example 4, the two positions in which a screen may be placed so as to receive equal amounts of light from the lamp and the candle.

11. Two gas-flames, of 16 and 9 candle-power respectively, are 140 cm. apart. Show that there are two positions, on the line joining the flames, in which a screen can be placed so as to be equally illuminated by each flame, and determine these positions.

12. What do you understand by the terms "real image," "virtual image"? Draw a figure showing how an image is formed in a plane mirror, and prove that the object and its image are equally distant from the mirror.

13. A man stands in front of a looking-glass of the same height as himself. Show that he only requires about one-half of it in order to see a full-length image of himself.

14. A candle is placed in any position between two plane mirrors at right angles; show that its two secondary images (produced by reflection from both mirrors) will exactly coincide.

15. A luminous object is placed between two plane mirrors inclined at 60° : find the number of images, and show that they all lie on a circle.

16. A ray of light is reflected successively from two mirrors inclined at right angles to each other. Prove that the ray after a second reflection is parallel to its original direction.

17. An object is placed between two mirrors inclined at 45° : show by a figure how an observer could see an image after four successive reflections.

18. A candle is placed between two parallel mirrors; draw a sketch showing the path of a ray of light which proceeds from the candle, and, after undergoing two reflections at one mirror, and three at the other, enters the eye of an observer looking toward the mirrors.

19. Two plane mirrors, A and B, are placed vertically upon a horizontal table. A ray of light PB falls upon the mirror B, and is reflected to the mirror A; show that the ray AQ reflected from the latter makes with PB an angle which is double of the angle between the mirrors.

20. If the mirrors of a kaleidoscope are placed at an angle of 45°, how many images will there be of an object (1) placed close to one of the mirrors, (2) placed midway between the mirrors?

21. Prove that if an object in front of a plane mirror moves through a distance d away from the mirror, the image will move through the same distance; whereas if the mirror moves parallel to itself through a distance d (the object remaining fixed) the image will move through a distance $2d$.

22. AB and AC are two plane mirrors inclined at an angle of 15°, and P is a point in AB. At what angle must a ray of light from P be incident upon AC, in order that after three reflections it may be parallel to AB?

23. An object is placed in front of a convex spherical mirror: find by a geometrical construction the position

and nature of the image; and show that the sizes of image and object are as their distances from the centre of the mirror.

24. Assuming that in the case of a spherical mirror the sizes of image and object are proportional to their distances from the centre of the mirror, show that they are also proportional to their distances from the mirror itself.

25. An object is placed at a distance p from a concave mirror of focal length f: prove that the image is formed at a distance p', which is given by the equation

$$1/f = 1/p + 1/p'.$$

Rules for Optical Calculations.—In order to avoid confusion as to plus and minus signs, or positive and negative focal distances, beginners will find it well to follow strictly some such rules as the following, and to employ the equations for conjugate positions always in the same form.

I. All distances are to be reckoned from the mirror or lens: if measured towards the right hand they are to be considered positive (+); if towards the left hand, negative (−).

It is convenient to suppose that the luminous object, or source of light, is always placed on the right hand of the mirror or lens: its distance will thus always be positive. This is equivalent to saying that all distances measured from the mirror or lens on the same side as the object or source of light are to be regarded as positive; and all distances in the opposite direction as negative.

II. The equation

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{p'} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

holds good for all mirrors, both concave and convex, f

denoting the focal length, p the distance of the object, and p' the distance of the image, reckoned from the mirror itself.

For all lenses, convex and concave, the equation for conjugate points is

$$\frac{1}{f}=\frac{1}{p'}-\frac{1}{p} \quad . \quad . \quad . \quad . \quad (2)$$

the letters denoting the same quantities as before, *and being always supposed to carry their own signs.*

III. The focal length of a concave mirror is positive; of a convex mirror negative. The focal length of a convex (convergent) lens is negative; of a concave (divergent) lens positive, *i.e.*,

$$\text{for}\left\{\begin{array}{l}\text{concave mirrors}\\ \text{concave lenses}\end{array}\right\} f \text{ is positive } (+),$$

$$\text{for}\left\{\begin{array}{l}\text{convex mirrors}\\ \text{convex lenses}\end{array}\right\} f \text{ is negative } (-),$$

IV. Whenever you have to work out any optical calculation, begin by writing down the proper equation, and do not alter the signs of the letters until you come to insert their values. Now substitute in the equation the numerical values of the known quantities, *each with its proper sign.* Want of attention to this point is the commonest cause of errors and incorrect answers.

When any two of the three quantities f, p, and p' are given, the third can be found by means of equation (1) or (2). If f and p are given, the numerical value of p' indicates the distance from the mirror or lens at which the image is formed: if p' is positive, the image is formed on the same side as the object (to the right of the mirror or lens, or in front of it); but if it carries the minus sign, the image is formed on the opposite side (to the left of the mirror or lens, or behind it). When the distances of the object and image (p and p') are given, the nature of the mirror or lens is indicated by the sign of f,

and the focal length by its numerical value. The mirror or lens is concave if f is positive, and is convex if f is negative.

In solving any problem it is generally advisable to draw a diagram roughly to scale for the purpose of guiding the algebraical work and preventing errors.

26. A candle-flame 1 cm. long is 36 cm. in front of a concave mirror, whose focal length is 30 cm. Find the nature, position, and size of the image.

Here $p = 36$, $f = 30$; and both are positive. The value of p' (the distance of the image from the mirror) is given by the equation

$$1/f = 1/p + 1/p'.$$

Substituting the above values we have

$$1/30 = 1/36 + 1/p',$$

or $$\frac{1}{p'} = \frac{1}{30} - \frac{1}{36} = \frac{6-5}{180} = \frac{1}{180},$$

and $$\therefore p' = 180.$$

The distance of the image from the mirror is 180 cm. Since p' is positive, the image is formed in front of the mirror, and is real and inverted. (Verify this by sketching the diagram.)

The relative sizes[1] of image and object are as their respective distances from the mirror, or

$$\text{I} : \text{O} = p' : p.$$

Hence $$\frac{\text{I}}{\text{O}} = \frac{180}{36} = 5.$$

The image is five times as large as the object, and is 5 cm. long.

27. The candle-flame is placed at a distance of 15 cm. from the same mirror. What sort of an image is now produced, and what is its size?

[1] When not otherwise stated, it may be assumed that the word "size" refers to *linear* dimensions.

As before, we have

$$1/30 = 1/15 + 1/p',$$

and $\quad \therefore 1/p' = 1/30 - 1/15 = -1/30.$

Hence $p' = -30$ cm. This means that the image is formed 30 cm. *behind* the mirror, and is virtual and erect.

The distances of the image and object from the mirror are as 30 to 15, or as 2 to 1. The image is, therefore, twice the size of the object, or is 2 cm. long.

28. A luminous point is 24 cm. in front of a concave mirror of 6 cm. focal length: where is the image formed? If the point moves through a small distance d away from the mirror, through what distance will the image move?

(1.) The value of p' is given by the equation

$$1/6 = 1/24 + 1/p',$$

from which we have $p' = 8$. The image is therefore formed 8 cm. in front of the mirror.

(2.) The new value of p is $24 + d$, and the corresponding value of p' is given by

$$1/6 = 1/(24 + d) + 1/p',$$

$$\therefore p' = 6(24 + d)/(18 + d).$$

This is less than 8, and the distance through which the image moves is

$$8 - 6(24 + d)/(18 + d) = 2d/(18 + d).$$

29. You are required to throw upon a wall a real image of a gas-flame which stands 8 ft. from the wall, and the image is to be three times the size of the flame: what sort of mirror would you choose, and where would you hold it?

Suppose the mirror to be placed x feet from the object on the side farther from the wall: it will then be $(8 + x)$ ft. from the wall, and, since the image is to be three times the size of the object, we must have $p' = 3p$, or

$$8 + x = 3 \times x, \text{ and } \therefore x = 4.$$

Thus $p = 4,\ p' = 8 + 4 = 12,$

and $1/f = 1/4 + 1/12 = 1/3.$

The mirror required is a concave mirror of 3 ft. focal length, and it must be held 4 ft. from the object.

[Observe that the distances of image and object from the mirror bear the right ratio to one another, being as 12 to 4, or 3 to 1. Check your results in this way whenever you can.]

30. At what distance from a concave mirror must an object be placed so that its image shall be magnified n times?

The distance will depend upon the focal length of the mirror. Let this be denoted by f. Since the image is to be n times the size of the object, its distance from the mirror must be n times as great, or $p' = np$ (numerically).

It must, however, be pointed out that in making use of the equation

$$\frac{\mathrm{I}}{\mathrm{O}} = \frac{p'}{p},$$

care should be taken to notice the *signs* of the quantities p' and p. When the image is *real*, both p and p' are positive, and $p' = np$. But when the image is virtual, p' is *negative*, and therefore $p' = -np$.

Thus there are two solutions :—

(1.) Image real—

Inserting in the equation

$$1/f = 1/p + 1/p'$$

the value $p' = np$, we get

$$1/f = 1/p + 1/np = (n + 1)/np,$$

and $\therefore\ np = (n + 1)f,$

or $p = (n + 1)f/n,$

which gives the required distance in terms of the focal length of the mirror.

(2.) Image virtual—

Inserting in the same equation the value $p' = -np$, we get

$$1/f = 1/p - 1/np = (n - 1)/np,$$

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refers to ι.

and $\therefore np = (n-1)f$,

or $p = (n-1)f/n$.

31. An object 3 in. in length is held 6 in. in front of a convex mirror whose radius of curvature is 2 ft. Find the position and size of the image.

The focal length is one-half the radius of curvature, and is therefore 1 foot or 12 inches. But, since the mirror is convex, its focal length is negative. Thus

$$f = -12, \text{ and } p = +6.$$

The distance of the image from the mirror is given by the equation

$$-1/12 = 1/6 + 1/p',$$

or $1/p' = -1/12 - 1/6 = -3/12$,

and $p' = -4$.

Thus the image is virtual and is formed 4 inches *behind* the mirror.

The relative sizes of image and object are as their distances from the mirror, *i.e.* as 4 to 6, or 2 to 3. The image is therefore 2 inches long.

32. Rays of light diverging from a point 3 ft. in front of a mirror converge, after reflection, to a point 1 ft. in front of the mirror. Is the mirror concave or convex, and what is its focal length?

33. The radius of curvature of a concave mirror is 30 cm. Rays of light diverge from a point 60 cm. in front of it: to what point will they converge after reflection?

34. A luminous point is 60 cm. in front of a mirror, and its image is found to be 20 cm. from the mirror, and on the same side: find the nature and focal length of the mirror.

35. An object is placed at a distance of 10 cm. in front of a convex mirror of 30 cm. focal length: where will its image be formed?

36. A candle-flame 1 in. long is 18 in. in front of a concave mirror whose focal length is 15 in.: find the position and size of the image.

37. An object 6 cm. in length is placed at a distance of 30 cm. from a concave mirror of 12 cm. focal length: find the nature, position, and size of the image.

38. The radius of curvature of a concave mirror is 60 cm. A small source of light is placed at the following distances in front of it: 120 cm., 60 cm., 30 cm., and 20 cm. Discuss the behaviour of the reflected beam in each case.

39. A gas-jet is placed on the principal axis of a spherical mirror and 1 ft. in front of it. A real and inverted image is produced on a screen held in front of the mirror, but at a greater distance than the gas-jet. If the image is twice as long as the flame, what is the focal length of the mirror?

40. An object is placed 5 in. from a concave mirror of 6 in. focal length: where is the image produced, and what is the magnification?

41. Prove that when an object is placed midway between a concave mirror and its principal focus, the image is twice as large as the object. What is the nature of the image?

42. A real image produced by a concave mirror is found to be three times the size of the object: if the mirror is one foot from the object, what is its focal length?

43. An object stands 8 in. in front of a certain concave mirror which produces a real image of it magnified three times. Prove that if the object is moved an inch farther from the mirror, the real image will approach 6 in. towards the mirror, and that it will now be only twice as large as the object.

44. It is desired to throw upon a wall an image of an object magnified twelve times, the object being 11 ft. from the wall: find the focal length of a concave mirror

that may be used for the purpose, and state where it must be placed.

45. An image produced by a concave mirror is found to be twice the size of the object: if the focal length of the mirror is 1 ft., where are the object and image situated? What would be the relation between the sizes if their positions were reversed?

46. Prove that when an object is at a distance $2f/3$ from a concave mirror of focal length f, the image produced is erect and virtual, and magnified three times.

47. An object is placed at a distance $3f/2$ in front of a concave mirror of focal length f: what is the nature and size of the image? Is there any other position of the object which will produce the same degree of magnification?

48. At what distance from a concave mirror (of focal length f) must an object be situated in order that the image may be half the size of the object?

49. An object 3 in. long is held 6 in. in front of a convex mirror whose radius of curvature is 2 feet. Find the nature, position, and magnitude of the image.

Where will the image be produced when the object is held 2 ft. in front of the same mirror, and what will be its size?

50. An object 4 cm. long is placed at a distance of 10 cm. from a convex mirror of 30 cm. focal length. Find the position and size of the image.

51. An object is held in front of a convex mirror at a distance equal to its focal length: what is the size of the image?

52. An image produced by a convex mirror is $1/n$th the size of the object: prove that the latter must be at a distance $(n-1)f$ from the mirror.

53. Enunciate the laws of refraction of light, and explain what is meant by the index of refraction of a substance.

54. A ray of light is incident upon the surface of still water, and the index of refraction for air and water is $\frac{4}{3}$. Show how the path of the ray in water can be found when its path in air is known: and discuss any special cases which may occur when the direction of the ray is reversed, *i.e.* when it travels upwards towards the surface of the water.

55. Describe an experiment to show total internal reflection, and point out the conditions under which it occurs. What is the *critical angle* for two media?

56. Given the indices of refraction from a medium A to another B, and from B to a third C: show how to determine the index of refraction from A to C.

57. The refractive index for air and water is $4/3$, and for air and glass $3/2$: find the index from glass to water and from water to air.

58. A ray of light passes from one medium into a second, the angle of incidence being $60°$ and the angle of refraction $30°$: show that the index of refraction is $\sqrt{3}$.

59. Find the index of refraction when the angles of incidence and refraction are $45°$ and $30°$ respectively.

60. The critical angle for a certain medium is $45°$: what is its refractive index?

61. Rays of light are emitted upwards in all directions from a luminous point at the bottom of a trough containing a layer 3 in. in depth of a transparent liquid, whose index of refraction is $5/4$. Show that all rays which meet the surface outside a certain circle, whose centre is vertically above the point, will be totally reflected, and find the radius of this circle.

62. A microscope is placed vertically above a short glass tube cemented to a flat piece of glass (so as to form a tube open at the top), and is focussed upon a mark on the glass slip. A layer of liquid of depth h is poured into the tube, and it is now found that the image of the mark is displaced through a distance d which is deter-

mined by refocussing the microscope. Prove that the refractive index of the liquid is $\mu = h/(h-d)$.

63. In an experiment made by the above method water was poured in to a depth of 4·6 cm., and the resulting displacement of the image was 1·15 cm. Calculate the refractive index of water.

64. Calculate the refractive index of a glass prism for sodium light from the following observations:—

Refracting angle of prism . . .	45° 4′.
Minimum deviation	26° 40′.

It can be shown that when a ray of light passes through a prism of angle A placed in the position of minimum deviation, the index of refraction of the prism is given by

$$\mu = \sin\frac{A+D}{2}\Big/\sin\frac{A}{2},$$

where D is the angle of deviation.

Here
$$\begin{aligned} A &= 45^\circ\ 4' \\ D &= 26^\circ\ 40' \\ \hline A+D &= 71^\circ\ 44' \end{aligned}$$

and
$$(A+D)/2 = 35^\circ\ 52'.$$

From the table of natural sines we find that

$$\sin 35^\circ = 0{\cdot}574, \text{ and } \sin 36^\circ = 0{\cdot}588.$$

Assuming that the rate of change of the sine is proportional to that of the angle within these limits, we have

$$\sin 35^\circ\ 52' = 0{\cdot}586 \text{ approximately.}$$

In the same way we obtain 0·383 as the approximate value of sin 22° 32′. Thus

$$\mu = 0{\cdot}586/0{\cdot}384 = 1{\cdot}53.$$

65. Find the refractive index of the same prism for lithium light, the minimum deviation produced being 26° 30′.

66. The minimum deviation produced by a hollow prism filled with a certain liquid is 30° : if the refracting angle of the prism is 60°, what is the index of refraction of the liquid?

67. The refracting angle of a prism is 60° and its index of refraction is $\sqrt{2}$. Show how to find the path of a ray through the prism, and calculate the angle of emergence and the deviation when the angle of incidence is 45°.

68. A prism is to be made of crown glass, the refractive index of which is known to be $1 \cdot 526$, and it is required to produce a minimum deviation of $17^\circ\ 20'$: to what angle must it be ground?

69. A ray of light is incident almost perpendicularly upon a prism of angle α and refractive index μ: show that if α is small the deviation is given by

$$\delta=(\mu-1)\alpha.$$

70. The refractive index of rock-salt is $1 \cdot 54$: what deviation would be produced by a rock-salt prism of $1^\circ\ 30'$ angle? and what should be the angle of a rock-salt prism which is required to produce a deviation of $48'$?

71. Taking the usual values for the refractive indices of water and glass, prove that the deviations produced by thin prisms of water and glass are one-third and one-half of the angles of the prisms respectively.

Note.—The refractive index for water and air is $4/3$ ($1 \cdot \dot{3}$), and for glass and air $3/2$ (or $1 \cdot 5$).

72. Find the angle of a water prism which will produce the same deviation as that given by a glass prism of 2° angle.

73. In order to determine the refractive index of a double convex lens, its focal length and the radii of curvature of its faces were measured, and were found to be

$$f=30 \cdot 6 \text{ cm.}, \ r_1=30 \cdot 4 \text{ cm.}, \ r_2=34 \cdot 5 \text{ cm.}$$

What was the index of refraction of the glass?

It can be proved[1] that the focal length of a lens, in terms of

[1] Aldis, *Geometrical Optics*, Art. 67.

its refractive index and the radii of curvature of its surfaces, is given by the formula $1/f=(\mu-1)(1/r_1-1/r_2)$.

In applying this we must remember that the radius of curvature of the first surface is negative, or $r_1=-30\cdot4$; and since the lens is convex, its focal length is also negative.

Thus $\quad -1/30\cdot6=(\mu-1)(-1/30\cdot4-1/34\cdot5)$,

or $\quad \mu-1=30\cdot4\times34\cdot5/30\cdot6\times64\cdot9=0\cdot528$,

and $\quad \therefore \mu=1\cdot528$.

74. Explain what is meant by the optical centre of a lens, and prove that the optical centre of a plano-convex or plano-concave lens lies on the curved surface.

75. Taking the usual values for the refractive indices of water and glass, prove that the focal length of a glass lens when immersed in water is four times its focal length in air.

76. Prove that the focal length of a plano-concave glass lens is equal to twice the radius of the concave surface.

77. A hemispherical lens of 3 in. radius is made of glass of refractive index 1·5: show that rays of light proceeding from a point on its axis, and 4 in. in front of its plane surface, will be rendered parallel by passing through it.

78. A double convex lens is to be made of glass of refractive index 1·5, and the radius of curvature of one of its faces is 20 cm. If the lens is to have a focal length of 30 cm., what must be the radius of curvature of the other face?

79. The radii of curvature of the two faces of an equi-convex lens are each equal to 46·5 cm., and it is made of glass of refractive index 1·532: show that its focal length is 43·7 cm.

80. An object is placed at the distance of 3 ft. in front of a lens and the image is formed 1 ft. behind the lens. What is the focal length of the lens? and what kind of lens is it?

Here $p=3$ and $p'=-1$.

Thus
$$\frac{1}{f}=\frac{1}{p'}-\frac{1}{p}$$
$$=-\frac{1}{1}-\frac{1}{3}=-\frac{4}{3},$$
and
$$f=-\frac{3}{4}.$$

The focal length is $\frac{3}{4}$ ft. or 9 in., and since it is negative the lens is convex.

81. An object whose length is 5 cm. is placed at a distance of 12 cm. from a convex lens of 8 cm. focal length: where is the image formed and how long is it?

Here $f=-8$ (since the lens is convex) and $p=+12$.
The distance of the image from the lens is given by the equation
$$-1/8=1/p'-1/12,$$
or
$$1/p'=1/12-1/8=-1/24.$$
Thus $p'=-24$. The image is formed at a distance of 24 cm. on the other side of the lens. Since its distance from the lens is twice that of the object, its length is also twice that of the object, or $=10$ cm.

82. A candle stands at a distance of 3 ft. from a wall: in what position must a convex lens of 8 in. focal length be placed between them so as to produce upon the wall a distinct image of the candle?

Let p denote the distance (in inches) of the candle from the lens; then $36-p$ will be the distance of the screen from the lens *irrespective of sign*. Remembering that p' is negative, we have $p'=p-36$, and now substituting in equation 2 (p. 197) we have
$$-1/8=1/(p-36)-1/p,$$
$$\therefore p^2-36p=8p-8p-288,$$
or
$$p^2-36p+288=0.$$
This may be written in the form
$$(p-24)(p-12)=0,$$
and
$$\therefore p=24 \text{ or } 12.$$

A distinct image will therefore be produced when the candle is placed either 1 ft. or 2 ft. from the lens.

83. If an object at a distance of 3 in. from a convex lens has its image magnified three times, what is the focal length of the lens?

There are two solutions to this problem, for the image may be either real or virtual. In the first case it is formed on the other side of the lens; in the second case on the same side as the object.

In both cases, since the image is three times as large as the object, its distance from the lens must be three times that of the object; but this only gives us the *numerical* value of p' in terms of p.

(1) Image real—

$$p' \text{ is negative and} = -3p \text{ or } -9 \text{ in.}$$

Thus
$$\begin{aligned} 1/f &= 1/p' - 1/p, \\ &= -1/9 - 1/3 = -4/9, \end{aligned}$$

and the focal length of the lens is $-2\frac{1}{4}$ inches.

(2) Image virtual—

$$p' \text{ is positive and} = 3p = +9 \text{ in.}$$
$$1/f = 1/9 - 1/3 = -2/9,$$

so that the focal length in this case is $-4\frac{1}{2}$ inches.

84. I have a convex lens of 15 cm. focal length. How far from an object must I hold it so that it may produce a real image of the object magnified three times?

Suppose p and p' to be the required distances of the object and image from the lens. Since the image is to be three times the size of the object, its distance from the lens must be three times as great, or $p' = 3p$ (numerically). But since the image is to be real, it must be formed on the *opposite* side of the lens; thus p' is *negative* and $= -3p$ (cp. last Example).

In the usual equation we have now to put $f = -15$ and $p' = -3p$. We thus get the equation

$$-1/15 = -1/3p - 1/p,$$

from which
$$p = +20.$$

Thus the lens must be held at a distance of 20 cm. from the object. The image is formed at a distance of 60 cm. (3×20) on the other side of the lens.

85. Rays of light diverging from a point 1 ft. in front of a lens are brought to a focus 4 in. behind it. What is the nature of the lens and what is its focal length?

86. The focal length of a convex lens is 9 in. If an object is placed 1 ft. in front of the lens where is the image formed?

87. Rays of light diverge from a point 20 cm. in front of a convex lens. The focal length of the lens is 4 cm. How do the rays behave after refraction through it?

88. Explain the difference between real and virtual images, and give examples of each. If you had a convergent lens of 1 ft. focal length, where would you place an object so as to produce by means of the lens (1) a real and diminished image, (2) an erect and virtual image? Give sketches showing how the image is produced in each case.

89. Rays of light diverging from a point 6 in. before a lens are brought to a focus 18 in. behind it: what is the focal length of the lens?

90. An object is placed at a distance of 60 cm. from a convex lens of 15 cm. focal length: where is the image formed? Compare its size with that of the object.

91. An object whose length is 5 cm. is placed at a distance of 12 cm. from a convex lens of 8 cm. focal length: what is the length of the image?

92. A candle is placed at a distance of 10 ft. from a wall, and it is found that when a convex lens is held midway between the candle and the wall a distinct image is produced upon the latter. Find the focal length of the lens and the relative sizes of the object and image.

93. A coin half an inch in diameter is held on the axis of a convergent lens, and 1 ft. in front of it: if the focal length of the lens is 8 in., find the position and magnitude of the image.

94. Draw figures, approximately to scale, showing the paths of the rays of light, and the positions of the images formed when a luminous object is placed at a distance of (1) 1 inch, (2) 6 inches from a convergent lens of 2 in. focal length.

95. An object is placed 8 in. from a convex lens, and its image is formed 24 in. from the lens on the other side. If the object were placed 4 in. from the lens, where would the image be?

96. The distance of an object from a convergent lens is double the focal length of the lens: prove that the image and object are of the same size.

97. A candle stands at a distance of 2 metres from a wall, and it is found that when a lens is held half a metre from the candle a distinct image is produced upon the wall: find the focal length of the lens, and also state the relative sizes of image and object.

98. A lens of 9 in. focal length is to be used for the purpose of producing an inverted image of an object magnified three times: where must each be situated?

99. A convex lens is held 5 ft. in front of a wall, and it is found that there is one position in which an object can be held in front of the lens such that an inverted image six times as large as it is thrown upon the wall. Determine this position, and also find the focal length of the lens.

100. At what distance from a convex lens must an object be placed so that the image may be half the size of the object?

101. You are provided with a convex lens of 18 in. focal length, and are required to place an object in such a position that its image will be magnified three times:

find the positions which will give (1) a real, and (2) a virtual image of the required size.

102. A candle stands a yard away from a wall. I wish to throw on the wall an image of the candle-flame magnified five times. What kind of a lens must I use, and where must I hold it?

103. Three successive measurements of the focal length of a convex lens were made with the following results:—

Exp. 1	$p=40$ cm.	$p'=66$ cm.
,, 2	$p=45$,,	$p'=60$,,
,, 3	$p=62$,,	$p'=43$,,

Calculate the focal length according to each measurement; also the mean value.

104. When an object is placed 9 inches in front of a certain convex lens, a real image magnified twice is produced. Where must the object be placed so that its real image may be three times as large as itself?

105. A convex lens produces a real image n times as large as the object: prove that the latter must be at a distance $(n+1)f/n$ from the lens.

106. A glass scale, 4 cm. long, was held in front of a convergent lens, and on holding a screen 90 cm. behind the lens, an image of the scale, 20 cm. in length, was produced upon the screen: prove that the lens had a focal length of 15 cm.

107. The focal length of a concave lens is 3 inches. Rays of light diverge from a point on its axis, and 6 inches from it. How will they proceed after refraction through the lens?

108. A virtual image of an object 30 cm. from a lens is formed on the same side of the lens, and at a distance of 10 cm. from it. What kind of a lens is it?

109. The focal length of a concave lens is 25 cm. Where must an object be placed so that its image may be one-sixth of its own size?

110. In order to find the focal length of a concave lens, it was blackened, with the exception of a circle 4 cm. in diameter at its centre. A beam of sunlight was allowed to pass through this, when it was found that an illuminated circle of 20 cm. diameter was formed on a screen held 64 cm. behind the lens and parallel to it. What was the focal length of the lens?

111. Show how to find the focal length (F) of a combination obtained by placing two thin lenses of focal lengths f_1 and f_2 in contact. Prove that for any number of such lenses placed in contact $1/F = \Sigma(1/f)$.

112. What is the focal length of a lens which is equivalent to two thin convergent lenses of focal lengths 15 cm. and 30 cm. placed in contact?

113. A concave lens of 8 cm. focal length is combined with a convex lens of 6 cm. focal length: what is the focal length of the combination?

114. A convex lens of focal length 16 cm. was placed in contact with a concave lens, and the focal length of the combination was found to be 48 cm. Calculate the focal length of the concave lens.

115. A candle is held 1 foot in front of a convex lens, and a distinct image of the flame is formed on a screen placed 4 inches behind it. A concave lens is now placed in contact with it, and it is found that the screen has to be moved 8 inches farther off in order to receive the image. What is the focal length of the concave lens?

116. Explain the action of a condensing lens when used as a magnifying glass. Give a sketch showing how the image is produced, and prove that the magnifying power is approximately equal to Δ/f, where Δ is the distance of most distinct vision.

117. Describe the action of the eye, considered as an optical instrument, and explain the causes of abnormal vision. Will the magnifying effect of a given reading-

lens be greater when used by a long-sighted or a short-sighted person?

118. A person whose distance of most distinct vision is 20 cm. uses a lens of 5 cm. focal length as a reading-glass: at what distance from a book must he hold it? Also what will be its magnifying power?

119. A long-sighted person can only see distinctly objects which are at a distance of 48 cm. or more: by how much will he increase his range of distinct vision if he uses convex spectacles of 32 cm. focal length?

120. An engraver uses a magnifying glass of 4 inches focal length, holding it close to the eye. At what distance from the work must it be so that the magnification may be four-fold?

121. A long-sighted person uses convex glasses of 40 cm. focal length, and finds that he cannot comfortably read print through them when it is held nearer than 30 cm. What is his nearest point of distinct vision?

122. A short-sighted man can read printed matter distinctly when it is held at 15 cm. from his eyes: find the focal length of the glasses which he must use if he wishes to read with ease a book at a distance of 60 cm.

123. A convex lens produces an image of a candle-flame upon a screen whose distance from the candle is l; the lens is displaced through a distance d, when it is found that a distinct image is again produced upon the screen. Show that the focal length of the lens is $(l^2 - d^2)/4l$.

124. Prove that the size of the object in the last question is a geometrical mean between the sizes of the two images produced.

125. In an experiment made according to the method of Ex. 123, the distance between the candle and screen was 255 cm. and the lens had to be shifted through a distance of 73·7 cm. What was its focal length?

126. Calculate the mean value of the focal length of

a convex lens which gave the following results by the method of displacement :—

Exp. 1 . . .	$l = 85$ cm.	$d = 38{\cdot}7$ cm.
„ 2 . . .	$l = 80$ „	$d = 33{\cdot}0$ „
„ 3 . . .	$l = 70$ „	$d = 14{\cdot}5$ „

127. *A diffraction-grating, ruled with 100 lines to the millimetre, is placed in front of a slit illuminated with sodium light. The minimum angular deviations of the images of the first and second order are found to be $3^\circ\ 23'$ and $6^\circ\ 47'$ respectively. What value does this give for the wave-length of sodium light?

128. *Calculate the wave-length for the D line from the following data of experiments with a diffraction-grating :—

Distance between lines of grating . 0·0009023 cm.
Direct reading of telescope . . $137^\circ\ 20'$.
Readings of telescope for diffracted images—
First order : $141^\circ\ 4'\ 30''$ and $133^\circ\ 36'\ 30''$.
Second order : $144^\circ\ 50'\ 45''$ and $129^\circ\ 50'\ 10''$.

129. *A hemispherical glass lens, of radius r, is silvered on its curved surface. A luminous point is situated on its axis at a distance d outside the plane surface. The image formed by reflection from the latter coincides in position with the image which is formed by the rays which pass into the glass and are reflected at the spherical surface. Prove that the index of refraction of the glass is $\mu = r/(r - 2d)$.

EXAMINATION QUESTIONS

130. Taking the refractive index from air to glass as $\frac{3}{2}$; draw an accurate picture of the path of a ray of monochromatic light, which falls at an incidence of 60° on the face of a prism whose vertical angle is 30°.

K. 1894.

131. Draw carefully, and as nearly as you can to

scale, the course of a pencil of light, starting from an object 3 inches from a convex lens whose focal length is 2 inches. Find the relation between the size of the object and that of the image. Matric. 1894.

132. A bright object, 4 inches high, is placed on the principal axis of a concave spherical mirror, at a distance of 15 inches from the mirror. Determine the position and size of its image, the focal length of the mirror being 6 inches. Matric. 1888.

133. How far from a concave mirror of radius 3 feet would you place an object to give an image magnified three times? Would the image be real or virtual? Int. Sc. 1888.

134. A concave mirror of 2 feet focal length is placed 1 foot from an object: find the change in the position of the image produced by moving the object 1 inch nearer the mirror. Matric. 1890.

135. An object, 2 cm. high, is placed 1 metre away from a spherical concave mirror of 23 cm. radius of curvature. Calculate the height of the image. Will it be real or virtual? Matric. 1891.

136. State the laws of reflection of light, and explain with the aid of a diagram the formation of images by concave mirrors. An object is placed 28 cm. from a concave mirror whose focal length is 10 cm.: find where the image is. Is it real or virtual? Is it erect or inverted? And what is its size, if the object be 4·2 mm. broad by 14 mm. long. Int. Sc. 1891.

137. A system of rays emitted from a point close to the inner surface of a hollow sphere is reflected from the inside of the sphere: draw carefully the system of reflected rays, and show how to find the positions of the image of the point as seen by an eye in two or three different positions. Int. Sc. (Hons.) 1890.

138. Explain clearly how to determine the real position of an object seen straight down under water, from observation of its apparent position.

Does it appear at the same place when viewed per-

pendicularly to the surface, and when viewed in an oblique direction? Give reasons for your answers, with diagrams. Matric. 1892.

139. What is meant by saying that the refractive indices of glass and of water are 1·5 and 1·33 respectively? Show for which of these substances the *critical angle*, or *limiting angle of refraction*, is the greater. Matric. 1885.

140. What must be the refracting angle of a prism, for which the refractive index is $\sqrt{2}$, so that rays which are incident on one of the faces at angles greater than $45°$ cannot emerge from the other face? Oxf. Schol. 1890.

141. A ray of light is incident at an angle of $60°$ on a sphere made of material whose index of refraction is $\sqrt{3}$ and emerges after one internal reflection. Show that its direction on emergence is parallel to its original direction. Camb. B.A. 1893.

142. Explain the apparent raising of a picture stuck at the bottom of a cube of glass, till it appears to an eye looking down as if it were in the glass. If the index of refraction is 1·8, how much does the picture appear raised to perpendicular vision? S. K. 1889.

143. A small air-bubble in a sphere of glass 4 inches in diameter appears, when looked at so that the bubble and the centre of the sphere are in a line with the eye, to be 1 inch from the surface. What is its true distance? ($\mu = 1·5$.) Int. Sc. 1887.

144. Show how to draw the path of a ray of light carefully through a thick lens by help of focal planes. What is meant by the "optical centre" of such a lens? How may its position be found? Draw a figure to scale, for the case of a meniscus of common glass with radii 2 ft. and 3 ft. respectively. Vict. B.Sc. 1890.

145. State the laws of refraction of light, and how to find the apparent position of an object under water to an eye outside. A small object is enclosed in a sphere

of solid glass of 7 cm. radius. It is situated 1 cm. from the centre, and is viewed from the side to which it is nearest. Where will it appear to be, if the refractive index of the glass be 1·4? [Small angles may be taken as proportional to their sines.]

Int. Sc. 1891.

146. Construct the path of a ray passing through a spherical boundary of a dense medium of given refractive index.

Calculate the position of the place to which parallel rays passing nearly perpendicularly through the surface would converge, if the refractive index be 1·7 and the radius of curvature 6 feet. Int. Sc. 1893.

147. Illustrate by a figure the action of a simple convex lens of 6 inches focal length, placed close in front of an eye whose distance of distinct vision is 14 inches, and find the magnifying power. S. K. 1893.

148. Draw carefully, and as nearly as you can to scale, the course of a pencil of light, starting from an object 3 inches from a convex lens whose focal length is 2 inches.

Find the relation between the size of the object and that of the image. Matric. 1894.

149. Draw a diagram, as accurately as you can, to scale, showing the formation of an image twice the size of an object by a lens of focal length 1·5 inches. State whether the image is real or virtual, and whether the lens is convex or concave. Write down the dimensions on the diagram. Can you get more than one case?

Prel. Sc. 1892.

150. A man who can see distinctly at a distance of 1 foot, finds that a certain lens when held close to his eye magnifies small objects 6 times; determine the focal length of the lens. Camb. M.B. 1893.

151. A short-sighted person has distinct vision at 5 inches. What kind of a lens should he use, and of what

focal length, to enable him to read a book 20 inches from his eyes? Int. Sc. 1895.

152. A short-sighted person, who can see most distinctly at a distance of 6 in. from his eye, wishes to see an object 5 ft. off. What sort of a lens should he use, and what must be its focal length? Illustrate your answer with a figure. Matric. 1890.

153. A certain short-sighted person can see most distinctly at a distance of 5 in. from his eye; what must be the focal length of a lens that will give him distinct vision of an object $2\frac{1}{2}$ ft. away? Draw a figure illustrating the action of the lens. Prel. Sc. 1889.

154. Explain the action of a lens when used as an eye-glass. A man who can see most distinctly at a distance of 5 in. from his eye, wishes to read a notice at a distance of 15 ft. off. What sort of spectacles must he use, and what must be their focal length? Int. Sc. 1889.

155. Why does a short-sighted person use a concave lens? The focal length of such a lens is 6 in., and a small object is placed 18 in. from the lens; draw a figure showing the path of the rays by which the image is formed and determine its position. Prel. Sc. 1890.

156. A lens forms an image one-third the size of an object, and 2 ft. distant from itself. What is the focal length of the lens? Where is the object? Consider the case of a virtual as well as of a real image. Draw diagrams illustrating your answers. Int. Sc. 1890.

157. A person who can see most clearly at a distance of 4 in. requires spectacles enabling him to see clearly things at a distance of 1 ft. Calculate the focal length of spectacles required, and show by a diagram how they act in the case. Prel. Sc. 1891.

158. The focal length of a magnifying glass is 3 inches; it is held at a distance of half an inch from an eye whose distance of most distinct vision

is 10 inches: at what distance should the object be placed?

Give a carefully drawn diagram, showing the passage of a pencil of light through the lens, from a point of the object to the eye. Matric. 1895.

159. A man's least distance of distinct vision is $8\frac{1}{2}$ inches. He looks at a small arrow through a convex lens of 1 inch focus, held $\frac{1}{2}$ inch from his eye. Where must he place the arrow to see it best?

Draw a figure, as nearly as you can to scale, showing the positions of the lens, the object, and the image, and the course of the pencil by which the tip of the arrow is seen. Prel. Sc. 1894.

160. *The minimum deviation of a ray of light produced by passing through a prism of angle $60° \ 6' \ 20''$ is $42° \ 40' \ 20''$. Show how to use these results to determine the refractive index of the glass prism, and find it, having given—

$$L \sin 51° \ 24' = 9{\cdot}89294, \quad L \sin 30° \ 4' = 9{\cdot}69984,$$
$$L \sin 51° \ 23' = 9{\cdot}89284, \quad L \sin 30° \ 3' = 9{\cdot}69963,$$
$$\log 1{\cdot}5610 = {\cdot}19340, \quad \log 1{\cdot}5600 = {\cdot}19312.$$

Int. Sc. (Hons.) 1886.

161. An object is placed at a distance of 9 inches from a concave lens of 6 inches focal length. Make a sketch, approximately to scale, showing the paths of the rays. What is the nature of the image?

If a convex lens of 6 inches focus is placed 12 inches beyond the concave lens, find where the final image will be. Camb. Schol. 1894.

162. *A microscope is made up of an objective of $\frac{1}{2}$-inch focal length and an eyepiece of 1 inch focal length placed 6 inches apart. A person uses it to look at a small arrow-shaped object, his distance of distinct vision being 8 inches. Where must the object be placed?

Draw a careful figure showing the successive images

and the complete course of the pencil by which a point in the object is seen. Int. Sc. (Hons.) 1894.

163. A convex lens of focal length 40 cm. is placed in contact with a concave lens of focal length 66 cm. Trace the path of a pencil of rays through the combination from an object at a distance of 204 cm., and explain how it is that if the lenses are of proper materials the arrangement is achromatic for light from a distant object. Camb. Schol. 1891.

164. *What is meant by the chromatic aberration of a lens, and how is it corrected in the object-glass of a telescope? The mean refractive indices of two specimens of glass are 1·52 and 1·66 respectively; the differences in the indices for the same two lines of the spectrum are ·013 for the first and ·022 for the second; find the focal length of a lens of the second glass, which, when combined with a convex lens of 50 cm. focal length of the first, will make an object-glass achromatic for these two lines. B.Sc. 1889.

165. *Calculate the focal lengths of the separate lenses in an achromatic combination for which the focal length is to be 30 inches from the following spectroscopic measurements:—

$$\text{Crown glass } \mu_C = 1{\cdot}516, \quad \text{Flint glass } \mu_C = 1{\cdot}619,$$
$$\mu_F = 1{\cdot}525, \qquad\qquad \mu_F = 1{\cdot}636.$$

Vict. B.Sc. 1891.

166. Explain the action of Nicol's prism. You are given two such prisms: show how to place the second so that the light which emerges from it may be half as intense as that which falls on it. B.Sc. 1889.

167. Find the distance of the focal plane from a convex spherical boundary of a dense medium of index μ.

Construct the path of a ray through a glass sphere of index 1·5, and calculate its focal length in terms of its radius. Int. Sc. (Hons.) 1890.

168. Define the dispersive power of a substance.

On what principle can an achromatic lens be constructed?

If an achromatic lens of 10 feet focus has to be made of substances whose dispersive powers are as 2 to 3, find the focal lengths of each constituent lens.

Int. Sc. (Hons.) 1890.

169. *Describe the optical arrangements in an ordinary opera glass, and mention the advantages and disadvantages of this telescopic arrangement over others. If the focal length of the objective be 4″, and that of the eye-lens be $1\frac{1}{2}''$, what will be the magnifying power, and what the distance between the objective and eye-lens when focussing on a distant object? Vict. Int. Sc. 1890.

170. *The focal length of the object-glass of a microscope is half an inch, and that of the eye-piece is 1 inch. Taking the least distance of distinct vision as 12 inches, find the distance between the object-glass and the eye-piece when the object viewed is $\frac{3}{4}$ of an inch from the object-glass. Camb. Schol. 1891.

CHAPTER VIII

SOUND

Velocity of Sound.—Newton proved that the velocity of sound in any medium is given by the equation $V = \sqrt{E/D}$, E denoting the elasticity and D the density of the medium.

The *elasticity* of a fluid is defined as being the ratio of any small increase of pressure to the proportional decrement of volume thereby produced. It can be shown that the elasticity of a perfect gas is equal to its pressure, provided that its temperature remains constant during the compression.

A geometrical proof of this important proposition is given in Maxwell's *Theory of Heat.* It may also be proved as follows:—

Let V be the volume of a given mass of gas under the pressure P. Now suppose the pressure to increase by a small amount p, and let v be the decrement of volume thereby produced; the pressure is now $P+p$, and the volume $V-v$. If the gas obeys Boyle's law, the product of these two quantities is equal to the product of the original pressure and volume, or

$$\begin{aligned} PV &= (P+p)(V-v), \\ &= PV + Vp - Pv - pv. \end{aligned}$$

Since both the quantities p and v are small, their product may be neglected (p. 19); thus

$$Vp = Pv.$$

Now the *proportional decrement of volume* (or decrement per

unit volume) is $\frac{v}{V}$, so that the elasticity (by definition) is $p\Big/\frac{v}{V}$ or Vp/v. But by the last equation

$$Vp/v = P,$$

and therefore the elasticity is equal to the pressure.

We have seen (p. 93) that if the barometer stands at h cm., the atmospheric pressure is $\Pi = h\rho g$ dynes per sq. cm. This gives us for the velocity of sound in air at 0°

$$V = \sqrt{h\rho g/D},$$

or, at the normal pressure,

$$V = \sqrt{1{,}013{,}663/0{\cdot}001{,}293},$$
$$= 27{,}999 \text{ cm. per sec.},$$

whereas the velocity found by experiment is 332 metres per sec. This discrepancy between the calculated and observed values remained unexplained until Laplace's time. He pointed out that the compression produced by a sound-wave takes place so rapidly that any heat which is developed by it cannot be conducted away: the elasticity cannot therefore be calculated on the supposition that the temperature remains constant. If we assume that no heat is allowed to escape, it can be proved that the elasticity is equal to $\gamma\Pi$, Π being the pressure and γ the ratio between the specific heat of air at constant pressure and its specific heat at constant volume. Introducing this correction (known as Laplace's correction) we have

$$V = \sqrt{\gamma h\rho g/D},$$

or taking $\gamma = 1{\cdot}4$,

$$V = \sqrt{1{\cdot}4 \times 1013663/0{\cdot}001293},$$
$$= 33{,}129 \text{ cm. per sec.},$$

which is almost identical with the velocity found by experiment.

1. Explain, on general principles, why the velocity of

sound in air increases with the temperature, but is independent of the pressure; and calculate the velocity at 15°.

Note.—The velocity of sound in air at 0° is 332 metres per second.

2. Find the temperature at which the velocity of sound in air is 350 metres per second.

3. An observer sets his watch by the sound of a signal-gun fired at a station 1500 metres off: find, to a hundredth of a second, the error due to distance, the temperature being 15°.

4. Show that the left and right hand members of the equation $V = \sqrt{E/D}$ are of the same dimensions.

5. A tuning-fork is held over a tall glass jar, into which water is gradually poured until the maximum reinforcement of the sound is produced. This is found to be the case when the length of the column of air is 64·8 cm. What is the vibration number of the fork?

6. Calculate the velocity of sound in hydrogen gas, assuming its velocity in air, and having also given that 1 litre of hydrogen = 0·0896 gm., and 1 litre of air = 1·293 gm.

7. An open organ-pipe 83 cm. long is blown with air at 0°: what is the frequency of the note produced?

8. Find the length of a closed organ-pipe, which when blown with air at 0° will give the note C (256 vibrations per second).

9. Calculate the velocity of sound in water at 10°, its coefficient of elasticity at this temperature being $2{\cdot}1 \times 10^{10}$.

The velocity of sound in liquids is given by the same expression $\sqrt{E/D}$, E denoting the coefficient of elasticity (or the reciprocal of the coefficient of compressibility) and D the density.

At 10° the density of water is sensibly equal to unity, and the required velocity is

$$V = \sqrt{2{\cdot}1} \times 10^5 = 144{,}900 \text{ cm. per sec.}$$

10. By experiments made in the Lake of Geneva, Colladon and Sturm found that sound travelled in water at 8°·1 with a velocity of 1435 metres per sec. What value does this give for the elasticity of water?

11. It is found that a force equal to the weight of 3000 lbs. is required to elongate a bar of iron 1 sq. inch in section by 1/10,000 of its original length: calculate from this the velocity of sound in iron. [1 cub. ft. of iron = 480 lbs.]

In calculating the velocity of sound in solids, E is to be taken as denoting Young's modulus of elasticity, the value of which for iron, *in poundals per square foot*,[1] is $10,000 \times 3000 \times 32 \times 12^2$.

The density of iron, in lbs. per cub. ft., is given as 480. Thus

$$V = \sqrt{E/D},$$
$$= \sqrt{9 \cdot 6 \times 10^8 \times 12^2/480} = 17,000 \text{ ft. per sec.}$$

12. Calculate the value of Young's modulus for steel, having given that its density is 7·8, and that sound travels in it with a velocity of 5200 metres per second.

13. A glass rod 70 cm. long is clamped at its centre and rubbed with a wet cloth so as to throw it into longitudinal vibration. The pitch of the note indicates that the number of vibrations per second produced is 3000: find from this the velocity of sound in glass.

14. What will be the pitch of the note emitted by a wire 50 cm. in length, when stretched by a weight of 25 kilogrammes, if 2 metres of the wire are found to weigh 4·79 gm.?

1 The modulus of elasticity (Young's modulus) is defined as follows:—Suppose a bar or wire of length L and cross-section σ to be stretched by a force F, and let l denote the elongation produced; then the modulus of elasticity for the material of which it is composed is $E = LF/l\sigma$. The modulus is frequently expressed in lbs. per sq. in., or kilogrammes per sq. cm.; in such cases it must be multiplied by g, and care should be taken to use the same units consistently in the calculation, *e.g.* if we wish to find the velocity in centimetres per sec., the modulus must be expressed in dynes per sq. cm. and the density in grammes per c.c.

It can be proved that a sound-wave travels along a stretched wire or string with a velocity $v=\sqrt{F/M}$, where F is the stretching force, and M is the mass of the wire per unit of length.[1]

If λ be the wave-length of the note, and n the number of vibrations per second producing it, then $v=n\lambda$: also the length l of the stretched wire must be an exact multiple of $\lambda/2$. When the string is sounding its fundamental note, l is equal to $\lambda/2$, and

$$n=\frac{v}{\lambda}=\frac{v}{2l}=\frac{1}{2l}\sqrt{\frac{F}{M}}.$$

The stretching force in the present example is the weight of 25 kgm. $=25{,}000\times 981$ dynes; and $M=4{\cdot}79/200=0{\cdot}02395$ gm. Thus $n=\frac{1}{100}\sqrt{\frac{25{,}000\times 981}{0.02395}}=320$; and if we take the vibration number of C as 256, the note emitted will be E, for $320/256=5/4$, and this interval[2] is a major third.

15. Two similar wires of the same length are stretched—the one by a weight of 4 lbs., and the other by a weight of 9 lbs. What is the interval between the notes which they produce?

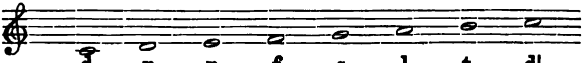
16. A stretched string 3 feet long gives the note c when vibrating transversely: what note will be given by a string 1 foot long stretched by the same weight and

[1] If the wire be of density d, $M=\pi r^2 d$, so that if we denote by t the tension of the wire per unit of sectional area, $t=F/\pi r^2$, and therefore $v=\sqrt{t/d}$. The complete expression for the number of vibrations per second produced by a wire of density d stretched by a weight P is

$$n=\frac{1}{2lr}\sqrt{\frac{Pg}{\pi d}}.$$

[2] The intervals of the diatonic scale (from C to c), and the vibration numbers of the notes, taking C=256, are as follows:—

C D E F G a b c



d r m f s l t d'

$1 : \frac{9}{8} : \frac{5}{4} : \frac{4}{3} : \frac{3}{2} : \frac{5}{3} : \frac{15}{8} : 2$

256 : 288 : 320 : 341·3 : 384 : 426·6 : 480 : 512

made of the same material, but of one-quarter the thickness?

17. A vibrating string is found to give the note *f* when stretched by a weight of 16 lbs. What weight must be used to give the note *a*? and what additional weight will give *c′*?

18. A wire 50 cm. in length and of mass 80 gm. is stretched so that it vibrates eighty times per sec.: find the stretching force in dynes.

19. A copper wire (density 8·8) 1 metre in length and 1·8 mm. in diameter is stretched by a weight of 20 kilogrammes. Calculate the number of vibrations which it makes per second when sounding its fundamental note.

20. The *e* string of a violin is tuned so as to vibrate 640 times per second; the vibrating portion has a mass of $\frac{1}{8}$ of a gramme and a length of 33 cm. What is the stretching force?

21. What is the frequency of a note which is a major third above another whose frequency is 464?

22. The vibration-numbers of three notes are 504, 630, and 945 respectively: what are the intervals between them?

23. One end of a violin-string passes over a smooth peg and carries a weight, while the other end is attached to a tuning-fork, as in Melde's experiments. The string vibrates in three segments when the weight suspended is 32 grammes: what weight must be used in order to make it divide into (1) four and (2) five segments?

24. An observer listens to a whistle sounded on a railway train as it comes toward him, and the pitch of the whistle appears to be *f*♯, but just as the train passes him the pitch falls to *f*. Show the speed of the train may be deduced from these observations.

The note actually emitted by the whistle is *f*, but while the train is approaching the observer the apparent pitch is heightened, because a larger number of sound-waves enter his ear per second (Döppler's principle).

Suppose the train to be at a distance d, and moving with a velocity of v ft. per sec. toward the observer; also let n be the vibration number of the note f. Consider what happens while the engine moves through v ft. (*i.e.* during 1 sec.) Assuming that the velocity of sound in air is 1100 ft. per sec., the first vibration, produced at a distance d, will reach the observer in a time $d/1100$. When the nth vibration is produced the train is v ft. nearer, and this last vibration will reach him in a time $d/1100 - v/1100$. Thus his ear receives n vibrations in $(1 - v/1100)$ sec., or $n/(1 - v/1100)$ per sec., which is therefore the vibration number of the note $f\sharp$. The interval between this note and f is a minor semitone, and is equal to $25/24$, so that

$$\frac{n}{1 - v/1100} = \frac{25n}{24},$$

and $$v = 1100/25 = 44.$$

Thus the speed of the train is 44 ft. per sec., or 30 miles per hour.

25. An observer listening to the whistle of an engine which is approaching him at the rate of 45 ft. per sec., notices that the pitch of the note which he hears is the same as that of a tuning-fork which makes 458 vibrations per sec. What is the actual pitch of the whistle? (Velocity of sound in air = 1100 ft. per sec.)

26. Give a graphical illustration of the manner in which "beats" are produced, and show that the number of beats per second can be calculated from the vibration numbers of the two notes producing them.

Two open pipes are sounded together, each note consisting of its first two harmonics, together with the fundamental. One note has 256 vibrations per second, the other 170. Show that two of the harmonics will produce beats at the rate of two per second.

27. A smoked-glass plate is held vertically in front of a vibrating fork provided with a style, and the plate is allowed to fall freely under the action of gravity, so that the style traces a wavy line upon it. Prove that if the number of waves marked in a distance d (starting from

rest) be n, then the vibration-number of the fork is $n/\sqrt{2d/g}$.

28. In an experiment made according to this method, it was found that in a distance of 10·9 cm. (measured from the position of rest) $68\frac{1}{2}$ waves were included. Find the vibration-number of the fork, taking $g = 980$.

EXAMINATION QUESTIONS

29. Calculate the velocity of sound at 0° C., having given the following data :—

Height of barometer, 760 mm. ; density of mercury, 13·6 grammes per c.c. ; ratio of specific heats, 1·41 ; mass of 1 litre of dry air at 0° C., 1·29 grammes ; acceleration due to gravity, 981 cm. per sec. per sec.
S. K. 1894.

30. Prove that the velocity of sound in a gas at given temperature is independent of its state of compression or rarefaction. Calculate the velocity of sound in hydrogen at − 100° C. Given that at 0° and 75 cm. of mercury, 11·4 litres of hydrogen weigh 1 gramme.
Int. Sc. (Hons.) 1892.

31. Water is compressed by 1/21,000 per unit volume by the pressure of a column of itself, 1033 centimetres high. How is this ascertained? Find from it the velocity of sound in water. Edinb. M.A. 1890.

32. The specific gravity of a certain gas, under a pressure of 75 centimetres of mercury and at 0° C., is one-thousandth that of water. What is the velocity of sound in it at that temperature? What is it also at the temperature 100°? Examine whether altering the pressure on the gas will affect the velocity.

[Take the ratio of the two specific heats as 1·4.]
S. K. (Hons.) 1888.

33. A sounding organ-pipe is warmed by means of boiling oil from 16° to 127° Centigrade. What is the effect on the note which it emits? Int. Sc. 1891.

34. What are the influences of temperature and of density on the rate of propagation of sound through a gas? The velocity of sound in a gas at 16° C. is 340 metres per second. What will it be when the pressure of the gas is doubled and its temperature raised to 168° C.? Prel. Sc. 1892.

35. The velocity of sound in water is 1440 metres per second: calculate the least length of a tube, open at one end, and closed at the other, which, when filled with water, would resonate to a tuning fork that gives 256 complete vibrations per second? What would be the length of the next longer similar tube that would resonate to the same note? Prel. Sc. 1892.

36. A rod is held in the middle and rubbed longitudinally with a resined rag till it emits a note. Describe carefully the way in which it is vibrating. How can the velocity of sound in oak be compared with that in deal? Int. Sc. 1890.

37. Prove an expression for the rate of vibration of stretched flexible strings. Calculate the pitch of the note emitted by a string half a metre long, and weighing 6 milligrammes, when stretched by a weight of 4 kilogrammes. Int. Sc. (Hons.) 1892.

38. A string is stretched by such a weight that a hump raised upon it runs along at the rate of 64 feet a second. Two points on this string, 4 feet apart, are firmly clamped to a sound-board without altering the tension of the string; if this part of the string be tweaked, what is the pitch of the note it will emit? Int. Sc. 1890.

39. A certain length of an inextensible string vibrates as a whole when stretched by a weight of 100 pounds. With what weight must double that length be stretched in order that it may vibrate at the same rate? Prel. Sc. 1891.

40. If a string 24 inches long weighs half an ounce, and is stretched with a weight of 81 lbs., find its rate of vibration when bowed or struck S. K. 1889.

41. How does the time of the fundamental transverse vibration of a wire depend—(1) on the length of the wire; (2) on the tension; (3) on the weight of and length of the wire?

A wire 25 cm. long, weighing 2 grammes, is stretched by a weight of 10 kilos. Find the period of the fundamental transverse vibration. Prel. Sc. 1894.

42. Three strings, A, B, and C, of the same length are stretched on a sonometer. Their relative weights are A : B : C :: 2 : 8 : 18, and the tensions in them are A : B : C :: 12 : 12 : 27. Calculate the ratios of their rates of vibration. Prel. Sc. 1889.

43. Find the time of the slowest vibrations of a pianoforte wire weighing 0·002 of a pound per foot, stretched between two points 5 feet apart, with a tension of 100 pounds weight. Edinb. M.A. 1890.

44. How does the velocity of transmission of a transverse wave in a stretched string depend on the tension of the string and its mass per unit of length? Given a string half a metre long, and weighing four-fifths of a gramme, what fundamental note will it emit when stretched with a weight of 10 kilogrammes?

Vict. B.Sc. 1890.

45. What is the value in C.G.S. units of Young's modulus of elasticity for a metal of which the density is 7·5, and in which sound travels with a velocity of 5600 metres per second? S. K. 1893.

46. A string weighing 1 oz. and 3 feet long is stretched by a force equal to the weight of 100 lbs.; find the frequency of the note emitted. Int. Sc. (Hons.) 1889.

47. A string 70 centimetres long and weighing 54 milligrammes is stretched by a weight of 4·04 kilogrammes and bowed transversely. What note does it give?

Int. Sc. (Hons.) 1890.

48. Calculate the velocity with which a lateral disturbance runs along a stretched string; and thus obtain

the note emitted by a string 2 feet long, weighing $\frac{1}{4}$ oz., and stretched with a weight of 256 lbs.

S. K. (Hons.) 1891.

49. If the middle C corresponds to 256 vibrations per second, how many beats would be heard per second if the note F sharp were simultaneously sounded in the natural scale and in the scale of equal temperament?

S. K. 1892.

50. *Describe the principal modes of vibration possible to a free solid bar, about 1 metre long, 10 cm. broad, and 5 cm. thick. How would you exhibit them?

If the bar were eight times as heavy as its own volume of water, and if, when vibrating longitudinally, its lowest note was three octaves above "middle C" (256 vib. per sec.), what should be the value of its Young's modulus or longitudinal elasticity? B.Sc. 1891.

51. *Find the ratio of the specific heats of a gas from the following data: A flask of 10 litres capacity weighs, when exhausted, 160 grammes; filled with the gas at a pressure of 75 cm. of mercury it weighs 168 grammes. The column of the gas which, at the same temperature as the weighing, in a tube closed at one end, responds best to a fork of 223·6 vibrations per second, is 50 cm.

Int. Sc. (Hons.) 1890.

52. What are the properties which determine the velocity of sound in a solid, a liquid, or a gas? Explain why Newton's value of the velocity of sound in air differs from the true value. Calculate the Newtonian velocity of sound in a gas whose density at standard pressure and temperature is 1 kilogram per cubic metre. What would you expect the true velocity to be?

S. K. (Hons.) 1891.

53. A locomotive whistle emitting 2000 waves per second is moving towards you at the rate of 60 miles an hour, on a day when the thermometer stands at 15° C. Calculate the apparent pitch of the whistle, and explain precisely why it is not the true pitch. S. K. 1891.

CHAPTER IX

MAGNETISM

Note.—All quantities are expressed in terms of the C.G.S. units. For the definitions of magnetic units and their dimensions, see pp. 5 and 16.

1. A magnetic pole of strength 90 is found to attract another pole 2 cm. from it with a force equal to the weight of a gramme: what is the strength of the second pole?

By the law of inverse squares, the force exerted between the two poles is equal to the product of their strengths divided by the square of their distance apart. This is equal to 981 dynes (the weight of a gramme), so that if P be the strength of the second pole,

$$981 = P \times 90/2^2,$$

and

$$P = 4 \times 981/90 = 43{\cdot}6.$$

2. The strength of a certain magnet-pole is 27: find the intensity of the magnetic field 3 cm. away from it, assuming the magnet to be so long that the influence of the other pole may be neglected. What force would be exerted by it upon a pole of strength 32 at a distance of 12 cm.?

The intensity of the magnetic field at any point is measured by the force experienced by a unit magnetic pole placed at the point. The force exerted by the given magnet-pole on an unit pole 3 cm. away from it is $27 \times 1/3^2 = 3$ dynes; and hence the strength of the field at this distance is 3.

In the second case the force would be $27 \times 32/(12^2) = 6$ dynes.

3. What is the force exerted between two poles of strength 32 and 36 units at a distance of 12 cm. from one another?

4. The repulsive force between two poles is 20 dynes when they are 4 cm. apart: what will it be when the distance between them is increased by 1 cm.?

5. A magnet-pole of strength 10 attracts another pole 5 cm. from it with a force of 2 dynes: what is the strength of the second pole?

6. The distance between two equal magnet-poles is 8 cm., and they repel one another with a force of 5 dynes: find the strength of each.

7. A magnet-pole of strength 3 placed in a certain field is acted on by a force of 8·1 dynes: what is the strength of the magnetic field?

8. Find the strength of a magnet-pole which is urged with a force of 9 dynes when placed in a magnetic field of intensity 0·18.

9. A magnet 8 cm. in length lies in a field of intensity $H = 0{\cdot}18$, and the strength of each of its poles is 5. Find the moment of the couple required to deflect it (1) through an angle of 30° from the magnetic meridian, (2) at right angles to the magnetic meridian.

The force acting on each pole in both cases is $m\mathrm{H} = 5 \times 0{\cdot}18 = 0{\cdot}9$.

(1) The arm of the couple, or perpendicular distance between the forces acting on the two poles, is $l \sin \delta = 8 \sin 30° = 8/2 = 4$, and the moment of the couple is $m\mathrm{H}l \sin \delta = 0{\cdot}9 \times 4 = 3{\cdot}6$.

(2) When the needle is at right angles to the meridian the arm of the couple is equal to the length of the needle, and the moment of the couple is $0{\cdot}9 \times 8 = 7{\cdot}2$.

10. Given that the dimensions of strength of pole are $\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}\mathrm{T}^{-1}$, show that the dimensions of strength of field are $\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{-\frac{1}{2}}\mathrm{T}^{-1}$. What is the strength of a pole which

is urged with a force of 9 dynes when placed in a field of intensity 0·5 ?

11. A freely suspended magnetic needle is deflected (1) through an angle of 45°, (2) through an angle of 60° from the magnetic meridian. Compare the couples which tend to bring the needle back to its position of rest in the two cases.

12. Prove that the magnetic force (or intensity of field) due to a bar magnet at any point on the axis of the magnet produced is equal to $2Md/(d^2 - l^2)^2$, where M is the magnetic moment of the magnet, $2l$ its length, and d the distance of the point from its centre.

Find the force due to each pole separately and subtract. Observe that since the length of the bar magnet is $2l$, its magnetic moment is $M = 2ml$.

13. Prove that the magnetic force due to the same magnet at a point opposite to its centre and at a distance d from it is equal to $M/(d^2 + l^2)^{\frac{3}{2}}$, and acts parallel to the axis of the magnet.

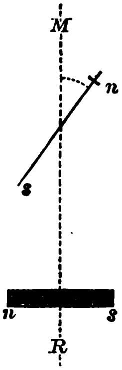


Fig. 7.

The forces due to the two poles are equal, and their components perpendicular to the axis are equal and opposite. Find the components parallel to the axis and add.

14. A magnetic needle is deflected through an angle α from the meridian by a bar magnet of magnetic moment M placed "broadside on" (*i.e.* in the position shown in Fig. 7, the line joining the centres of the two magnets being in the meridian MR, and the axis of the bar-magnet perpendicular to it.) Prove that if H be the strength of the field, and d the distance between the centres of the magnets, $M/H = (d^2 + l^2)^{\frac{3}{2}} \tan \alpha$.

Find the moments of the couples due to the action of the bar-magnet and of the earth's field, and equate.

15. Prove that the equation of equilibrium in the preceding example would take the form

$$M/H = (d^2 - l^2)^2 \tan a/2d,$$

if the bar-magnet were placed "end on," *i.e.* in an east and west position, with the centre of the deflected magnet lying on the axis of the bar-magnet produced.

16. A bar magnet forms the base of an equilateral triangle, the poles being at the extremities of the base. Find the magnitude and direction of the force at the apex of the triangle.

Note on the Time of Vibration of a Magnet.—The small oscillations of a magnet are governed by laws similar to those which regulate the vibrations of a pendulum swinging through a small arc. The time of a complete vibration of a simple pendulum is $t = 2\pi\sqrt{l/g}$: the time of a complete vibration of a magnetic needle suspended horizontally is $t = 2\pi\sqrt{I/K}$, where I is the moment of inertia of the needle, and K is the moment of the directive force tending to restore the needle to its position of rest. The quantity I depends upon the mass and dimensions of the needle; K is the product of two factors, M and H, M being the magnetic moment of the magnet, and H the horizontal intensity of the earth's magnetic force. The force acting on a needle is therefore proportional to the square of the number of oscillations which it makes in a given time.

17. A magnetic needle is suspended horizontally at a considerable height above a bar-magnet which lies on the floor underneath it. When the north pole of the bar-magnet points southwards, the needle makes 14 vibrations per minute; and when its north pole points northwards, the needle makes 8 vibrations per minute. At what rate would the needle vibrate under the action of the earth's force alone?

Let n denote the number of oscillations made per minute by

the needle under the action of the earth's force H. Then $H = kn^2$, where k is a constant.

In the first position of the bar-magnet the needle is vibrating in a field the strength of which is H + F, where F is the force due to the bar-magnet. In the second position of the bar-magnet its action is opposed to that of the earth, and the resultant field is of strength H − F.

Thus $\quad H + F = k \times (14)^2 = 196k,$

and $\quad H - F = k \times (8)^2 = 64k.$

$$\therefore\ H = 130k.$$

Again, since $H = kn^2$, it follows that $n = \sqrt{130} = 11{\cdot}4$.

18. A compass needle makes 50 oscillations per minute at a place where the dip is 64°, and 48 oscillations per minute at another place where the dip is 71°. Compare the value of the total magnetic force at the two places.

Let I denote the total force which acts along the line of dip, H the horizontal component, and θ the angle of dip; then $H/I = \cos\theta$, or $I = H/\cos\theta$ (Fig. 8). Thus at the first place we have $I = H/\cos 64° = H/0{\cdot}438$, and at the second place $I' = H'/\cos 71° = H'/0{\cdot}326$.

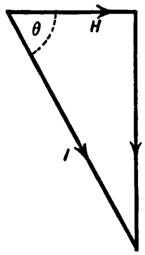


Fig. 8.

The vibrations of the compass-needle are controlled by the *horizontal component* of the total force, therefore

$$H : H' = (50)^2 : (48)^2 = 2500 : 2304,$$

and

$$I : I' = (2500/0{\cdot}438) : (2304/0{\cdot}326)$$
$$= 1 : 1{\cdot}238.$$

19. The horizontal component of the earth's magnetic force at Kew is 0·18 and at Dundee 0·16. If a needle which makes 40 oscillations per minute at Kew is transferred to Dundee, what will be its rate of oscillation?

20. A magnetic needle makes 100 oscillations in 8 min. 20 sec. under the action of the earth's force alone. Under the combined action of the earth and a magnet A

it makes 100 oscillations in 7 min. 30 sec., and when the magnet A is replaced by another B the needle makes 100 oscillations in 6 min. 40 sec. Compare the magnetic moments of A and B.

21. A compass needle makes 10 oscillations per minute under the influence of the earth's magnetism alone. When the north pole of a long magnet A is held 1 ft. south of it, the needle makes 12 oscillations per minute; and when the north pole of another magnet B is held in the same position, the number of oscillations per minute increases to 15. Compare the pole-strengths of the magnets A and B.

22. At Berlin the total magnetic intensity is 0·48 (in C.G.S. units) and the dip is 64°: at New York the total intensity is 0·61 and the dip 72°. If a magnet vibrating horizontally at Berlin makes 20 oscillations in a minute, how many oscillations would it make in the same time at New York?

23. At a certain place a dip-needle makes an angle of 60° with the horizontal. When a weight of 1 gm. is attached to the upper end, the inclination is reduced to 30°: what weight would make it horizontal?

24. A dip-circle is rotated (in azimuth) through an angle α from the magnetic meridian, and the apparent angle of dip under these conditions is θ': prove that the true dip (θ) at the place is given by the equation

$$\tan\theta = \tan\theta' \cos\alpha.$$

25. Discuss the precise advantages of the method usually adopted for determining the magnetic dip (*i.e.* by observing the position in which the needle points vertically downwards, and then rotating the dip-circle through 90°); and prove that the true dip may be found from observations in *any* two azimuths at right angles by the formula

$$\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2,$$

θ_1 and θ_2 being the observed angles of dip in any two planes at right angles, and θ being the true dip.

EXAMINATION QUESTIONS

26. A bar magnet, hung horizontally by a fine wire, lies in the magnetic meridian when the wire is without twist. It is then found that when the top of the wire is twisted through 120° the magnet is deflected through 30°. Through what further angle must the top of the wire be twisted in order to turn the magnet perpendicular to the magnetic meridian? S. K. 1893.

27. A bar magnet is suspended horizontally in the magnetic meridian by a wire without torsion. To deflect the bar 10° from the meridian the top of the wire has to be turned through 180°. The bar is removed, remagnetised, and restored, and the top of the wire has now to be turned through 250° to deflect the bar as much as before. Compare the magnetic moments of the bar before and after remagnetisation. S. K. 1894.

28. What is meant by the horizontal intensity of the earth's magnetic field?

A horizontally suspended magnet vibrates 12 times per minute at a place where the horizontal intensity is 0·180. How many times per minute will it vibrate at a place where the horizontal intensity is 0·245?

Int. Sc. 1893.

29. Two bar magnets, the moments of which are in the ratio 8 : 27, are placed with their centres 3 feet apart, their magnetic axes being in the same straight line, which is perpendicular to the magnetic meridian. If their north poles are turned towards each other, find the position which a small compass needle must occupy on the line joining the magnets in order that it may point in the same direction as if the magnets were not there. S. K. 1889.

30. A magnetic needle makes a complete vibration

in a horizontal plane in 2·5 seconds under the influence of the earth's magnetism only, and when the pole of a long bar-magnet is placed in the magnetic meridian in which the needle lies, and 20 cm. from its centre, a complete vibration is made in 1·5 seconds. Assuming $H = \cdot 18$ (C.G.S.), and neglecting the torsion of the fibre by which the needle is suspended, determine the strength of the pole of the long magnet. Int. Sc. (Hons.) 1886.

31. Two small straight magnets, of magnetic moments M, M′, are placed in the same straight line with similar poles facing each other. Show that, if their dimensions are negligible in comparison with r, the distance between their centres, the force tending to separate them is $6MM'r^{-4}$. S. K. 1894.

32. The north pole of a very long vertical magnet whose strength was 250, was placed at a perpendicular distance of 20 cm. from the centre of a horizontal magnetic needle whose length was 5 cm. and strength 50. Find the moment of the couple acting upon the small magnet. Camb. Schol. 1891.

33. Explain how the law of force between two magnetic poles has been established.

A small galvanometer needle, swinging freely under the earth's magnetic force alone, makes 3 complete oscillations per sec.; the strength of the field in which the needle hangs is diminished by the control magnet of the galvanometer until the time of a complete oscillation is 2 secs. Find how much the "zero-position" of the needle will be altered by a change in the magnetic declination of 1 min. of arc. By how many millimetre-scale-divisions of a reflecting galvanometer will the change of zero be represented, if the scale be 50 cm. from the mirror? N. S. Tripos 1890.

CHAPTER X

ELECTROSTATICS

Note.—All quantities are expressed in terms of the C.G.S. units. For the definitions of the electrostatic units and their dimensions, see pp. 4 and 17.

1. Two small spheres are at a distance of 5 cm. apart: one has a charge of 10 units of electricity, the other a charge of 5 units. What is the force exerted between them?

It follows from Coulomb's law, and from the definition of the unit quantity of electricity, that the force (in dynes) is equal to the product of the charges divided by the square of the distance between the spheres.

Thus $F = 10 \times 5/5^2 = 50/25 = 2$ dynes.

If the two charges are of the same kind (*i.e.* both positive or both negative) the force will be one of repulsion; if the one charge is positive and the other negative, the force will be one of attraction.

2. Two small electrified bodies at a distance of 12 cm. apart are found to attract one another with a force of 6 dynes. The one has a positive charge of 32 units: what is the charge of the other?

3. What is the distance between two small spheres which have charges of 32 and 36 units respectively, and repel one another with a force of 8 dynes?

4. Express in dynes the repulsive force exerted be-

tween two small spheres 15 cm. apart, and charged respectively with 40 and 45 units of electricity.

5. Two small spheres are 10 cm. apart, and one of them has a charge of 45 units: what must be the charge on the other so that the force exerted between them may be equal to the weight of 5 milligrammes?

6. Determine the relation between the electrostatic unit of quantity in the metre-milligramme-minute system and the corresponding C.G.S. unit.

7. An electrified ball is placed in contact with an equal and similar ball which is unelectrified: on being separated 8 cm. from one another the force of repulsion between them is equal to 16 dynes. What was the original charge on the electrified ball?

Since the balls are of equal size the charge will be equally shared between them when they are placed in contact. Let q be the charge on each: then the repulsive force between them is $(q^2/8^2)$, and this is equal to 16 dynes. Thus $q^2=8^2\times 16$, and $q=8\times 4=32$. The original charge on the electrified ball was $2q=2\times 32=64$ units.

8. Two small equal balls, one having a positive charge of 10 units and the other a negative charge of 5 units, are 5 cm. apart: what is the attractive force between them? If they are made to touch, and again separated by the same distance, what will be the force of repulsion?

9. A small uncharged sphere is placed in contact with an equal charged sphere, and then removed to a distance of 6 cm. They repel each other with a force of 4 dynes: what was the original charge? At what distance must they be placed so that the force may be diminished to 1 dyne?

10. Two small spheres, each charged with 50 units of electricity, are placed at two of the corners of an equilateral triangle 1 metre on the side: what is the magnitude and direction of the resultant electric force at the third corner?

11. What charge is required to electrify a sphere of 25 cm. radius until the surface-density of the electrification is $5/\pi$?

The *surface-density* is the quantity of electricity per unit of surface. Thus if S be the area of the surface, and σ the surface-density, the charge is $Q = S\sigma$. The area of the surface of a sphere of radius r is $4\pi r^2$: thus $S = 4\pi \times (25)^2$, and $Q = 4\pi \times (25)^2 \times 5/\pi = 20 \times 625 = 12500$.

12. A sphere of 5 cm. radius has a charge of 1000 units of electricity: what is the surface-density of the charge?

13. What charge must be imparted to a spherical conductor of 3 cm. diameter in order that the superficial density of the electrification may be 7? [Take $\pi = 22/7$.]

14. A magnetised knitting-needle, carrying a small gilt pith-ball at one end, is suspended horizontally by a silk fibre: a second pith-ball, of the same size as the first, is electrified and brought into contact with it. Prove that the charge on the second pith-ball is proportional to $\left(\sin \frac{\alpha}{2}\right)^{\frac{3}{2}}$, where α is the angle through which the knitting-needle is deflected from the magnetic meridian.

15. Three small electrified spheres, A, B, and C, have charges 1, 2, and 4 respectively. Find the position in which B must be placed between A and C in order that it may be in equilibrium. Prove also that there is another position along the line CA produced in which B will be equally repelled by A and C.

16. A small conductor A is held at a distance of 30 cm. from a suspended pith-ball, both being positively electrified. When a second small conductor B, negatively electrified, is placed 20 cm. from the pith-ball on the line joining it to A, the ball is neither attracted nor repelled. Compare the charges of A and B. How

would the pith-ball be affected if B were placed (1) close to A but not touching it, (2) in contact with A?

17. The bob of a seconds pendulum consists of a sphere of mass 16 grammes, and it is suspended by a silk thread. Vertically beneath it is placed a second sphere, which is positively electrified, and when the pendulum-bob is negatively electrified its time of oscillation is found to be 0·8 sec. Prove that the attractive force between the two spheres is equal to the weight of 9 grammes. (The arc of vibration is supposed to be so small that the attractive force is always along the vertical.)

Potential and Capacity.—It can be shown [1] that if a quantity q of electricity be collected at a given point, the difference of potentials due to it at any two given points A and B, whose distances from the given point are r and r' respectively, is

$$V_A - V_B = q/r - q/r'.$$

If the point B is removed to an infinite distance, or is connected with the earth, r' becomes ∞, and $q/r' = 0$. If we agree to regard the potential of the earth as zero, the expression for the difference of potentials between A and the earth, or, more briefly, the potential of the point A, reduces to

$$V_A = q/r.$$

If instead of a single quantity of electricity there are several charges q_1, q_2, q_3, . . . whose distances from the given point are r_1, r_2, r_3, . . . respectively, then the potential at A due to all these charges is

$$V_A = \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ . \ . \ . \ = \Sigma\left(\frac{q}{r}\right).$$

The external action of an electrified spherical con-

[1] Clerk Maxwell, *Elementary Treatise on Electricity*, Art. 86; Silvanus P. Thompson, *Electricity and Magnetism*, Art. 238.

ductor is the same as if all the charge were collected at its centre. If the charge be Q, the potential due to it at any external point, whose distance from the centre of the sphere is r, is Q/r. This is only true when r is not less than the radius R of the sphere. At the surface of the sphere $r = R$, and the potential is Q/R. Now the capacity (C) of the sphere is measured by the charge required to raise its potential from zero to unity, or

$$C = Q/V,$$

where Q is the charge and V the potential due to it. But we have seen that $V = Q/R$. Hence $C = R$, or—

The capacity of a spherical conductor (placed in air at a considerable distance from other conductors) is numerically equal to its radius.

18. A hollow spherical conductor, whose radius is 1 decimetre, is charged with 10 units of electricity: find the potential (1) at the surface of the sphere, (2) inside it, (3) at points distant 15 and 25 cm. from its centre. If the sphere is connected by a long thin wire with a second conducting sphere of 1 cm. radius, what will be the charge and potential of each sphere?

(1) At the surface $V = Q/C = Q/R = 10/10 = 1$.
(2) The potential is constant and equal to unity throughout the whole interior of the sphere.
(3) At the first point $V = 10/15 = 0\cdot\dot{6}$; at the second $V = 10/25 = 0\cdot4$.

After connection is made the two spheres will be reduced to a common potential. The capacity of the system (neglecting the capacity of the fine wire) is equal to the sum of the capacities of the two spheres $= 10 + 1 = 11$. Their joint charge is equal to the original charge $= 10$. Hence their common potential is $V = Q/(C + C') = 10/11 = 0\cdot\dot{9}\dot{0}$. The charge of the first sphere is $q_1 = CV = 10 \times 10/11 = 9\cdot\dot{0}\dot{9}$; the charge of the second sphere is $q_2 = C'V = 1 \times 10/11 = 0\cdot\dot{9}\dot{0}$.

Observe that the charge is shared in the ratio of 10 to 1; *i.e.* in the ratio of the capacities of the conductors.

19. Charges of 50 units of electricity are placed at each of the corners of a square whose side is 1 metre: find the potential at the point of intersection of the diagonals.

20. Two conductors, of capacity 10 and 15 respectively, are connected by a fine wire, and a charge of 1000 units is divided between them: find the charge which each takes, and the potential to which it is raised.

21. A conductor of capacity 75 is charged to a potential 20, and is then made to share its charge with a second conductor of capacity 25: what will be the final charge and potential of each?

22. Three spheres of capacity 1, 2, and 3 are charged to potentials 3, 2, and 1 respectively, and are then connected by a fine wire: what is their common potential?

23. Is it correct to say that the electric potential of a body is a *condition* of that body, just as its temperature is, and if not, why not? How could you alter the potential of a body without touching it or altering its charge?

24. Find the dimensions of capacity in the electrostatic system and calculate the capacity of a sphere of 5 metres radius, in that system in which the decimetre, gramme, and minute are taken as units.

25. Two small uncharged metallic spheres are hung up by silk threads in an electrical field; they are connected together for a moment by a fine wire, which is then removed. If the spheres are now examined, what will be their electrical state? If no charge is found upon either of the spheres, what conclusion would you draw as to the relative potentials of the points occupied by the centres of the spheres at the instant when the wire was removed?

26. Two spheres, each of 1 cm. diameter, are connected by a wire, and are at the same potential 40. The force of repulsion between them is 5 dynes: what is their distance apart? What will be the force when the distance between them is half a metre?

27. What is an equipotential surface? Show that the work done by or against the electrical forces during the transference of a charged body from one equipotential surface to another is independent of the path along which the body moves.

28. Two spheres, of 2 and 3 cm. radius, are charged respectively to potentials 5 and 10: what will be their common potential if they are placed in electrical connection?

29. An insulated spherical conductor is charged with electricity: how is the charge distributed over its surface? Sketch and describe the form of the lines of force and the equipotential surfaces in its neighbourhood. Explain how these and the distribution of the charge will be affected by bringing near to the sphere an uncharged conductor (1) when the latter is insulated, (2) when it is connected to earth.

30. Two spheres, of 2 and 6 cm. radius, are charged respectively with 80 and 30 units of electricity: compare their potentials. If they are connected by a fine wire, how much electricity will pass along it?

31. Two small electrified spheres of the same size are placed at a distance of 4 cm. apart, and the charge of the one is double that of the other. The two spheres are brought into contact, and are then removed to a distance of 6 cm. from one another: find the ratio between the repulsive forces in the two cases.

32. A charged sphere of radius r is made to share its charge with a second sphere of radius r'; prove that the density of the electrification on the first is to that on the second as $r' : r$. (See Ex. 11.)

33. A conducting sphere of 25 cm. radius is electrified to potential 100. It is then made to share its charge with a Leyden jar of unknown capacity, and the potential is found to fall to 20. What is the capacity of the Leyden jar?

34. The diameters of two spheres are 4 cm. and 6

cm. respectively, and the potential of the second is one-third greater than that of the first: compare the surface-densities of their charges.

35. A small sphere, charged with 4 units of positive electricity, is 12 cm. from a second insulated sphere charged with 9 units of negative electricity. The second sphere is removed, touched with an unelectrified sphere of one half its diameter, and is then placed 8 cm. from the first sphere. Prove that the attractive forces in the two cases are as 2 : 3.

36. A sphere of 25 cm. radius is charged until its surface density is $5/\pi$: what is its potential?

37. A number of small insulated spheres situated on the circumference of a circle are charged to different potentials: prove that the potential at the centre of the circle will not be altered if all the spheres are connected together by fine wires whose capacity may be neglected.

38. A conducting sphere of radius r is charged to potential v: a second conducting sphere of radius r', which is hollow, is charged to potential v'. The second sphere is now opened so as to admit the first, which is allowed to touch it. Find the potential and charge of each of the spheres.

39. Two small insulated bodies, of capacities a and b, receive charges A and B respectively. What will be their common potential if they are connected by a long fine wire, and how much electricity will flow along the wire?

40. In the preceding question, find the point on the line joining the two small bodies at which the potential is the same before and after contact is made by means of the wire.

Capacity of a Spherical Air-Condenser.—The capacity of a spherical air-condenser, consisting of an insulated sphere of radius r, and a concentric spherical shell of radius r', is $rr'/(r'-r)$.

For let M (Fig. 9) be the common centre, and suppose a charge Q of positive electricity (" + Q ") to be imparted to the inner sphere A. This will induce an equal negative charge, – Q, upon the internal surface of the outer spherical shell B. If B is insulated there will also be a charge + Q repelled to its outer surface, but this positive induced charge can be removed by placing B in contact with the earth. The potential of the outer sphere will now be zero, and so also will be the potential at any external point N.

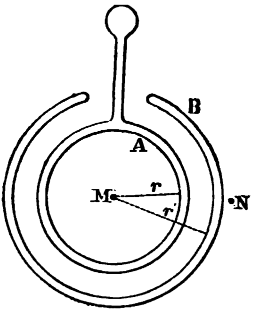


Fig. 9.

For if d be the distance of N from M, the potential at N due to the charge on A (see p. 245) is $+Q/d$: the potential at N due to the charge on B is $-Q/d$, and the sum of these two quantities is zero.

At the surface of A the potential due to its own charge is Q/r, and that due to the charge on B is $-Q/r'$ (for the potential due to the charge on the outer sphere B at any point inside it is constant, and is equal to the potential at the surface of B due to this charge).

Thus the resultant potential of A is $Q/r - Q/r'$. The capacity of the condenser is numerically equal to the charge required to raise the potential of the inner sphere from zero to unity, or

$$C = \frac{Q}{Q/r - Q/r'} = \frac{1}{1/r - 1/r'} = \frac{rr'}{r' - r}.$$

Specific Inductive Capacity.—The *Specific Inductive Capacity* of a substance is the ratio between the capacity of a condenser containing the given substance as dielectric and that of an equal and similar air-condenser. If the capacity of the latter be C, then the capacity of

the former is $C' = Ck$, where k is the specific inductive capacity of the dielectric which it contains. If the condenser consist of two concentric spherical surfaces, the full expression for its capacity is $C' = k \cdot rr'/(r' - r)$.

41. An uncharged condenser containing a solid dielectric is placed in electrical connection with an equal and similar air-condenser charged to the potential V. If the common potential after sharing the charge is V′, what is the specific inductive capacity of the dielectric?

If C be the capacity of the air-condenser, its original charge was $Q = VC$. The joint capacity of the system after connection is made is $(C + C')$, where C' is the capacity of the uncharged condenser: and the total charge is $V'(C + C')$. This is also equal to Q, for no electricity is supposed to be lost in sharing the charge.

$$\therefore\ VC = V'(C + C').$$

If k be the specific inductive capacity of the dielectric, $C' = Ck$,

$$\text{and} \qquad VC = V'C(1 + k),$$
$$\text{or} \qquad k = (V - V')/V'.$$

42. A charge Q is imparted to an uninsulated air-condenser consisting of two concentric spherical surfaces of radii r and r'. Prove that at any point between the two surfaces, and at a distance d from their common centre, the value of the potential is $Q/d - Q/r'$.

43. Find the capacity of a spherical condenser, the inner coating of which has a diameter of 60 cm., and the dielectric being glass of specific inductive capacity 6·8 and thickness 1·5 cm.

44. Two condensers are similar in every respect, excepting that one contains air as the dielectric, and the other turpentine of specific inductive capacity 2·16. If a charge of 500 units is divided between them, what charge will each take?

45. Two metal spheres, whose radii are 10 cm. and 5 cm. respectively, are connected by a long fine wire, and the smaller sphere is surrounded by a concentric

spherical shell of 5·5 cm. internal radius. If a charge of 520 units is divided between them, what will be the charge and potential of each?

46. A condenser consists of two concentric spherical surfaces whose radii are 100 mm. and 101 mm. respectively. The space between them is filled with sulphur of specific inductive capacity 3·4. Find the radius of an insulated spherical conductor which would have the same capacity as this condenser.

Energy of a Charged Condenser.—It follows from the definition of difference of potential that if Q units of electricity are transferred from a place at which the potential is V to another place at which the potential is V′, the work done by the electrical force is $Q(V-V')$ ergs: if $V'=0$ (as, for example, if the charge is removed to an infinite distance, or allowed to escape to earth), then the work done is QV. But all this is on the supposition that the potential at both places remains constant and is unaltered by the transference of the charge; which would be approximately correct if the charge were transferred from one conductor of large dimensions (and therefore large capacity) to another conductor of large dimensions, such as the earth.

The case is different when the potential of the charged body is due to its own charge; for if the conductor or condenser is of finite capacity C, and if a charge Q is imparted to it, it is raised to a definite potential V, which is given by the equation $C=Q/V$. If the electricity is allowed to escape gradually to earth, then as the charge diminishes from Q to zero, so the potential falls from V to zero, the *average* potential during the process being $V/2$. It can be proved that the work done in the discharge is measured, not by the product of the charge into the potential, but by the product of the charge into the *average* potential during the discharge, and is therefore equal to $QV/2$.

For the discharge may be supposed to take place by n separate steps, such that in each of these separate discharges a quantity q of electricity is removed. The potential is Q/C at the beginning, and $(Q-q)/C$ at the end of the first discharge; hence the work done in this first discharge is *less than* qQ/C and *greater than* $q(Q-q)/C$. The difference between these quantities is q^2/C, and the difference between either of them and

the correct expression for the work done in this partial discharge is *less than* q^2/C.

The potentials at the beginning of the first, second, nth discharges are Q/C, $(Q-q)/C$, . . . $(Q-\overline{n-1}\cdot q)/C$ respectively. The amounts of work done in these separate discharges may be represented by qQ/C, $q(Q-q)/C$, . . . $q(Q-\overline{n-1}\cdot q)/C$ respectively. If we add together the n terms so as to find the total work done in the discharge, we shall make an error which is *less than* $n\times q^2/C$. Now the whole charge Q is equal to $n\times q$, so that the error is less than qQ/C; and if n be made very large q becomes so small that the quantity qQ/C may be neglected.

The total work done is

$$W=\frac{q}{C}\left\{Q+(Q-q)+\ .\ .\ .\ (Q-\overline{n-1}\cdot q)\right\},$$

$$=\frac{q}{C}\left\{nQ-(1+2+\ .\ .\ .\ +\overline{n-1})q\right\}.$$

Now the sum of the first n natural numbers is $n(n+1)/2$, and the sum of the first $(n-1)$ natural numbers is $(n-1)n/2$.

Thus

$$W=\frac{Q^2}{C}-\frac{(n-1)nq^2}{2C}=\frac{Q^2}{2C}+\frac{Qq}{2C},$$

and, as we have seen, the last term can be neglected when n is made very large.

Since $Q=VC$, the energy of the charge (*i.e.* the work done in the discharge) is equal to $Q^2/2C$, or $QV/2$, or $CV^2/2$.

The work done against the electrical forces in charging the conductor or condenser is also equal to $\frac{1}{2}QV$. This follows necessarily from the principle of the conservation of energy: it may also be proved independently by a method similar to that which we have adopted above.

47. A Leyden jar of capacity C_1 is charged and discharged. It is again charged to the same potential, and partially discharged by allowing it to share its charge with an empty jar of capacity C_2. Lastly, the two jars are separately discharged. Compare the energies of the four discharges.

The energy of the original charged jar is $Q^2/2C_1$, Q denoting the quantity of electricity which it contains.
The same quantity is next shared between the two jars whose joint capacity is C_1+C_2, so that their common potential after the charge is shared is $V=Q/(C_1+C_2)$.
The energy of the charge of the first jar is now

$$\frac{C_1V^2}{2}=\frac{C_1}{2}\left(\frac{Q}{C_1+C_2}\right)^2,$$

and that of the second jar is

$$\frac{C_2V^2}{2}=\frac{C_2}{2}\left(\frac{Q}{C_1+C_2}\right)^2.$$

The energy lost in the partial discharge (*i.e.* in sharing the charge) is

$$\frac{Q^2}{2C_1}-\frac{C_1+C_2}{2}\cdot\left(\frac{Q}{C_1+C_2}\right)^2=\frac{C_2Q^2}{2C_1(C_1+C_2)}.$$

Thus the energies of the four discharges, taken in the order in which they occur, are as

$$\frac{1}{C_1}:\frac{C_2}{C_1(C_1+C_2)}:\frac{C_1}{(C_1+C_2)^2}:\frac{C_2}{(C_1+C_2)^2},$$

i.e. as $(C_1+C_2)^2:C_2(C_1+C_2):C_1^2:C_1C_2$.

48. An air-condenser, whose armatures are concentric spheres of diameter 20 and 24 cm. respectively, is charged to potential 50: find the work done in charging it.

49. A condenser of capacity 10 is raised to a potential 30: what is its charge, and how much work is done in charging it?

50. A spherical air-condenser, the coatings of which have radii of 16 and 17·5 cm. respectively, is charged to a potential 12, its outer coating being in contact with earth. Calculate its energy.

51. Two insulated spheres, of 12 cm. and 3 cm. radius, are charged respectively with 36 and 24 units of electricity: compare their potentials, and the energies of their charges.

52. Compare the energy of a condenser of capacity 9 charged to potential 3, with that of a condenser the charge of which is 18, and the potential 12.

53. Compare the work done in charging a Leyden jar of capacity 30 to potential 15 with that required to charge a jar of capacity 20 to potential 45.

54. The armatures of a condenser are concentric spheres of diameter 20 and 24 cm. respectively, and the space between them is filled with shellac of S.I.C. = 3. The condenser has a charge of 7560 units. If it is discharged in such a way that all the energy is converted into heat, how much heat will be produced? [$J = 4 \cdot 2 \times 10^7$.]

The capacity of the condenser is $3 \times 10 \times 12/2 \times 180$. The energy of its charge is $Q^2/2C = (7560)^2/360 = 158760$ ergs.

Let H denote the amount of heat (in terms of the calorie or gramme-degree) resulting from the discharge: then $JH = E$, or

$$4 \cdot 2 \times 10^7 \times H = 158760,$$

and $\therefore H = 0 \cdot 003782$ calorie.

55. The capacity of a condenser is 700; to what potential must it be charged in order that the energy of its discharge may be equivalent to one heat-unit?

56. Two equal and similar Leyden jars have their outer coatings connected to earth: one is uncharged, the other is charged to potential V. Find the energy to each after the charge has been shared, and show that one-half of the energy of the charged jar is dissipated in the spark which passes when their knobs are connected.

57. The coatings of a charged Leyden jar are connected with those of an uncharged jar of double the capacity. Compare the energy of the system with that of the original charged jar.

58. What would be the solution of the preceding example if the linear dimensions of the uncharged jar were double those of the charged jar, other things being equal?

59. A, B, and C are three equal conductors, of which A alone is charged. A is first made to share its charge with B, and then the remainder is shared with C. Compare the energy of each of the three charges with that originally possessed by A.

60. A conductor of capacity 100 is charged to potential 50. Show that its energy is the same as that of a stone of mass 400 gm. moving at the rate of 25 cm. per second.

61. A charged sphere of radius r is made to share its charge with an uncharged sphere of radius r': prove that the energy of the original distribution is to that of the final distribution as $(r+r') : r$.

62. A Leyden jar is charged and then connected up with nine uncharged jars so as to form a battery of ten equal jars: show that the energy of the whole battery is only one-tenth that of the single jar.

63. A condenser is charged to a given potential and then discharged. It is again charged to the same potential, and made to share its charge with another condenser of half its capacity, after which the jars are separately discharged. Compare the energy of discharge in each case.

64. A Leyden jar is charged with electricity; an equal charge is imparted to a battery consisting of four equal and similar jars, the inner coatings of which are connected together. Compare the energies of the two charges.

65. The outer armatures of two spherical air-condensers are of the same radius, viz. 20 cm. The first is charged, and the radius of its inner armature is 15 cm.: the second is uncharged, and its inner armature has a radius of 18 cm. Prove that if the first condenser is made to share its charge with the second, three-fourths of its energy will be lost.

66. A given quantity of electricity is to be shared between a number of conductors of different capa-

cities. Prove that the energy of the system is a minimum when all the conductors are charged to the same potential.

67. *The upper disc of an absolute electrometer has a diameter of 8 centimetres, and the distance between it and the lower disc is 0·45 centimetre. Find the difference of potential between the two discs when a force equal to the weight of 3·6 grammes is required to balance the attraction between them. $(g = 981)$.

EXAMINATION QUESTIONS

68. How much energy is expended in carrying a charge of 50 units of electricity from a place where the potential is 20 to another where it is 30?

What is meant by saying that the potential of a conductor is 20? S. K. 1894.

69. The inner coating of one spherical Leyden jar, whose surfaces have radii 12 and 14 respectively, is charged with 25 units of positive electricity, and the inner coating of another, with surfaces of radii 8 and 12, is charged with 5 positive units, the outer coatings of both being earth connected. Their inner coatings are then momentarily joined by a fine wire; in which direction will electricity pass, the dielectric in both jars being air, and the distance between the jars considerable? Give full reasons for your answer. S. K. 1894.

70. A condenser is composed of two square plates each 10 cm. in side; the plates are 1 mm. apart. It is found that 500 ergs are needed to charge it to a certain difference of potential. Find the difference of potential and the charge on either plate. Camb. Schol. 1893.

71. Two insulated and widely separated metallic spheres receive charges of positive electricity which raise

their potentials to 4 and 5 respectively. The densities of the charges being in the ratio 4 : 9, compare the radii of the balls. S. K. 1889.

72. Two equally charged spheres repel each other when their centres are half-a-metre apart with a force equal to the weight of 6 milligrams. What is the charge on each, in electrostatic units? S. K. 1892.

73. Define electrostatic unit quantity of electricity. Two pith-balls each having 20 units are hung up from the same point by silk threads 100 centimetres long. The pith-balls weigh each $\frac{1}{10}$ gramme. At what distance will they keep each other by their mutual repulsion? Glasg. M.A. 1890.

74. Two spheres, of diameters 9 and 3, are connected by a long thin wire, and 144 units of electricity are shared between them. Compare their charges; and describe how to study the distribution of electricity on their surfaces when they are brought nearer together. From which sphere would a brush discharge first occur? Give a reason. Int. Sc. 1890.

75. A spherical conductor of 4 cm. radius which, if isolated, would be at a potential of 200 units, is placed inside an uncharged spherical conductor of 8 cm. radius, and they are brought into contact. Find the potential of the larger sphere.

The smaller sphere is then removed and brought into contact with the larger sphere externally: find the final charge of the smaller sphere. Camb. Schol. 1895.

76. A sphere of radius 40 mm. is surrounded by a concentric sphere of radius 42 mm., the space between the two being filled with air. What is the relation between the capacity of this system and that of another similar system in which the radii of the spheres are 50 and 52 mm. respectively, and the space between them is filled with paraffin of specific inductive capacity 2·5? S. K. 1894.

77. Find the number of volts in an electrostatic unit of potential. S. K. (Hons.) 1892.

78. *What is a farad? Calculate in terms of a microfarad the capacity of an air condenser in which the plates are parallel and each 1 square metre in area, the distance between them being 1 millimetre.

Vict. B.Sc. 1891.

79. *Describe some form of Thomson's guard-ring electrometer, and how it is to be used to measure differences of potential in absolute electrostatic units.

Define this absolute unit of potential, and show how the instrument's determinations are in the units specified in your definition.

If the trap-door is 5 centimetres diameter, is 3 millimetres from the under plate, and is at 1200 volts potential difference; what force of attraction would you expect between them? Int. Sc. (Hons.) 1891.

80. *Electric tension being defined as 2π times the square of the electric density, show that it must be greater on projecting and highly convex portions of a conductor than elsewhere. If its value is known on an isolated sphere of known size, how can you estimate the potential of such a sphere? What potential would enable the electric tension on an isolated sphere 3 centimetres radius to balance the two-thousandth part of atmospheric pressure? B.Sc. 1891.

81. *Describe and give the theory of the trap-door electrometer.

A thunder-cloud is over still water. What is the electrostatic surface-density of the charge on the water if it rises under the cloud 0·1 cm. above its previous level? B. Sc. 1893.

CHAPTER XI

CURRENT ELECTRICITY

Ohm's Law.—The current which flows along any conductor is directly proportional to the electromotive force (or difference of potential) between its ends, and is inversely proportional to its resistance. Thus if C denote the current and E the electromotive force (or E.M.F.),

$$C \propto \frac{E}{R}, \text{ or } C = k \cdot \frac{E}{R},$$

R being the resistance of the conductor, and k a constant.

If we agree to define the resistance of a conductor as being the ratio between the E.M.F. along it and the current thereby produced ($R = E/C$), the constant k becomes equal to unity, and Ohm's law may be expressed by the equation

$$C = \frac{E}{R}.$$

This equation holds good when C, E, and R are expressed in terms of the C.G.S. electromagnetic units defined on p. 5, and also when these three quantities are expressed in terms of the so-called practical units (current in ampères, E.M.F. in volts, and resistance in ohms).

1. An incandescent lamp takes a current of 0·7 ampère, and the E.M.F. between its terminals is found to be 98 volts: what is its resistance?

Since C is expressed in ampères and E in volts, the resistance of the lamp, in ohms, will be given by the equation

$$R = E/C = 98/0 \cdot 7 = 140.$$

2. The E.M.F. of a battery (or difference of potential between its poles on open circuit) is 15 volts: when the poles are connected by a copper wire a current of 1·5 ampères is produced, and the potential difference between the battery poles falls to 9 volts. Find the resistance of the wire and the internal resistance of the battery.

The resistance (R) of the wire is the ratio of the difference of potential between its ends (9 volts) to the current thereby produced (1·5 ampère),

$$i.e.\ R = V/C = 9/1 \cdot 5 = 6 \text{ ohms.}$$

Notice that 9 volts is the potential difference causing the flow of current through the wire of resistance to 6 ohms; the *total E.M.F.* acting round the circuit is 15 volts. Call this E, and the resistance of the battery B: applying Ohm's law to the complete circuit, we have $E = C(B + R)$,

$$i.e.\ 15 = 1 \cdot 5(B + 6) = 1 \cdot 5B + 9,$$

and $$B = 6/1 \cdot 5 = 4 \text{ ohms.}$$

3. The wire used on Indian telegraph lines is iron wire of No. 2 B.W.G., having a resistance of 4·6 ohms per mile. The batteries consist of Minotto cells of 1·04 volt E.M.F. and 30 ohms resistance per cell. Assuming that the resistance of the instruments is 80 ohms, and that a current of 8 milli-ampères is required to work them, find how many cells should be employed on a line 200 miles in length.

If n be the number of cells required, the E.M.F. of the battery is 1·04 n volt, and its internal resistance is 30 n ohms. The resistance of the line is $4 \cdot 6 \times 200 = 920$ ohms, and that of the instruments is 80 ohms; the total external resistance (assuming that of the return circuit through the earth to be negligible) is 1000 ohms.

A *milli-ampère* is one-thousandth of an ampère, so that the required current is 0·008 ampère. By Ohm's law,

$$C = 1{\cdot}04\,n/(30\,n + 1000), \text{ and this} = 0{\cdot}008,$$
$$\therefore 1{\cdot}04\,n = 0{\cdot}24\,n + 8,$$
$$\text{i.e. } 0{\cdot}8\,n = 8, \text{ and } n = 10.$$

4. A current of 8·5 ampères flows through a conductor, the ends of which are found to have a difference of potential of 24 volts: what is its resistance?

5. If an incandescent lamp of 80 ohms resistance takes a current of 0·75 ampère, what E.M.F. is required to work it?

6. A glow lamp takes a current of 1·32 ampère and the E.M.F. between its terminals is found to be 66 volts: what is its resistance while hot?

7. A battery consists of 5 Daniell cells, each having an E.M.F. of 1·08 volt and an internal resistance of 4 ohms: what current will the battery produce with an external resistance of 7 ohms?

8. You are required to send a current of 2 ampères through an electromagnet of 3·5 ohms resistance, and are supplied with a number of Grove cells each of 1·9 volt E.M.F. and 0·25 ohm internal resistance: how many cells are required?

9. One end (A) of a wire ABC is connected to earth; the other end (C) is kept at a constant potential of 100 volts. If the resistance of the portion AB is 9·6 ohms and that of BC 2·4 ohms, what current will flow along the wire, and what will be the potential of the point B?

10. A Bunsen cell has an internal resistance of 0·3 ohm and its E.M.F. on open circuit is 1·8 volt. The circuit is completed by an external resistance of 1·2 ohm: find the current produced and the difference of potential which now exists between the terminals of the cell. [See Ex. 2.]

11. On adding 3 ohms to the resistance of a certain circuit the current is diminished in the ratio of 6 to 5: what was the original resistance, and how much should be added to this in order to bring the current down to half its original value?

12. How many cells, each of 1·8 volt E.M.F. and 1·1 ohm internal resistance will be required to send a current of 0·5 ampère through an external resistance of 50 ohms?

13. Four Grove cells, each having a resistance of 0·25 ohm, are connected up with an electromagnet of 5 ohms resistance, and the current has only half the required strength: how many additional cells of the same kind must be used to produce the desired effect?

14. Two cells, of E.M.F. 1·8 volt and 1·08 volt respectively, are placed in a certain circuit in opposition (*i.e.* with their poles in such positions that the cells tend to send currents in opposite directions). The current is found to be 0·4 ampère: what current will be produced if the cells are placed properly in series?

15. The poles of a battery are connected up to the electrodes of a reflecting galvanometer. When the battery is on open circuit the galvanometer reading is 300; but when the circuit is completed by a wire of 10 ohms resistance the deflection falls to 200. What is the internal resistance of the battery?

16. The poles of a battery of 5 cells are connected by a wire 8 metres long, having a resistance of 0·5 ohm per metre; each cell has an E.M.F. of 1·4 volt and a resistance of 2 ohms: find the distance between two points on the wire such that the E M.F. between these points is 1 volt.

17. The poles of a battery of 4 cells are connected by a wire 8 ft. in length, and the resistance of each cell of the battery is equal to that of 1 foot of the wire. Compare the E.M.F. acting along a portion of the wire, 3 ft. in length, with the E.M.F. of a single cell on open circuit.

18. A current is sent by a battery of constant E.M.F. (1) through a resistance of 20 ohms, (2) through a wire of unknown resistance, and (3) through a resistance of 40 ohms. The currents produced are in the ratio of

10 : 9 : 8; find the resistance of the battery and that of the wire.

19. The poles of a battery consisting of 40 Daniell cells in series are connected by a resistance of 280 ohms, and the current produced is 0·0535 ampère; when the external resistance is increased to 1080 ohms the current is reduced to one half: find the average resistance and E.M.F. of each cell of the battery, and determine the difference of potential existing between the poles of the battery when the external resistance is 280 ohms.

20. A Bunsen cell of 1·8 volt E.M.F. and a Planté cell (or accumulator) are connected up with a resistance of 400 ohms, the cells being in opposition, and the strength of the current is observed. On rearranging the cells, so that they tend to send currents in the same direction, it is found that the resistance has to be increased to 4000 ohms in order to reduce the current to its former value. Assuming that the resistances of the cells may be neglected, find the E.M.F. of the accumulator and the current produced.

21. With an external resistance of 9 ohms a certain battery gives a current of 0·43 ampère: when the external resistance is increased to 32 ohms the current falls to 0·2 ampère. What is the resistance of the battery?

22. The external resistance in a certain circuit is two-thirds that of the battery: what change will be produced in the current by reducing the internal resistance to half its former value, the E.M.F. remaining unchanged?

23. A No. 7 Brush dynamo gives an E.M.F. of 830 volts when running at the rate of 750 revolutions per minute, and its internal resistance is 11 ohms: show that such a machine can supply 16 arc lamps in series, each lamp offering a resistance of 4·5 ohms and requiring a current of 10 ampères.

24. The instruments on a telegraph circuit have a resistance of 230 ohms, and the line itself offers a resist-

ance of 13 ohms per mile: how many Daniell cells, each of E.M.F. = 1·06 volt and internal resistance = 4 ohms, will be required to send a current of 10 milli-ampères through 100 miles of such a line?

25. The current along a telegraph line is tested at two stations whose respective distances from the sending battery are 50 and 150 miles. The current in the latter case is one-half that in the former. If the galvanometer has a resistance equal to that of 15 miles of the line wire, prove that the battery resistance is equal to that of 35 miles of wire.

Resistance.—The resistance of a conductor (of uniform section) is directly proportional to its length and inversely proportional to the area of its cross-section.

If l_1 and l_2 are the lengths of two uniform conductors made of the same material, s_1 and s_2 the areas of their cross-sections, the ratio of their resistances (R_1 and R_2) is

$$\frac{R_1}{R_2} = \frac{l_1/s_1}{l_2/s_2} = \frac{l_1 s_2}{l_2 s_1} \qquad (1)$$

If the conductors are cylindrical wires of radii r_1 and r_2 respectively, than $s_1 = \pi r_1^2$, and $s_2 = \pi r_2^2$,

$$\therefore \frac{R_1}{R_2} = \frac{l_1 r_2^2}{l_2 r_1^2}. \qquad (2)$$

The resistance of a conductor further depends upon the material of which it is made. The *specific resistance* of a substance in the C.G.S. system is defined to be the resistance between the opposite faces of a cube, 1 cm. in the side, made of the substance.

If ρ be the specific resistance of a conductor of length l and cross-section s, its resistance, *in C.G.S. units*, is $\rho \times l/s$: expressed in *ohms* its resistance is $\rho \times l/s \times 10^9$, for one ohm $= 10^9$ C.G.S. units of resistance (p. 6).

If the conductor is a cylindrical wire of radius r its resistance is

$$R = \rho l / \pi r^2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

TABLE OF (C.G.S.) SPECIFIC RESISTANCES.

Silver	1,520
Copper	1,600
Aluminium	2,900
Platinum	9,000
Iron	9,700
Lead	19,500
Mercury	94,340

The numbers in the above table are only given for the purposes of calculation, as representing approximately the specific resistances of pure metals at 0° : the resistance of a conductor depends to some extent upon its physical state, and commercial metals usually contain impurities which considerably increase their resistance.

The specific resistance of a metal can be determined by measuring the resistance of a length l of wire made of the metal and then finding its diameter : this can be done directly by a micrometer wire-gauge, or, more accurately, by weighing the wire in air and in water so as to find its mass m and density δ. The cross-section of the wire is then given by the equation $(\pi r^2) l \delta = m$, from which we have $\pi r^2 = m / l \delta$.

By equation (3) the resistance of the wire is $R = \rho . l / \pi r^2$,

$$\left. \begin{aligned} \therefore\ R &= \rho . l^2 \delta / m, \\ \text{and} \quad \rho &= m R / l^2 \delta \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If R_1 and R_2 are the resistance of two wires made of the same substance, l_1 and l_2 their lengths, m_1 and m_2 their masses, then

$$\frac{R_1}{R_2} = \frac{l_1^2 / m_1}{l_2^2 / m_2} = \frac{l_1^2 m_2}{l_2^2 m_1} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

26. Find the resistance at 25° of a copper wire 10 metres long and 1 mm. in diameter. The resistance of copper increases by 0·39 per cent for each degree rise in temperature.

The resistance of the wire at 0° is

$$R_0 = 1600 \times 1000/\pi \times (0{\cdot}05)^2,$$
$$= 2{\cdot}037 \times 10^8 \text{ C.G.S. units} = 0{\cdot}2037 \text{ ohm.}$$

At 25° the resistance is

$$R_{25} = 0{\cdot}2037(1 + \overline{0{\cdot}0039 \times 25}) = 0{\cdot}2236 \text{ ohm.}$$

27. A uniform glass tube 92·1 cm. in length was filled with mercury, and the resistance of the column of mercury was measured and found to be 1·059 ohm; the weight of the mercury contained in the tube was 10·15 gm. Calculate from this experiment the specific resistance of mercury, taking its sp. gr. as 13·6.

Expressed in C.G.S. units the resistance of the column is $1{\cdot}059 \times 10^9$, and therefore by equation (4) the specific resistance is

$$\rho = 10{\cdot}15 \times 1{\cdot}059 \times 10^9/(92{\cdot}1)^2 \times 13{\cdot}6 = 93177.$$

28. Two wires of the same length and material are found to have resistances of 4 and 9 ohms respectively: if the diameter of the first is 1 mm., what is the diameter of the second?

29. The resistance of a bobbin of wire is measured and found to be 68 ohms: a portion of the wire 2 metres in length is now cut off, and its resistance is found to be 0·75 ohm. What was the total length of wire on the bobbin?

30. Compare the resistances of two wires A and B, given that they are of the same weight and material, but that B is nine times as long as A.

31. Copper wire one-twelfth of an inch in diameter has a resistance of 8 ohms per mile: what is the resistance of a mile of copper wire the diameter of which is $\frac{1}{36}$ in.?

32. Calculate the resistance of a lead wire 5 metres long and 1 mm. in diameter.

33. If copper wire of No. 21 B.W.G. (diameter

= 0·032 in.) has a resistance of 54 ohms per mile, what is the resistance of a mile of No. 13 copper wire (diameter = 0·096 in.)?

34. A mile of telegraph wire 2 mm. in diameter offers a resistance of 13 ohms; what is the resistance of 440 yards of wire 0·8 mm. in diameter made of the same material?

35. The resistance at 0° of a column of mercury 1 metre in length and 1 sq. mm. in cross-section is called a "Siemen's Unit." Find the value of this unit in terms of the ohm.

36. Copper wire of No. 20 on the new standard wire-gauge has a resistance of 0·026 ohm per metre, and weighs 5·84 gm. per metre: what is the resistance of a metre of No. 32 S.W.G. copper wire weighing 0·524 gm.?

37. Compare the resistances of two wires, one of which weighs 20·5 gm. and is 4·5 metres long, while the other weighs 82 gm. and is 18 metres long.

38. A trough 2 cm. deep and 2·5 cm. broad is cut in a wooden board. The trough is 2 metres long and is half filled with mercury: find the resistance between its ends.

39. What length of platinum wire 1 mm. in diameter is required in order to make a 1-ohm resistance coil?

40. A wire m metres in length and $1/n$th of a millimetre in diameter is found to have a resistance r: what is the specific resistance of the material of which it is made?

41. Find the resistance of an iron wire ABC which consists of two parts, the first (AB) being 60 cm. long and 1 mm. in diameter, and the second (BC) being 3 metres long and 1·6 mm. in diameter.

42. The poles of a battery are connected by a wire whose resistance is equal to that of the battery, and the poles of a second exactly similar battery are connected by a wire of the same weight and material, but three

times as long as the first. Compare the currents in the two cases.

43. Find the length of an iron wire one-twentieth of an inch in diameter which will have the same resistance as a copper wire one-sixtieth of an inch in diameter and 720 yards long, the conducting power of copper being six times that of iron.

44. Express in microhms (or millionths of an ohm) the resistance of a strip of silver 10 cm. long, 0·5 cm. broad, and 0·1 mm. thick.

45. A wire made of platinoid (German-silver containing a small percentage of tungsten) is found to have a resistance of 0·203 ohm per metre. The cross-section of the wire is 0·016 sq. cm.: express the specific resistance of platinoid in microhms.

46. The density of aluminium is 2·7: what is the resistance of an aluminium wire 1 metre long and weighing 1 gm.?

47. Two wires, the one of aluminium and the other of copper, are of the same length and have the same resistance. Compare their weights, having given that the densities of aluminium and copper are as 1 to 3·6, and their specific resistances as 1 to 0·55.

48. The central conductor of the Bessbrook and Newry electrical tramway is made of steel having a specific resistance of 0·0000121 ohm, and costing £7 : 10s. per ton: high conductivity copper of 0·0000016 ohm specific resistance would have cost £84 per ton. The cross-section of the steel conductor was 8·817 sq. cm.: calculate the cross-section of a copper conductor of the same resistance, and show that it would have cost half as much again as the steel conductor.

The Tangent Galvanometer.—A tangent galvanometer consists of a circular coil of wire placed with its plane in the magnetic meridian, and at the centre of which is suspended a small magnetic needle.

The force exerted by a current flowing through the coil upon a magnet pole placed at (or very near to) its centre is directly proportional to the current-strength, the total length of the coil and the strength of the pole, and is inversely proportional to the square of the distance between each element of the circuit and the pole.

Let r be the radius of the coil, n the number of turns, and m the pole-strength of the magnet: then if the current C is expressed in terms of the C.G.S. electromagnetic unit (as defined on p. 5), the force exerted upon *each* pole of the magnet is equal to $2\pi nr \times Cm/r^2 = 2\pi nCm/r$.

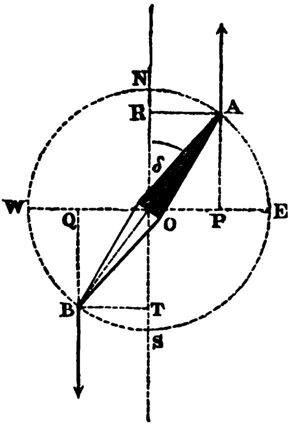


Fig. 10.

Suppose the needle to be deflected through an angle δ from the magnetic meridian NS (see Fig. 10) so as to lie along AB. It is acted upon by a *deflecting couple* due to the equal forces at A and B (along RA and TB respectively), and the arm of this couple is RT, which is equal to $l \cos \delta$, l being the length AB of the needle. Thus the moment of the deflecting couple is

$$2\pi nCm \times l \cos \delta/r.$$

The earth's field acts upon each pole with a force mH[1] in the direction of the arrows: the arm of this *directive couple* is $PQ = l \sin \delta$, and its moment is $mH \times l \sin \delta$.

[1] The letter H is the symbol usually adopted for representing the horizontal intensity of the earth's magnetic force.

For equilibrium these moments must be equal, or

$$2\pi n C m . l \cos \delta / r = m H . l \sin \delta,$$

and $$\therefore\ C = \frac{r}{2\pi n} . H \tan \delta \quad . \quad . \quad . \quad (6)$$

The quantity $rH/2\pi n$, by which the tangent of the deflection has to be multiplied in order to obtain the current-strength, may be called the *reduction-factor* of the galvanometer: but we shall use this term to denote the value of the quantity k in the equation $C = k . \tan \delta$, where the current C is understood to be expressed in *ampères* instead of C.G.S. units.

49. A current of 0·85 ampère flows through a tangent galvanometer consisting of three turns of wire, each of 30 cm. diameter: what is the strength of the field due to the circular current at its centre? If the value of H at the place is 0·17, through what angle will the needle be deflected?

The strength of the field produced by the current is measured by the force which would be experienced by a unit magnetic pole, and this by the preceding paragraph is $2\pi n C/r$. Since an ampère is one-tenth of the C.G.S. unit of current, the required strength of field is $2\pi \times 3 \times 0{\cdot}085/15 = 0{\cdot}1068$.

By equation (6)

$$\tan \delta = \frac{2\pi n}{r} . \frac{C}{H} = \frac{2\pi \times 3}{15} \times \frac{0{\cdot}085}{0{\cdot}17} = 0{\cdot}628.$$

Referring to the table of tangents we see that since $\tan 39° = 0{\cdot}629$, the angular deflection will be nearly 39°.

50. The same current is sent through two concentric circular wires of 1 and 3 decimetres radius respectively, and a small magnet is suspended at their common centre. The currents flow in opposite directions: compare their joint effect upon each pole of the magnet with that produced by a single coil of 2 decimetres radius traversed by the same current.

The force due to the coil of 1 dcm. radius is $f=2\pi Cm/10$, and that due to the coil of 3 dcm. radius is $f'=2\pi Cm/30$. Their joint effect (since the forces act in opposite directions) is

$$F=f-f'=2\pi Cm\left\{\frac{1}{10}-\frac{1}{30}\right\}=2\pi Cm\times\frac{2}{30}.$$

The force due to the coil of 2 dcm. radius is $F'=2\pi Cm/20$.

Thus $$F : F'=\frac{2}{30}:\frac{1}{20}=4:3.$$

[This is not the ratio of the *deflections*. If the deflections are δ and δ' respectively, then $\tan\delta : \tan\delta'=4:3$.]

51. Find the strength of a current which produces a deflection of 45° in a tangent galvanometer consisting of a single copper rod bent into a circle of 40 cm. diameter. [$H=0{\cdot}17$.]

52. A current of 0·04 ampère flows through a circular coil of wire consisting of 3 turns, each 2 dcm. in diameter: what force will be exerted by the current upon a magnetic pole of strength 10 placed at the centre of the coil?

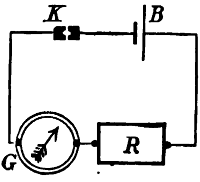


Fig. 11.

53. A battery is connected up in series with a known resistance R and a tangent galvanometer of resistance G (Fig. 11). The deflection of the galvanometer needle is α: on increasing the known resistance to R′ the deflection falls to α'. Show that the internal resistance of the battery is given by the equation

$$B=\frac{R'\tan\alpha'-R\tan\alpha}{\tan\alpha-\tan\alpha'}-G.$$

54. Calculate the internal resistance of a Grove's cell from the following data, obtained by the method of Ex. 53:—

Resistance of Galvanometer $=0 \cdot 102$ ohm.

(1) $R=2$ ohms, $\alpha=53°$.
(2) $R'=5$ ohms, $\alpha'=30°$.

55. A Daniell cell is connected up in series with a tangent galvanometer of 1 ohm resistance and a box of resistance coils. When a resistance of 2 ohms is taken out of the box the deflection of the galvanometer is 60°, and when the resistance in the box is increased to 20 ohms the deflection falls to 30°. Find the resistance of the cell.

56. What does the equation in Ex. 53 reduce to when the second deflection indicates a current of one-half the original strength?

A current from a battery of 4 Daniell cells was sent through a resistance-box and a tangent galvanometer of negligible resistance. With a resistance of 58 ohms a deflection of 55° was obtained, and when the resistance was increased to 131 ohms the deflection fell to $35\frac{1}{2}°$. Find the resistance of the battery, given that $\tan 55° = 2 \tan 35\frac{1}{2}°$.

57. A Minotto cell gave the following results, according to the method of Ex. 53:—

(1) $R=37$ ohms, $\alpha=55°$.
(2) $R'=151$ ohms, $\alpha'=35\frac{1}{2}°$.

Resistance of Galvanometer $=36$ ohms.

What was the resistance of the cell?

58. A deflection α is obtained with a tangent galvanometer G connected up as in Fig. 11 with a battery B and a known resistance R. When R is replaced by a wire of unknown resistance the galvanometer deflection is α': show that the value of the unknown resistance is $(\tan \alpha / \tan \alpha')(B+G+R)-(B+G)$.

59. Two single concentric circles of wire are placed with their planes in the magnetic meridian, and a small magnet carrying a mirror is suspended at their common centre. The same current is sent through both circles

successively, and the deflections of the spot of light are 40 and 65 : find the ratio of the diameters of the circles.

60. The terminals of a battery are connected by a wire 10 ft. long; the wire is wrapped once round the frame of a tangent galvanometer and a certain deflection is produced. The wire is now removed and replaced by another piece of the same wire 50 ft. long. It is found that this second wire has to be coiled three times round the galvanometer frame in order to produce the same deflection. Prove that the resistance of the battery is equal to that of 10 feet of the wire.

61. A tangent galvanometer has two coils, one a thick-wire coil of resistance G, the other a thin-wire coil of resistance G′, and the ratio of the reduction-factors (see p. 271) of the two coils is R. When the first coil is connected up through an unknown resistance with a battery of constant E.M.F. and negligible resistance, a deflection α is produced; and when it is replaced by the second coil the deflection is α'. Prove that the unknown resistance is equal to $(RG - R'G')/(R' - R)$, where $R' = \tan \alpha'/\tan \alpha$.

62. The battery used on a certain telegraph line has a resistance equal to that of 30 miles of the line wire. The current is tested at a station 120 miles from the battery by a galvanometer wound with twenty turns of wire having the same resistance per metre as the line wire. Show that in order to obtain the same deflection at a distance of 270 miles from the battery the number of turns of wire on the galvanometer would have to be doubled. (Neglect the resistance of the return circuit through earth, and assume that the coils of wire on the galvanometer have the same radius throughout.)

63. The current from a battery is sent through a tangent galvanometer, the reduction-factor (p. 271) of which is k, and it produces a deflection α: when an additional resistance r is introduced into the circuit the deflection falls to α'. Show that the E.M.F. of the battery is equal to $kr \,.\, \tan \alpha \,.\, \tan \alpha' / (\tan \alpha - \tan \alpha')$.

64. The poles of a battery of negligible resistance are connected by a tangent galvanometer of resistance G and a wire of unknown resistance. A deflection α is produced: on increasing the resistance of the circuit by a known amount R, the deflection is diminished to β. Show that the resistance of the wire is equal to

$$R \tan \beta/(\tan \alpha - \tan \beta) - G.$$

Electrolysis.—The weight of an element which is set free by electrolysis in one second by a current of unit strength is called the electro-chemical equivalent of that element. In giving the value of the electro-chemical equivalent of an element it is necessary to state whether the current strength is expressed in C.G.S. units or ampères. According to Lord Rayleigh's experiments a current of one ampère deposits 0·001118 gm. of silver, or 0·0003296 gm. of copper per second.

The weight of an element which is set free by a current of strength C in time *t* is given by the equation

$$W = Cet \quad . \quad . \quad . \quad . \quad (7)$$

e denoting the electro-chemical equivalent of the element.

Electrolysis provides us with a convenient means of finding the reduction-factor (p. 271) of a tangent galvanometer. Connect up the galvanometer G, as in Fig. 12, with an electrolytic cell or voltameter V and a constant battery B consisting of one or more Daniell cells or accumulators. The voltameter may consist of a pair of copper plates immersed in a beaker containing a saturated solution of copper sulphate: for more accurate work a silver voltameter should be employed. Let W denote the weight of metal deposited in time *t* by a constant current C which produces a deflection α of the gal-

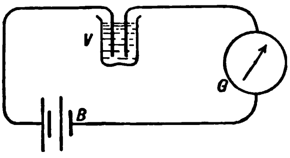


Fig. 12.

vanometer needle. If k is the reduction-factor of the galvanometer, $C = k \cdot \tan \alpha$: also, by equation (7),

$$C = W/et,$$

$$\therefore k \cdot \tan \alpha = W/et,$$

and $$k = W/et \cdot \tan \alpha.$$

65. A current of 1 ampère is used for the purpose of preparing pure silver. How long will it take to deposit a gramme?

66. How much copper will be deposited by a current of one ampère in an hour?

67. A current of 0·5 ampère is used for preparing pure silver by electrolysis: how long must the current be allowed to flow in order to obtain a deposit of 4 grammes?

68. What is the strength of a current which deposits a milligramme of copper per minute?

69. Express in ampères the value of a current which deposits a gramme of silver in 20 minutes.

70. It is found that a current of 1·868 ampère deposits 1·108 gm. of copper in half an hour: what value does this give for the electro-chemical equivalent of copper?

71. What is the strength of a current which deposits 0·935 gm. of copper in an hour and 10 minutes?

72. A Siemen's C_2 dynamo is capable of depositing 6 kilogrammes of copper per hour: what is the strength of the current produced by it?

73. A battery of three Daniell cells is connected up in series with a copper voltameter in which 31·7 gm. of copper is deposited in an hour. How much copper is deposited and zinc dissolved in the whole battery in the same time. [Atomic weight of copper = 63·4; of zinc = 65.]

74. A constant current was sent for half an hour through the thick coils of a Helmholtz galvanometer,

producing a deflection of 40°, and also through a copper voltameter in which 1·325 gm. of copper was deposited. What was the reduction-factor of the galvanometer.

75. Assuming the value of the electro-chemical equivalent of silver as given on p. 275, find how much water will be decomposed by a current of 1 ampère in an hour. [Atomic weight of silver = 108, of oxygen = 16, of hydrogen = 1.]

Kirchoff's Laws.—I. In any branching network of wires, the algebraical sum of all the currents which meet at a point is zero, or

$$\Sigma(C)=0.$$

II. The electromotive force acting around any closed circuit is found by multiplying the resistance of each portion of the circuit into the current which flows through it, and adding together the products thus obtained, or

$$\Sigma(E)=\Sigma(CR).$$

Divided Circuits and Shunts.—Suppose a circuit to divide as at A in Fig. 13, part of the current flowing through a conductor of resistance r and part through a conductor of resistance r', reuniting at B; then the two wires are said to be arranged in parallel or in multiple arc, and the resistance of the divided circuit between A and B is equal to the product of the resistances of the two branches divided by their sum.

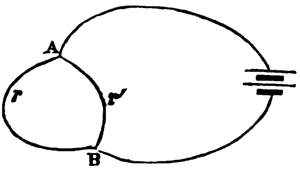


Fig. 13.

For let e be the E.M.F. between A and B, c the current through the branch r, and c' the current through the branch r'. By Ohm's law

$$c=e/r, \text{ and } c'=e/r' \quad . \quad . \quad . \quad (8)$$

Now apply Kirchoff's first law to the point A. Let C be the current in the main circuit flowing *towards* A; then c and c' flow *away* from A,

$$\therefore C - c - c' = 0,$$

or

$$C = c + c' = e/r + e/r'.$$

The equivalent resistance of the divided circuit is

$$R = \frac{e}{C} = \frac{e}{e/r + e/r'} = \frac{1}{1/r + 1/r'} = \frac{rr'}{r + r'} \quad . \quad . \quad (9)$$

It is evident from the above equation (8) that the currents in the two branches are inversely proportional to their resistances.

Also, by equation (9),

$$e = CR = C \cdot rr'/(r + r'),$$

therefore the current in the branch r is

$$c = e/r = C \cdot r'/(r + r'),$$

and the current in the branch r' is

$$c' = e/r' = C \cdot r/(r + r').$$

By similar reasoning it can easily be proved that the resistance of a multiple arc consisting of n conductors, each offering a resistance r, is $R = r/n$.

When the terminals of a galvanometer are connected directly by a wire, or through a resistance-box, the galvanometer is said to be *shunted*, and the connection is called a *shunt*. If C is the current in the main circuit, G the resistance of the galvanometer, and S that of the shunt, the current through the shunted galvanometer is

$$C_g = C \cdot \frac{S}{G + S},$$

and the current through the shunt is

$$C_s = C \cdot \frac{G}{G + S}.$$

76. Eight incandescent lamps are arranged in four

parallel groups of two each (as in Fig. 14) between two electric light leads. The difference of potential between the leads is 108 volts, and each lamp takes a current of 1·2 ampère. What is the equivalent resistance between the leads?

Since the current through each of the four branches is 1·2 ampère, the resistance of each group (of two lamps in series) is $108/1 \cdot 2 = 90$ ohms, or the resistance per lamp is 45 ohms. The joint resistance of the four groups in parallel is $90/4 = 22 \cdot 5$ ohms. (Here we assume that the resistance of the leads is very small, and that the lamps are placed close together, so that the E.M.F. is constant.)

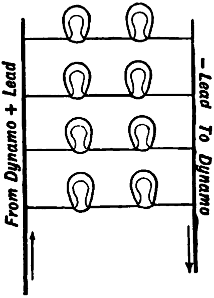


Fig. 14.

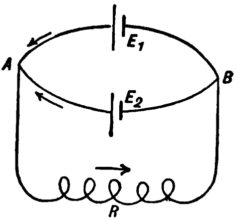


Fig. 15.

77. Two batteries of E.M.F.'s E_1 and E_2, and resistances r_1 and r_2, are arranged in parallel so that they tend to send currents in the same direction through an external resistance R (Fig. 15). What is the strength of the current in the external circuit?

Let C be the current through the external resistance, c_1 the current through the battery E_1, and c_2 the current through the battery E_2.

By Kirchoff's second law, we have in the circuit E_1CR

$$c_1r_1 + CR = E_1,$$

and in the circuit E_2CR

$$c_2r_2 + CR = E_2.$$

By Kirchoff's first law $C = c_1 + c_2$,

$$\therefore C = (E_1 - CR)/r_1 + (E_2 - CR)/r_2,$$

$$Cr_1r_2 = E_1r_2 + E_2r_1 - C(r_1R + r_2R),$$

and $$C = (E_1r_2 + E_2r_1)/(r_1r_2 + r_1R + r_2R).$$

78. A 1-ohm coil is tested and found to have a resistance of 1·004 ohm. What length of platinoid wire having a resistance of $16\frac{2}{3}$ ohms per metre must be wound in multiple arc between the terminals in order to make it a correct ohm?

If x is the required resistance of the platinoid wire, we have, by equation (9),

$$1 = 1{\cdot}004x/(1{\cdot}004 + x),$$

$$\therefore 1{\cdot}004 + x = 1{\cdot}004x,$$

or $$1{\cdot}004 = 0{\cdot}004x, \text{ and } x = 251.$$

The shunt wire must therefore have a resistance of 251 ohms, and the required length is $251 \times 3/50 = 15{\cdot}06$ metres.

79. A uniform wire is bent into the form of a square: find the resistance between two opposite corners in terms of the resistance of one of the sides.

80. What is the resistance between the extremities of one of the sides in the preceding example?

81. Twelve incandescent lamps are arranged in parallel between two electric light leads. The difference of potential between the leads is 99 volts, and each lamp takes a current of 0.75 ampère: what is the equivalent resistance between the leads?

82. A divided circuit consists of two wires of the same material whose lengths are l and l', and cross-sections s and s' respectively: compare the currents in the two wires.

83. The resistance of a circuit, including that of the battery, is 18 ohms. What change will be produced in the current through the battery if two points of the circuit, between which the resistance is 12 ohms, are connected by a wire of 4 ohms resistance?

84. A standard coil having a resistance of one legal ohm is connected in multiple arc with a resistance-box, and you are required to take plugs out of the box until the joint resistance of the two is exactly equal to a B.A. unit (see p. 6). A legal ohm is equal to 1·0112 B.A. units: how much resistance must be taken out?

85. Three wires whose resistances are r_1, r_2, and r_3 respectively are connected in multiple arc: prove that the resistance of the multiple arc is $r_1r_2r_3/(r_1r_2 + r_2r_3 + r_3r_1)$.

86. Two points A and B are connected by three wires whose resistances are 1, 3, and 6 ohms respectively: find the total current which passes through the multiple arc when the difference of potential between A and B is 3 volts.

87. A divided circuit consists of two equal and similar wires: what alteration will be produced in its resistance by making the wires touch so that a point one-quarter the length from an end of the one wire is in contact with a point three-quarters the length from the corresponding end of the other wire?

88. An equilateral triangle, one foot in the side, is made of wire of uniform material but of unequal thickness; the base being made of wire 0·8 mm. thick, and the sides being 1·0 mm. and 1·2 mm. respectively: find the resistance between the two extremities of the base, taking the resistance of an inch of the base wire as unity.

89. An uniform wire ABCD of resistance 12 ohms is bent into the form of a square of which the ends at A are soldered together and the opposite corners A and C are connected up to a battery of 6 ohms resistance. Compare the current in the battery with that which would pass through it if A and C were connected by a piece of the same wire.

90. A skeleton cube is made up of twelve uniform wires: prove that the resistance between diagonally opposite corners of the cube is equal to five-sixths of the resistance of one of the wires.

91. The resistances in the arms of a Wheatstone bridge are 5, 15, 20, and 60 ohms, so that the bridge is balanced and no current passes through the galvanometer. The E.M.F. of the battery is 2 volts and its resistance is 4 ohms: what current passes through it?

92. A tangent galvanometer of 10 ohms resistance is shunted by a shunt of resistance 0·64 ohm. A steady current is passed for an hour through the shunted galvanometer, and also through a silver voltameter in which 0·532 gm. of silver is deposited, the deflection of the galvanometer being 52°. Express the strength of the current in ampères, and calculate the reduction factor of the galvanometer.

If C be the strength of the current, then, by the equation $W = Cet$, we have

$$0{\cdot}532 = C \times 0{\cdot}001118 \times 3600 = C \times 4{\cdot}025,$$
$$\therefore C = 0{\cdot}532/4{\cdot}025 = 0{\cdot}1321 \text{ ampère.}$$

The current passing through the galvanometer is

$$C_g = C \times S/(G+S),$$

where S is the resistance of the shunt, and G that of the galvanometer. But if k be the required reduction-factor, we have also

$$C_g = k \,.\, \tan \delta,$$

and $\therefore k = C_g/\tan \delta = C \,.\, S/(G+S) \tan \delta,$

$$= 0{\cdot}1321 \times 0{\cdot}64/10{\cdot}64 \times 1{\cdot}28 = 0{\cdot}00621.$$

93. A battery B of low resistance is joined up in circuit with a resistance-box R and a shunted galvanometer as in Fig. 16. After noting the deflection the shunt is removed, and it is found that the resistance R has to be increased to a higher value R′ in order to reduce the galvanometer deflection to its original value. Prove that, if the internal resistance of the battery be neglected, the resistance of the galvanometer is equal to S(R′ − R)/R.

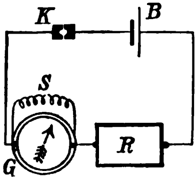


Fig. 16.

94. A battery of resistance B is connected up with a galvanometer of resistance G. The galvanometer is then shunted by a wire of resistance S. Compare the currents produced by the battery in the two cases, and show that a *compensating resistance* equal to $G^2/(G+S)$ must be introduced into the circuit if it is required to keep the current unaltered on shunting the galvanometer.

95. What shunt is required to reduce to one-hundredth the sensitiveness of a galvanometer of 396 ohms resistance?

96. A galvanometer of 45 ohms resistance is shunted by a shunt of 5 ohms. Find the resistance of the shunted galvanometer and the current which flows through it when a difference of potential of 22·5 volts is maintained between its terminals.

97. A battery of 20 ohms resistance is joined up in circuit with a galvanometer of 10 ohms resistance. The galvanometer is then shunted by a wire of the same resistance as its own: compare the currents produced by the battery in the two cases.

98. In the preceding example determine the ratio between the currents which flow through the galvanometer before and after it is shunted.

99. In measuring the resistance of a battery by Thomson's method, it is joined up in circuit with a galvanometer G and a resistance-box R as shown in Fig. 17, the battery itself being shunted by a wire of resistance S. On removing the shunt a larger current passes through the galvanometer; but by increasing the resistance R to some higher value R′ the same deflection can be obtained. Prove that the resistance of the battery is given by the equation

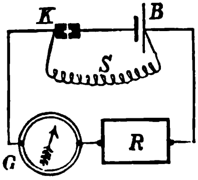


Fig. 17.

$$B = S(R' - R)/(R + G).$$

100. The two coils of a differential galvanometer are of equal resistance and exert equal effects upon the galvanometer needle. One of the coils, shunted by a wire of known resistance, is connected up with a battery, and the deflection is read off. Both coils are now connected up in series with the battery and are shunted by a wire of twice the original resistance: additional resistance is also introduced into the circuit until the galvanometer deflection is reduced to exactly its original value. Prove that the internal resistance of the battery is equal to the additional resistance introduced.

101. A battery is connected by wires of inappreciable resistance with a galvanometer, and the current is read off. The galvanometer is now shunted with a shunt having one-ninth of its own resistance, and it is found that the current through the galvanometer is now reduced to one-half. Prove that the resistance of the galvanometer is eight times as great as that of the battery.

102. It is found that on shunting a certain galvanometer the current through it is reduced to one-half, while the battery current is doubled: show that the resistance of the galvanometer is double that of the battery, and three times that of the shunt.

103. A galvanometer is shunted by a shunt of double its own resistance, and is connected up in circuit with a coil of fine wire and a battery of negligible resistance. The shunt is now removed and an additional resistance of R ohms introduced into the circuit, when it is found that the galvanometer deflection is the same as at first. What is the resistance of the coil of wire?

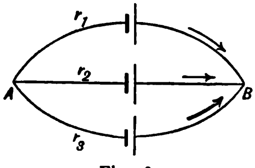


Fig. 18.

104. The like poles of three cells are connected in multiple arc (Fig. 18) by wires of inappreciable resistance, so that their E.M.F.'s all act towards the same

point B. Prove that the currents through the several branches are given by equations of the form

$$c_1 = \frac{e_1(r_2+r_3) - e_2r_3 - e_3r_1}{r_1r_2+r_2r_3+r_3r_1}, \text{ etc.},$$

where e_1, e_2, and e_3 are the E.M.F.'s of the three cells, and r_1, r_2, r_3 the resistances of the three branches respectively.

[Apply Kirchoff's laws. *Some* definite directions—say those indicated in the figure—must be *assumed* for the currents, but it is evident that all three currents cannot really flow from A to B.]

105. Two equal cells when connected in series with a given wire produce a current of 0·28 ampère; when connected in multiple arc through the same wire the current produced is 0·2 ampère. Prove that the resistance of the wire is three times that of either cell.

106. Two equal cells are connected (1) in series, (2) in parallel, the external resistance in both cases being the same. The currents produced are in the ratio of 4 to 3: prove that the resistance of the wire is two and a half times that of either cell.

107. You are provided with a delicate galvanometer of 48 ohms resistance, and are required to measure by means of it a current, the strength of which is supposed to be 2·5 ampères: if the largest current that can safely be sent through the galvanometer is 0·1 ampère, what resistance would you supply as a shunt?

If the deflection of the shunted galvanometer indicates a current of 0·085 ampère, what is the current in the main circuit?

108. Two direct-reading galvanometers G and G′ are connected up in series with a battery of constant E.M.F. and negligible resistance. The reading of G is 70 and that of G′ is 30. The two galvanometers are now connected up in multiple arc, when the reading of G is found to be 100. Find the reading of G′, and compare the constants of G and G′ and also their resistances?

109. If the points A and B in Fig. 18 are connected by a wire of resistance R, what currents will flow through the wire and through each of the cells?

110. A conductor of resistance R is connected up with a battery of six cells each having an internal resistance r, and the cells are arranged partly in series and partly in parallel, so that there are three rows containing 3, 2, and 1 cells respectively. Find the current in the external conductor.

111. You are required to replace the two cells in Ex. 77 (Fig. 15) by a single battery, which will send the same current through the external resistance R. Prove that this battery must have an E.M.F. equal to $(e_1r_2+e_2r_1)/(r_1+r_2)$, and that its internal resistance must be $r_1r_2/(r_1+r_2)$.

Arrangement of Cells for Maximum Current.—When the internal resistance of the cells of a battery is so small that it can be neglected in comparison with the external resistance of the circuit, the current produced is approximately proportional to the number of cells employed. But if the external resistance is small compared with that of a single cell, the current is only slightly increased by adding on cells in series: for although every additional cell increases the E.M.F., it also increases the resistance in nearly the same proportion. A stronger current can be obtained by arranging the cells either partly or wholly in parallel (as in Fig. 15), so as to diminish the internal resistance.

When the external resistance and the number of cells are given, the maximum current is obtained by grouping the cells in such a way that *the internal resistance of the battery is (as nearly as possible) equal to the external resistance of the circuit.*

For let R be the external resistance, N the number of cells, e the E.M.F. and r the resistance of each cell. Suppose the cells to be arranged in m rows, each row consisting of n cells in series, then

$$m \times n = \mathrm{N} \qquad . \quad . \quad . \quad . \quad (a)$$

The E.M.F. of the battery is the same as that of each row, and is equal to ne. Each row consisting of n cells has a resist-

ance nr: the equivalent resistance of the m rows arranged in parallel is nr/m.

Thus, by Ohm's law, the current produced is

$$C=\frac{ne}{\frac{nr}{m}+R}=\frac{e}{\frac{r}{m}+\frac{R}{n}}.$$

Now e is constant, and therefore C is a maximum when $r/m+R/n$ is a minimum. Observe also that the product rR/mn is constant, because r and R are given, and mn is a constant, being equal to the total number of cells. But if the product of two factors is constant it can be shown that their sum is a minimum when they are equal to one another.

Thus if pq is constant, $p+q$ is a minimum when $p=q$. For $p+q=(\sqrt{p}-\sqrt{q})^2+2\sqrt{pq}$, and, since the last term is constant, $p+q$ is a minimum when $(\sqrt{p}-\sqrt{q})^2$ is a minimum. Now the minimum value of a square is zero: hence $p+q$ is a minimum when $\sqrt{p}-\sqrt{q}=0$, *i.e.* when $p=q$.

If then we wish to give C its maximum value we must make

$$\frac{r}{m}=\frac{R}{n}, \text{ or } \frac{nr}{m}=R \quad . \quad . \quad . \quad . \quad (b)$$

But nr/m is the internal resistance, and R is the external resistance, so that the maximum current is obtained when the resistance of the battery is made equal to that of the external circuit.

112. What is the best way of arranging a battery of 18 cells, each having a resistance of 1·8 ohm, so as to send the largest current through an external resistance of 1 ohm?

By equation (a)

$$m \times n=18,$$

and by the condition for maximum current [equation (b)],

$$1\cdot 8n/m=1.$$

Thus $n^2=10$, and the nearest integer to $\sqrt{10}$ is 3. It follows that $m=18/3=6$, and that the cells must be arranged in 6 rows of 3 each.

113. The current from a battery of 4 equal cells is sent

through a tangent galvanometer, the resistance of which, together with the attached wires, is exactly equal to that of a single cell. Show that the galvanometer deflection will be the same whether the cells are arranged all abreast or all in series. What is the best arrangement that could be adopted, and what would be the corresponding deflection?

114. You are supplied with 12 exactly similar cells, the internal resistance of each of which is one-fourth of the external resistance of the circuit: how would you group the cells so as to obtain the maximum current?

115. How would you arrange a battery of 12 cells, each of 0·6 ohm internal resistance, so as to send the strongest current through an electromagnet of resistance 0·7 ohm?

116. You have a battery of 48 Daniell cells, each of 6 ohms internal resistance, and are required to send the largest possible current through an external resistance of 15 ohms: how would you group the cells? Find also the current produced and the difference of potential between the poles of the battery, assuming that the E.M.F. of a Daniell cell is 1·07 volt.

117. If there are 18 cells in a battery, each of resistance 1·5 ohm, how can they best be arranged so as to send a strong current through an external circuit of 3·5 ohms resistance?

Power and Heating Effect of a Current.—Suppose a current C to flow through a portion of a circuit, the ends of which are at a difference of potential E. The total quantity of electricity which passes any point in the circuit during a time t is $Q = Ct$. The work done by the current during the same time is measured by the product of the quantity of electricity thus transferred into the difference of potential (p. 5), or

$$W = QE = CEt. \qquad (10)$$

The *power* or *activity* of the current is measured by its rate of doing work, *i.e.*

$$P = QE/t = CE \quad . \quad . \quad . \quad (11)$$

If C and E are expressed in C.G.S. units the above equations give the work in ergs and the power in ergs per second. Since the ampère $= 10^{-1}$ C.G.S. units of current, and the volt $= 10^8$ C.G.S. units of E.M.F., a current of one ampère flowing through a difference of potential of one volt does work at the rate of $10^{-1} \times 10^8 = 10^7$ ergs per second. This is adopted as the practical unit of power, and is called a **Watt**. Thus the rate at which work is done in a circuit is measured in watts by the product of the current in ampères into the E.M.F. in volts. We have already shown (p. 15) that a horse-power is approximately equal to 746 watts.

If the portion of the circuit under consideration contains no other source of E.M.F. besides that which produces the current, it follows from Ohm's law that the above equations may be written in the form

$$W = C^2Rt \quad . \quad . \quad . \quad . \quad (12)$$

and

$$P = C^2R \quad . \quad . \quad . \quad . \quad (13)$$

On the other hand, if a voltameter, or an electromotor in motion, is included in the circuit, the current is diminished by an amount corresponding to the counter electromotive force thus set up, and Ohm's law cannot be applied; but in all cases CEt measures the work done, and CE the power.

When a current flows through a metallic conductor without doing any work beyond overcoming the resistance of the conductor, its energy is entirely converted into heat, which goes to raise the temperature of the conductor. If H is the amount of heat thus produced, its equivalent in ergs is JH, and therefore by equation (12)

$$JH = C^2Rt \quad . \quad . \quad . \quad . \quad (14)$$

This equation expresses what is known as *Joule's law*. Here again we have supposed that current and resistance are both expressed in C.G.S. units. If C is expressed in ampères, its equivalent in C.G.S. units is $C \times 10^{-1}$; and if R is expressed in ohms its equivalent in C.G.S. units is $R \times 10^9$. The numerical value of J (Joule's equivalent; see p. 176) is $4 \cdot 16 \times 10^7$: thus the amount of heat generated in time t by a current of C ampères flowing through a resistance of R ohms is

$$H = (C \times 10^{-1})^2 \times (R \times 10^9)t/4 \cdot 2 \times 10^7,$$

or

$$H = C^2Rt/4 \cdot 2 \quad . \quad . \quad . \quad (15)$$

118. The output of a dynamo is frequently expressed in "units" of 1000 watts each: thus a "four-unit" machine produces 4000 watts. How many 16-candle-power lamps, each requiring 3·5 watts per candle, can be run by such a dynamo? If the lamps used are 50-volt lamps, what current does each take?

Each lamp absorbs $16 \times 3 \cdot 5 = 56$ watts, and if n is the required number of lamps, $56n = 4000$, and $n = 71 \cdot 4$. If 71 lamps are used, and the dynamo is run at its normal speed, the lamps will be somewhat too bright: if 72 are used, they will be somewhat dull.

The number of watts absorbed by each lamp is equal to the product of the current in ampères into the E.M.F. in volts. If C is the current taken by each lamp,

$$C \times 50 = 56,$$

and

$$C = 56/50 = 1 \cdot 02 \text{ ampère.}$$

119. The poles of a battery, of internal resistance B, are connected successively by two wires, whose resistances are R_1 and R_2 respectively: if B is a mean proportional between R_1 and R_2, show that the quantities of heat developed in equal times in each of the two wires are the same.

Let E denote the E.M.F. of the battery. The currents produced in the first and second cases respectively are

$$C_1 = E/(B + R_1), \text{ and } C_2 = E/(B + R_2).$$

The quantity of heat developed in time t in the wire of resistance R_1 is

$$H_1 = \frac{1}{J} \cdot C_1^2 R_1 t = \frac{1}{J}\left(\frac{E}{B+R_1}\right)^2 R_1 t.$$

Substituting for B its value $\sqrt{R_1 R_2}$, this gives

$$H_1 = \frac{R_1 t}{J} \cdot \frac{E^2}{R_1 R_2 + 2R_1\sqrt{R_1 R_2} + R_1^2} = \frac{t}{J} \cdot \frac{E^2}{R_1 + 2\sqrt{R_1 R_2} + R_2}$$

Since this expression is symmetrical with respect to R_1 and R_2, it is clear that it also represents the heat developed in the same time in the wire R_2.

120. The current from a single secondary cell was sent through the thick coils of a Helmholtz galvanometer, and also through a coil of wire of 1 ohm resistance immersed in 100 grammes of water. In 40 minutes a rise in temperature of 15°·8 was produced, and the mean deflection of the galvanometer needle was 32°. Calculate the strength of the current, and the reduction-factor of the galvanometer.

The amount of heat produced is $H = 100 \times 15 \cdot 8$. The resistance of the coil is 10^9 C.G.S. units, and, according to equation (14), p. 289, the value of the current is given by the equation

$$(4 \cdot 2 \times 10^7) \times 100 \times 15 \cdot 8 = C^2 \times 10^9 \times 40 \times 60,$$

$$\therefore C^2 = 4 \cdot 2 \times 15 \cdot 8/2400 = 0 \cdot 02765,$$

and $$C = 1663.$$

Thus the current strength is 0·1663 C.G.S. unit or 1·663 ampère.

The reduction-factor (k) is defined by the relation $C = k \cdot \tan\delta$, and since $\tan 32° = 0 \cdot 625$, we have

$$k = C/\tan\delta = 1 \cdot 663/0 \cdot 625 = 2 \cdot 661.$$

[This is the factor for reducing readings to ampères; if the current is required in C.G.S. units the corresponding reduction-factor will be 0·2661.]

121. Compare the amounts of power required to send a current of constant strength through two wires of re-

sistances r_1 and r_2, (1) when they are arranged in series, and (2) when they are arranged in parallel.

122. If, instead of a constant current, a constant E.M.F. were used in the preceding problem, what would be the ratio between the amounts of power absorbed in the two cases.

123. Two wires of the same length and diameter, one being of platinum and the other of silver, are joined up in multiple arc with a powerful battery. Compare the amounts of heat generated in them. (See Table, p. 266.)

124. What horse-power is required to maintain a current of 4 ampères through a resistance of 37·3 ohms?

125. An arc lamp takes a current of 9·75 ampères, and the E.M.F. between its terminals is 50 volts: what power does it absorb?

126. In the Provisional Orders of the Board of Trade the unit of electrical supply is defined as 1000 *ampères flowing for one hour under a pressure of one volt.* Prove that this is equal to 1·34 horse-power working for one hour.

127. An incandescent lamp of 16 candle-power takes a current of 0·75 ampère with a difference of potential of 60 volts between its terminals: find the number of watts per candle-power absorbed by the lamp, and the amount of heat generated in it in an hour.

128. A 16 C.-P. incandescent lamp takes a current of 1·05 ampères with an E.M.F. of 50 volts. Find the number of watts absorbed per candle, and determine the H.-P. required to supply 100 of these lamps in parallel. (1 H.-P. = 746 watts.)

129. A house is lit by 23 Bernstein lamps of 20 candle-power, each lamp taking a current of 10 ampères, and requiring an E.M.F. of 10 volts. The total length of the conducting wires is 440 yards, and they are made of No. 12 B.W.G. copper wire having a resistance of 4·6 ohms per mile. Show that the amount of energy lost in

the conductors is one-twentieth of that used in the lamps.

130. The poles of two batteries, each consisting of 10 cells, are connected by thick copper wires of inappreciable resistance; the plates of the one battery are three times as large as the plates in the other, but the cells are similar in all other respects. Compare the intensities of the currents, and the amounts of heat generated in the same time in the two batteries.

131. A current of 1·4 ampère is allowed to flow for half an hour through a coil of wire of 5 ohms resistance immersed in 250 grammes of water. What elevation of temperature will be produced by it?

132. The terminals of a battery are connected to a wire spiral immersed in a calorimeter containing 200 grammes of water. A current of 2 ampères flows through the wire and causes the temperature of the water in the calorimeter to rise through 2° in quarter of an hour. What is the difference of potential between the terminals?

133. In order to determine the strength of a current it was made to pass through a coil of wire of 5 ohms resistance placed in a calorimeter: a steady stream of water was kept flowing through the calorimeter at the rate of 15 c.c. per minute, and the heating effect of the current was such that the water was 4° warmer on leaving the calorimeter than it was on entering. Find the strength of the current.

134. The amount of heat generated per second by a current of c ampères in a copper wire 1 metre long and d millimetres in diameter is given by the equation $H = k.(c/d)^2$. Calculate the value of the constant (k) in this equation, taking the specific resistance of copper in C.G.S. units to be 1600.

135. When a current divides between two wires arranged in multiple arc, the current in each branch is inversely proportional to its resistance (p. 278); prove that the total amount of heat generated in the two wires

is less than it would be if the current were to divide between them in any other proportion.

136. A steady current is passed for 20 minutes through (*a*) a spiral of platinum wire immersed in a calorimeter containing 300 c.c. of water, the temperature of which rises 3°; (*b*) a silver voltameter in which 0·9 gm. of silver is deposited; and (*c*) a tangent galvanometer which gives a deflection of 45°. Calculate the strength of the current, the constant of the galvanometer, and the resistance of the spiral.

Efficiency of Dynamos and Motors.—In the following examples the term efficiency will be used to denote the *efficiency of conversion*, which is the ratio between the electrical power developed and the horse-power required to drive the dynamo. A portion of the power developed in the circuit of a dynamo is always absorbed in overcoming the resistance of the dynamo itself; and the ratio of the power developed in the external circuit to the total power is sometimes called the *electrical efficiency* of the system. This is equal to $R/(R+r)$, where r is the resistance of the dynamo and R that of the external circuit (supposing the latter to contain no opposing E.M.F.) What we have called the efficiency (*i.e.* the efficiency of conversion) depends upon the construction of the dynamo itself, whereas the electrical efficiency depends not only upon the internal resistance of the dynamo but also upon that of the external circuit.

It is therefore clear that we should distinguish between the *gross* and *nett* efficiency of conversion. The gross efficiency is the ratio of the total electrical horse-power to the horse-power required to drive the dynamo: the nett efficiency is the ratio between the electrical horse-power in the external circuit and the horse-power absorbed by the dynamo. This latter quantity measures the economy of the system as a whole, and is commonly called the *commercial efficiency*.

The efficiency of a motor is the ratio between the horse-power developed by the motor and the electrical horse-power absorbed by it.

137. Find the useful commercial efficiency of an Edison No. 5 dynamo from the following tests made at the Franklin Institute (Philadelphia):—

E.M.F. at terminals of dynamo . .	125·2 volts.
Current in external circuit . . .	100·9 ampères.
Resistance of external circuit . .	1·241 ohm.
Power applied	18·89 H.-P.

The power developed in the external circuit is $CE = 100 \cdot 9 \times 125 \cdot 2 = 12633$ watts, or $12633/746 = 16 \cdot 94$ H.-P.

[The power can also be calculated from the given current and resistance: this gives the same result, for $C^2R = (100 \cdot 9)^2 \times 1 \cdot 241 = 12634$ watts.]

The commercial efficiency is $16 \cdot 94/18 \cdot 89 = 0 \cdot 8968$, or 89·68 per cent.

138. An engine of 130 H.-P. drives a dynamo which sends a current of 10 ampères through an external resistance of 100 ohms. What is the nett efficiency of the system?

139. A Weston No. 7 M dynamo tested at the Franklin Institute was found to give a current of 123·6 ampères through an external resistance of 1·224 ohm, the potential difference at the terminals of the dynamo being 151·2 volts. The power absorbed was 28 H.-P.; show that the commercial efficiency of the dynamo was 89·5 per cent.

140. A petroleum engine delivering 7 H.-P. was coupled up with a dynamo used for charging secondary batteries. The average current was 77 ampères, with an E.M.F. of 58 volts: find the electrical horse-power and the efficiency of the dynamo.

141. A dynamo driven by a gas-engine of 2 H.-P. (actual) is used for supplying a current to a circuit whose resistance (including that of the dynamo) is 3·357

ohms. Show that if the efficiency of the dynamo is 90 per cent, it will produce a current of 20 ampères.

142. The total weight of a tram-car driven by an electromotor (including the weight of the passengers) is 5 tons. The current for the motor is derived from a battery of accumulators, the average E.M.F. of which is 110 volts during the discharge. Assuming the motor to have an efficiency of 70 per cent, find the rate at which the car will run on the level when the discharge current is 18·65 ampères, the resistances due to friction, etc., being at the rate of 28 lbs. per ton.

The power developed by the motor is $18{\cdot}65 \times 110 \times 0{\cdot}7$ watts, which is equivalent to $18{\cdot}65 \times 110 \times 0{\cdot}7/746$ H.-P. Thus the motor does work at the rate of $18{\cdot}65 \times 110 \times 0{\cdot}7 \times 550/746$ foot-pounds per second. Suppose that it gives the car a velocity of x feet per second: the work done by it per second will be $5 \times 28 \times x$ ft.-lbs.

Equating these two expressions, we have

$$x = 18{\cdot}65 \times 110 \times 0{\cdot}7/746 \times 5 \times 28 = 7{\cdot}563.$$

Thus the car will run at the rate of 7·563 ft. per sec., or 5·16 miles per hour.

143. Two trials of an Immisch motor gave the following results:—

	Exp. I.	Exp. II.
Current . .	31·5 ampères.	31 ampères.
E.M.F. . .	134 volts	147 volts.
Power developed	4·7 H.-P.	5·2 H.-P.

Show that the mean efficiency of the motor was 0·84.

144. A smaller motor of the same kind was tested with a current of 25·5 ampères and an E.M.F of 40 volts, and was found to develop 0·98 H.-P. Show that its efficiency was 0·72, and that the electrical horse-power absorbed by it was 1·37.

145. The following results were obtained with an Ayrton and Perry motor, running at 2240 revolutions per minute:—

Current = 33 ampères.
E.M.F. = 32·5 volts.
Brake horse-power = 0·4586.

What was the efficiency of the motor?

146. An electromotor which is required to develop 2 H.-P. is to be introduced into an arc lighting circuit traversed by a constant current of 15 ampères. Assuming the motor to have an efficiency of 80 per cent, show that the power absorbed by it will be 1865 watts, and that the E.M.F. at its terminals will be 124·3 volts.

147. The total weight of a tram-car drawn by an electromotor is 5 tons. The current for the electromotor is derived from a battery of accumulators the E.M.F. of which is 110 volts during the discharge. The resistances to motion amount to 28 lbs. per ton, and the discharge current is 18·5 ampères. Assuming the motor to have an efficiency of 70 per cent: show that the car will run at the rate of 150 yards per minute.

148. Determine the power supplied in the following test, and also the efficiency of the dynamo:—The dynamo pulley was 9 in. in diameter and made 1200 revolutions per minute. The difference between the tensions of the taut and slack sides of the belt was measured by a transmission dynamometer and found to be 35 lbs. The current produced was 30 ampères and the E.M.F. 65 volts.

149. The current supplied to an electromotor varies according to the load between 60 and 120 ampères, but the E.M.F. is kept constant at 300 volts. The total resistance of conductors and motor is 1·5 ohms. Find how much power is utilised in the motor when the current is 60, 80, 100, and 120 ampères respectively. When is the available power a maximum, and how much power is then wasted as heat?

150. *A circular coil consisting of 100 turns of wire and of mean radius 10 cm. rotates about a vertical diameter at a place where the horizontal intensity of the

earth's magnetic field is 0·18 C.G.S. unit. Find (in volts) the mean E.M.F. generated in it when rotating at the rate of 1800 revolutions per minute.

EXAMINATION QUESTIONS

151. A Daniell cell, the internal resistance of which is 0·3 ohm, works through an external resistance of 1 ohm. What must be the resistance of another Daniell cell so that when it is joined up in series with the first and working through the same external resistance the current shall be the same as before? If the cells be joined up in parallel how will the current be modified?

S. K. 1893.

152. State Ohm's law. A current can go partly through a galvanometer, the resistance of whose wire is 297 ohms, and partly through a piece of wire of 3 ohms, arranged as a shunt (*i.e.* in parallel or multiple arc with the galvanometer wire); calculate the whole current when the galvanometer shows that a current of ·0182 ampère is passing through it. Matric. 1893.

153. The internal resistance of a Daniell's cell is 1 ohm; its terminals are connected (*a*) by a wire whose resistance is 4 ohms, (*b*) by two wires in parallel, one of the wires having a resistance of 4 ohms, the resistance of the other wire being 1 ohm. Compare the currents through the cell in the two cases. Matric. 1894.

154. A galvanometer coil has a resistance of 125 ohms; calculate the resistance of a wire which will shunt off nine-tenths of the main current from the galvanometer. Edinb. M.A. 1893.

155. A battery sends a current in succession through a tangent galvanometer which it deflects through 1°, a voltameter in which it decomposes 20 c.c. of hydrogen in a certain time, and a coil in a calorimeter in which the thermometer rises 0·2° in that time. The battery is then increased so that the current is five times as great.

Describe the effect of the increase on the three instruments. Matric. 1894.

156. A current produces 35 c.c. of mixed hydrogen and oxygen per minute in an electrolytic cell, and at the same time produces 155 units of heat per second in a wire; if the current be increased until it produces 105 c.c. of mixed gases per minute, calculate how much heat will now be produced per second in the wire.

Matric. 1892.

157. A circuit is formed of six similar cells in series and a wire of 10 ohms resistance. The E.M.F. of each cell is 1 volt and its resistance 5 ohms. Determine the difference of potential between the positive and negative poles of any one of the cells. S. K. 1894.

158. A current from three Daniell cells in series is passed through solutions of nitrate of silver, of sulphuric acid, and of sugar of lead, all in series; platinum electrodes being used. How much hydrogen is liberated, how much sulphuric acid decomposed, and how much lead, copper, and silver deposited by the time half a gramme of zinc has been fairly dissolved in each cell? If all this occurs in 30 minutes, what is the strength of the current, being given that unit current (one ampère) deposits 4 grammes of silver in an hour?

N.B.—Atomic weights: H = 1, O = 16, S = 32, Cu = 63·5, Zn = 65, Ag = 108, Pb = 207. Int. Sc. 1890.

159. A galvanometer, the resistance of which is $\frac{1}{3}$ ohm, being joined up in circuit with a cell by thick copper wires, the resulting current is noted; and it is found that the current in the galvanometer is halved if, without any other change being made, the terminals of the galvanometer are joined by a wire of resistance 0·1 ohm. What is the resistance of the cell? S. K. 1893.

160. State Joule's law connecting the rate of heat-production in a circuit traversed by an electric current with the strength of the current.

A Daniell's cell has an internal resistance of 2 ohms.

Compare the amount of heat produced in the cell for each gramme of zinc consumed in it (1) when the cell is short circuited; (2) when the terminals are connected by a resistance of 2 ohms; (3) when they are connected by a resistance of 100 ohms.

Int. Sc. 1894.

161. In order to obtain a current through a resistance of 5 ohms a number of cells each of E.M.F. 1·8 volt and resistance ·3 ohm are joined in series. Draw a diagram showing the relation between the number of cells employed and the current produced, and show that the current cannot exceed 6 ampères however great the number of cells may be. Camb. Schol. 1893.

162. Eight cells, each with half an ohm internal resistance, and 1·1 volt E.M.F., are connected (*a*) all in series, (*b*) all in parallel, (*c*) in two parallel sets of four cells each: calculate the current sent in each case through a wire resistance of ·8 ohm. Matric. 1891.

163. Define electrical resistance. Determine the number of cells required to send a current of half an ampère through a body whose resistance is 30 ohms, if each cell has an electromotive force of 1·25 volt and a resistance of 2 ohms. If each cell had a resistance of 3 ohms, show that some must be coupled in parallel.

Prel. Sc. 1892.

164. Describe a series of experiments to prove Ohm's law. A current passes from A to D through a circuit composed as follows: Between A and B is a resistance of 1 ohm, between B and D an unknown small resistance; A is joined to C by a resistance of 2 ohms, and B to C by one of 99 ohms. The terminals of an electrometer (or of a high-resistance galvanometer) are joined alternately to A and C and to B and D, and the deflections are the same in the two cases. Find the value of the unknown small resistance. B.Sc. 1886.

Between A and B there is a divided circuit, one branch of which (AB) has a resistance of 1 ohm, while the other branch

(ACB) has a resistance of 101 ohms. If C is the total current, then the current through ACB is C/102, and the potential difference between the points A and C is, by Ohm's law, equal to $2 \times C/102 = C/51$. The potential difference between the points B and D is equal to Cr, where r is the unknown small resistance. Since they are equal, it follows that the value of the unknown resistance is $\frac{1}{51}$ ohm.

165. State Faraday's law of electrolysis.

A certain tangent galvanometer has a current passed through it which deflects it 45°. The same current passes through a copper sulphate cell, where it deposits 0·3 gramme of copper in 30 minutes. Taking the electro-chemical equivalent of copper as 0·00033 gramme/ampère-second, find the value of the current, and show how to determine the current for any other reading of the galvanometer. Int. Sc. 1895.

166. Define the terms watt, joule, ampère, coulomb, volt, and ohm. How many ergs correspond to 4000 watt-hours? A lamp of 400 candle-power requires $2\frac{1}{2}$ watts per candle; how many ampères will it take at 100 volts? And what will be the loss of E.M.F. in passing the current through a conductor whose resistance is one-fifth of an ohm. Vict. B.Sc. 1890.

167. A battery of 12 equal cells in series screwed up in a box, being suspected of having some of the cells wrongly connected, is put into circuit with a galvanometer and two cells similar to the others. Currents in the ratio of 3 to 2 are obtained according as the introduced cells are arranged so as to work with or against the battery. What is the state of the battery? Give reasons for your answer. S. K. 1895.

168. How does the rate at which heat is developed by an electric current in a conductor depend on the current and on the resistance of the conductor?

The wires leading to a group of seven glow lamps, arranged parallel, have resistance 1 ohm, and the lamps

have each resistance 70 ohms. Find the ratio of the heat developed in the lamps to that developed in the leading wires. Int. Sc. 1893.

169. Find the power required to maintain a current of ·75 ampère in each of the following systems: (*a*) 100 lamps of 45 volts connected, each by leads of 1 ohm, in multiple arc with a dynamo; (*b*) the same number of lamps in ten rows of ten lamps each, the leads to each row having a resistance of 10 ohms. Camb. Sc. 1891.

170. Calculate the power required to light 80 incandescent lamps, if the electromotive force required be 65 volts, and the current required by each be 0·8 ampère. If the lamps be all in parallel, and the leads have a resistance of 0·5 ohm, calculate the power wasted in them. B.Sc. 1892.

171. An electric light installation consists of a group of lamps in parallel arc between the ends of leads. The leads have total resistance 0·4 ohm, and bring current from 60 accumulators, each with E.M.F. 2 volts and resistance 0·01 ohm. When 25 lamps are switched on each takes 0·4 ampère. Find the resistance of a lamp, and the watts used in each part of the circuit.
Int. Sc. (Hons.) 1894.

172. Two horizontal rods are placed parallel and at a distance of 1 metre apart. A third rod slides over them parallel to itself with an uniform velocity of 10 metres per second. Find in volts the E.M.F. between the ends of the fixed rods, assuming the earth's vertical magnetic force to be ·47 C.G.S. units. Camb. Schol. 1891.

173. State generally the magnitude of the electromotive force generated by the motion of a linear conductor in a magnetic field.

Apply the statement to find the strength of the current in a circuit formed by bending a straight uniform wire into the form of a V, laying another piece of the same wire across the first, and moving the latter parallel to itself, so that it always remains in contact with both sides

of the V, the plane of the wires being normal to the force of a uniform magnetic field, and the resistance of the sliding contacts negligible. N. S. Tripos. 1890.

174. *A coil of wire consisting of 50 turns in the form of a circle 30 cm. in diameter rotates 20 times per sec. about a vertical axis. Find the average value in volts of the E.M.F. produced if the earth's horizontal magnetic force be ·18 C.G.S. units. Explain how you obtain the result? Int. Sc. (Hons.) 1889.

175. *What is meant by a conductor moving so as to "cut lines of force?" On what does the electromotive force thus developed depend?

A copper disk of 10 centimetres radius spins on its axis (perpendicular to its plane) 3000 times a minute in the earth's horizontal field of force: find the E.M.F. between centre and circumference of disk. [$H = \cdot 18$.] Vict. B.Sc. 1890.

176. *Give an account of some one practical method of employing the temperature-variation of electrical resistance to indicate small differences of temperature.

The four branches of a Wheatstone quadrilateral, taken in order, are denoted by r_1, r_2, r_3, r_4. The resistance of each one of them at 15° C. is 10 ohms, and each has a temperature-coefficient ·007. The temperature of r_1 and r_3 is raised by exposure to radiation, while r_2, r_4 are screened, and the balance is restored by shunting r_1. Calculate the resistance of the shunt if the radiation raises the temperature of the wires exposed to it by the hundredth of a degree? N. S. Tripos. 1890.

177. *Show how to calculate the coefficient of self-induction of a long or endless solenoid. For instance, if a wooden curtain-ring, of mean diameter 21 cm., and sectional diameter 3 cm., be uniformly wound with 1000 turns of very fine wire, calculate its coefficient of self-induction. B.Sc. (Hons.) 1891.

CHAPTER XII

MISCELLANEOUS EXAMPLES

1. A body moving with uniform acceleration is found to have a velocity of 2 metres per second after moving 50 metres from rest. Express the acceleration in C.G.S. units.

2. A boy throws a cricket ball vertically upwards with a velocity of 112 feet per second: how high will it rise, and how many seconds will elapse before he catches it on the return?

3. Rain-drops falling vertically downwards with a velocity of $88\sqrt{3}$ feet per second are watched by a passenger in a train going 60 miles an hour. Find the angle at which the rain will appear to him to fall.

4. Two bodies A and B start from rest, and both are observed to move through 16 cm. in the first two seconds. In the third second A moves through 20 cm. and B through 25 cm. In the fourth second A moves through 28 cm. and B through 40 cm. Find in each case whether the acceleration is uniform or variable.

5. A force equal to the weight of an ounce pushes a mass of a pound along a smooth level surface. Find the velocity when the mass has been displaced through a distance of 3 yards ($g = 32$).

6. What force acting uniformly through a distance

of 50 metres will just stop a mass of one kilogramme moving with a velocity of 10 cm. per second?

7. A certain force acts upon a hundredweight for a minute, and imparts to it a velocity of a mile per hour. Express the force in poundals.

8. The weights in an Atwood's machine are 355 and 345 grammes. In the *second* second (starting from rest) the heavier weight descends through 21 cm. What value does this give for g?

9. An engine hauls a 90-ton train a distance of a mile up an incline of 1 in 125. Express in foot-pounds the work done.

10. The diameter of the cylinder of a steam-engine is 10 inches, the length of stroke is 20 inches, and the average steam-pressure during the stroke is 30 lbs. per square inch above the atmospheric pressure. Calculate the work done in each stroke on the piston.

11. A bullet of 2 oz. leaves the muzzle of a gun with a velocity of 800 feet per second. The barrel of the gun is 3 feet long. Calculate the average pressure on the bullet as it travels down the bore, and the amount of work done on it. ($g = 32$.)

12. A body is let fall from a height of 4008 cm. at the equator, where $g = 978$. How far out of the vertical will it alight?

13. A particle slides down a rough inclined plane of angle 45°. Prove that the proportion of its energy which is dissipated in friction is equal to the coefficient of friction.

14. The angle of inclination of a rough inclined plane is equal to the angle of friction: prove that the roughness reduces the mechanical advantage by one-half.

15. Two forces, of 3 lbs. and 5 lbs. respectively, act upon a particle and make with one another an angle of 60°. Find (by calculation, or by making a diagram to scale) the numerical value of their resultant.

16. Six parallel forces, of magnitudes 1, 2, 3, 4, 5 and 6 respectively, act at six points on a straight line. The distances between the successive forces are respectively 5, 4, 3, 2 and 1, and the first and sixth forces act in the opposite direction to the rest. Find the magnitude and point of application of the resultant.

17. A uniform heavy beam 10 feet long has weights of 10 lbs. and 100 lbs. respectively attached at its ends, and is found to balance about a point 7 ft. 3 in. from the 10-lb. weight. What is the weight of the beam?

18. Weights of 1, 5 and 5 lbs. respectively, are hung from the angular points of a plate in the form of an equilateral triangle. The plate itself weighs 9 lbs. Find the centre of gravity of the whole.

19. When placed in the left-hand pan of a balance a body appears to have a weight W; and in the right-hand pan a weight W′. Find the true weight of the body and the ratio of the arms of the balance.

20. If the apparent weights in the preceding question are 50 gm. and 50·0024 gm., find the true weight and the ratio of the arms.

21. A 7-ton gun is raised from a depth of 145 feet in sea water. If the material of the gun has 7·8 times the density of the water, how much more work would have to be done in raising it through the same distance in air?

22. A vessel contains water and mercury, and an iron sphere floats on the mercury and is completely covered by the water. The volume of the sphere is 100 c.c.; the density of mercury is 13·6, and of iron 7·8. What is the volume immersed in each liquid?

23. The weight of a Nicholson hydrometer is 90 grammes, and it floats in water up to a certain mark on the stem. What weight must be placed in the upper dish of the hydrometer in order to make it sink to the same mark in a salt solution of density 1·2?

24. The weight of a cork bung in air is 30 gm., and the apparent weight of a piece of lead in water is 100

gm. When the lead is attached (as a sinker) to the cork, the apparent weight of both together in water is 5 gm. What is the specific gravity of the cork?

25. A lump of silver weighing 100 gm. is suspended by a string in water. If the density of the silver is 10·5, what is the tension in the string?

26. If the pressure on 36 cubic centimetres of a gas at 91 centimetres of mercury be reduced to 72 centimetres, what will the volume become?

27. The volume of the receiver of an air-pump is nine times that of the barrel. How many strokes will be required to reduce the pressure of the air in the receiver from 76 cm. to 36·3 cm. of mercury? [Use logs.]

28. The apparent height of the barometer as measured on a certain scale at $t°$ is H_t. Show that the real height at which the column would stand at 0° is given by

$$H_0 = H_t \frac{1 + \alpha t}{1 + \kappa t},$$

where α is the coefficient of linear expansion of the scale, and κ the coefficient of cubical expansion of mercury.

29. From the preceding show that the correction to be applied to reduce the barometric height to zero is approximately $H_t(\alpha - \kappa)t$.

30. A barometer is provided with a brass scale which is correct at 0°. The apparent height is 76·3 cm. and the temperature is 14°. What is the corrected height? [The coefficient of linear expansion of brass is 0·000019 and the coefficient of cubical expansion of mercury is 0·000182.]

31. A column of mercury, contained in a vertical glass tube closed at the lower end, has a height of 100 cm. at 50°; what will be its height at 0°? [The coefficient of cubical expansion of the glass may be taken as 0·000027 and that of the mercury as 0·000182.]

32. The apparent loss of weight of a glass stopper is 4·45 gm. when weighed in petroleum at 0°, and 4·05

gm. when weighed in petroleum at 100°. The coefficient of cubical expansion of glass is 0·000024; what is that of the petroleum?

33. A glass vessel shaped like a thermometer, with a stem of uniform bore graduated in millimetres, is used as a dilatometer. In calibrating it, it is found to contain 160·1 grammes of mercury when filled up to the mark 15 mm. on the stem, and 168·6 gm. of mercury up to the mark 185 mm. The mercury is removed and a quantity of ether is introduced which fills the dilatometer to the mark 13 mm. at 0°. When raised to 30° the ether rises in the stem to 173 mm.; calculate its mean apparent coefficient of expansion in the glass.

34. A lecture-room containing forty students is to be supplied with air at 14° C., the external temperature being 4°. How much heat will be absorbed per hour in warming the air, assuming that the supply of 0·75 cubic metre per minute is required for each person present? [Specific heat of air = 0·237.]

35. It is found that when 7 gm. of steam at 100° is passed into 150 gm. of water at 8° the temperature rises to 36°. Calculate from this the latent heat of steam.

36. It is found that a plate of copper 1 square decimetre in area and 1 cm. thick allows 35,000 units of heat to pass through it per minute, when a difference of 6° in temperature is maintained between its opposite faces. Calculate its conductivity.

37. When the Centigrade scale is used the value of the mechanical equivalent of heat is $4{\cdot}2 \times 10^7$ and the latent heat of steam is 536. By what numbers will these quantities be represented when the Fahrenheit scale is used?

38. The density of water at 0° is 0·99988 and that of ice is 0·91674. Calculate the amount of external work done during the freezing of a litre of water under a pressure of one megadyne.

39. A train of 160 tons is drawn by an engine which

consumes 16 lbs. of coal per mile-run, the coal being of calorific power 8000. If the resistance to motion is $\frac{1}{200}$ of the weight, find the efficiency of the engine. [$J = 1400$.]

40. A machine works at the rate of 10 watts, and the energy it produces is entirely converted into heat, which is employed in raising the temperature of a litre of water. If the temperature of the water at the start is 0°, what will it be at the end of an hour, assuming that there are no losses. [Take $J = 4 \cdot 2 \times 10^7$.]

41. A lamp and a standard candle are 10 ft. apart. A screen, placed on a line between them, and 2 ft. from the candle, is equally illuminated on both sides. What is the candle-power of the lamp?

42. Two very thin watch-glasses, having each a radius of curvature of 60 cm., are joined together so as to make a convergent lens, the space between them being filled with benzene. The focal length of the lens is 60 cm.; what is the refractive index of the benzene?

43. A short-sighted person sees objects most distinctly at a distance of 13 cm. If he wears his spectacles at a distance of 1 cm. from the eye, what focal length should they have so as to enable him to see distinctly objects at a distance of 25 cm.?

44. What change should the person in the preceding question make in his spectacles when he finds his distance of most distinct vision reduced to 11 cm.?

45. How many vibrations will be made by an E tuning-fork of frequency 320 before its sound is audible at a distance of 166 metres? Take the velocity of sound in air as 332 metres per second.

46. Assuming the same value for the velocity of sound, find the shortest length of a tube, closed at one end, which will resound loudly when a tuning-fork giving 256 vibrations per second is held near the open end.

47. The interval between two notes is a semi-tone. When sounded together they give twenty beats per second. What are their vibration-numbers?

48. A magnet suspended in a torsion balance hangs in the magnetic meridian when the wire is free from torsion. When the torsion head is turned through 360° the magnet is deflected 45° from the meridian. How much further must the torsion head be turned so as to bring the magnet at right angles to the meridian?

49. At one end of the base of an equilateral triangle, one decimetre in the side, is placed a charge of 10 units of electricity, and at the other end a charge of −10 units. Find the potential and the force at the apex. In what direction does the force act?

50. The radii of two small conducting spheres are in the ratio of 2 to 3. The smaller one is charged with a single positive unit of electricity, while the larger one has an unknown negative charge. After connecting them by a fine conducting wire (without altering their distance) it is found that the force between them is the same in magnitude as before the connection was made. What was the charge on the larger sphere?

51. A battery consists of 7 cells in series, each having a resistance of 1·4 ohms and an E.M.F. of 1.7 volts. What current will it send through an external resistance of 14 ohms? And what will be the difference of potential between its terminals when sending this current?

52. How many cells of E.M.F. 1·8 volts and internal resistance 1·6 ohms are required to send a current of half an ampère through an external resistance of 16 ohms?

53. A current is sent through an external resistance of 2·6 ohms by six cells arranged (1) all in parallel, (2) all in series. Each cell has an E.M.F. of 1·4 volts and an internal resistance of 1·2 ohms. Find the current through the external resistance in each case.

54. A battery consists of ten small cells of high internal resistance. In order to measure this the poles are connected to a quadrant electrometer and the

deflection produced is 156 divisions. When the poles are also joined to a resistance of 100 ohms, so as to send a small current through it, the deflection is reduced to 80 divisions. What is the internal resistance?

55. A current of 1 C.G.S. unit is passing through a wire bent into a circle of 10 cm. radius. Find the force exerted on a unit magnetic pole at the centre of the circle.

56. A battery is connected up in series with a carefully calibrated ammeter and a tangent galvanometer; the latter consists of ten coils having a diameter of 40 cm. The deflection produced is 42° and the ammeter shows that the current is 0·5157 ampère. What is the value of H at the place? (See table of tangents.)

57. A wire of 9 ohms resistance is bent into the form of an equilateral triangle. Two of the corners are connected up to a cell having a resistance of 7 ohms and an E.M.F. of 1·8 volts. Find the current through the battery and through each side of the triangle.

58. A cable is composed of 17 equal copper wires each having a resistance of 8·6 ohms per mile and a carrying capacity of 2 ampères. Calculate the resistance per mile of the cable, and also the fall of potential per mile when it is carrying its full current.

59. A type of accumulator having an E.M.F. of 2·1 volts and a resistance of 0·01 ohm is to be used for supplying 100 incandescent lamps in parallel, each lamp requiring a current of 0·75 ampère at 65 volts. What is the smallest number of cells that will suffice?

60. The difference of potential between two leads is 2000 volts, and the insulation resistance between them is 100,000 ohms. Express the leakage in ampères and the loss of power in watts.

EXAMINATION QUESTIONS.

61. A snow slope rises a height of 50 feet in a slope of 200 feet. A sledge weighing 400 lbs. is drawn up it by a rope parallel to the surface of the snow. Find a triangle representing, in magnitude, the forces acting, and find the pull on the rope when the sledge is going steadily up. Find the work done in pulling the sledge up the slope. Friction is to be neglected.

Matric. 1894.

62. What is meant by acceleration?

A meteorite burst at a height of 57,600 feet, and one of the fragments was brought instantaneously to rest by the explosion. It then descended with an acceleration of 32 ft./sec.^{2}, while the sound of the explosion travelled with a velocity of 1100 ft./sec. Which reached the ground first, the fragment or the sound, and what time did each take? Matric. 1894.

63. What are meant by Energy of Position and by a Foot-Pound?

A reservoir of water of area 330,000 square feet is initially of uniform depth 10 feet. How many foot-pounds can it supply to a turbine on a level with the bottom of the reservoir; and what horse-power can it maintain on the average if it is emptied in 10 hours? [1 cubic foot of water weighs 62·4 lbs.; 1 horse-power is 33,000 foot-pounds per minute.] Int. Sc. 1894.

64. If the speed of a cricket-ball is changed from -50 to $+250$ foot-seconds in 0·004 sec., what is the time-average of the force applied, in terms of the weight of the ball? Edinb. M.A. 1893.

65. An ounce bullet has a velocity at the muzzle of the rifle of 1000 feet per second; supposing the acceleration of the bullet uniform while the bullet is in the gun, find the time taken to traverse the barrel, which is two feet long;

and calculate the average force on the bullet and the average activity in the barrel. Glasgow M.A. 1893.

66. Give a careful statement of Newton's Laws of Motion.

Calculate the momentum and the energy of a bullet weighing 20 grammes moving with a velocity of 400 metres per second. Find also the uniform force which would stop it in $\frac{1}{10}$ second. Camb. Schol. 1895.

67. In a certain wheel and axle the radius of the wheel is 10 in. and that of the axle 2 in. To the rope which passes round the wheel a mass of 20 lbs. is attached, and to the rope round the axle a mass of 80 lbs. How far will the latter mass have ascended, and what will be its velocity when the former mass has descended 400 feet? Camb. Schol. 1895.

68. A cannon ball of mass 4 lbs. leaves the muzzle of a cannon whose mass is 800 lbs. with a velocity of 1200 feet per second ($g = 32$).

What is (*a*) the momentum of the system after explosion? (*b*) the energy of the system after explosion?

Camb. Schol. 1895.

69. A string is placed straight on a smooth table, one end being fixed to the table, while a heavy ring is attached to the other end. An equal ring threaded on the string is then dropped through a narrow slit in the table: prove that its acceleration will be one-fifth of that of a body falling freely. Camb. Schol. 1892.

70. What is meant by the Kinetic Energy of a body? How is it calculated (1) when the body moves along a straight line; (2) when the body rotates about a fixed axis?

A thin hoop of mass 5 kilogrammes and radius 50 cm. rolls along the ground at the rate of 10 metres per second. Calculate in ergs its kinetic energy.

Camb. Schol. 1895.

71. A hoop 30 cm. in radius rotates ten times per second about an axis through its centre at right angles

to the plane of the hoop. Find how far the hoop must fall from rest under gravity for its energy of translation to be equal to its energy of rotation. N.S. Tripos. 1893.

72. An elastic string, 1 metre in length, is stretched 1 centimetre when a mass of 1 kilogramme is suspended from it. One end of the string is fixed, and the mass is raised until it reaches the fixed point. It is then dropped. Show that the lowest point it reaches is approximately 116·28 cm. below the fixed support. Camb. Schol. 1895.

73. *State the laws of friction and describe some method of verifying them experimentally.

Show that in the case of a rough screw with a rectangular thread the efficiency is greatest when the angle of the screw $= \pi/4 - \lambda/2$, where λ is the angle of friction.

Camb. Schol. 1895.

74. On a given sheet of squared paper draw a diagram representing the following forces acting at a point, and find approximately *from the diagram* the magnitude of their resultant:

(*a*) a force of 1 lb. to the East,
(*b*) a force of 2 lbs. to the North-East,
(*c*) a force of 3 lbs. to the North. Camb. Schol. 1894.

75. A uniform triangular plate is hung from a point by three unequal strings attached to its corners. Show that the centre of gravity of the triangle is vertically below the point of support, and that the tensions of the strings are proportional to their lengths.

Camb. Schol. 1895.

76. *Find the radius of a homogeneous sphere of lead so that its attraction upon a particle upon its surface may be 0·000001 of the weight of the particle. [Density of lead is twice the mean density of earth; radius of earth $= 21 \times 10^6$ feet.] Edinb. M.A. 1892.

77. *If a tunnel were bored through the earth down to the Antipodes, the value of gravity at any point in it would be proportional to the distance from the centre,

the earth being assumed to be of uniform density. Show that a body dropped down the tunnel would arrive at the Antipodes in about $42\frac{1}{2}$ minutes. [The earth's radius may be taken as 4000 miles.] B.Sc. 1892.

78. *Calculate the velocity, in feet per second, with which a particle must be projected from the surface of the moon in order to escape from the moon's attraction. [The earth's radius is 21×10^6 feet, the moon's radius is $\frac{1}{4}$ of this, and the moon's mass is $\frac{1}{80}$ that of the earth. The attraction of the *earth* on the particle is to be neglected.] B.Sc. 1893.

79. *In determining the time of vibration of a pendulum, an observation of 11 transits with a chronometer beating half-seconds gives 10 vibrations in 10·5 seconds. After a lapse of about 3 minutes from the "zero transit" another set of 11 transits are observed; the mean interval between corresponding transits of the two series is 182 secs. To what degree of accuracy can the time of vibration of the pendulum be determined by these observations? N. S. Tripos. 1889.

80. *Calculate the shear at each point of a uniform circular cylinder produced by applying a twist at each end.

If a load of 2 kgms. at the end of a lever 30 cm. long, applied at one end of a circular rod one-third of a centimetre in diameter and half a metre long, twists its point of application through 10°, the other end being firmly clamped, what is the rigidity of the material in C.G.S. units? B. Sc. (Hons.) 1891.

81. State fully the principle of Archimedes.

A body floats in a liquid of specific gravity s_1, with a certain fraction f of its volume immersed: in a liquid of specific gravity s_2 it floats with the fraction $1-f$ of its volume immersed. Show that the specific gravity of the solid is $s_1 s_2/(s_1+s_2)$. Glasgow M.A. 1893.

82. A cylinder, weighing 1 lb., floating in water with its axis vertical and each of its ends horizontal, requires

a weight of 4 oz. to be placed on its upper surface to depress it to the level of the water. Find the specific gravity of the cylinder. Matric. 1894.

83. If W_1, W_2 are the weights of the same body when immersed in fluids of specific gravities S_1, S_2, prove that its weight in vacuo is

$$\frac{W_1S_2 - W_2S_1}{S_2 - S_1}$$

Vict. Int. Sc. (1894).

84. Define the density of a substance.

A cylinder 5 cm. long and 4 cm. in diameter weighs in air 170 grammes, and in a liquid 120 grammes. Find the density of the cylinder and of the liquid, neglecting the buoyancy of the air ($\pi = 3{\cdot}142$).

Camb. Schol. 1895.

85. Define the metacentre of a body floating freely in a fluid, and explain how its position relative to the centre of gravity of the body determines whether the equilibrium is stable or unstable.

In a ship of 9000 tons displacement it was found that moving 20 tons from one side of the deck to the other, a distance of 42 feet, caused the bob of a pendulum 20 feet long to move through 10 inches. Prove that the metacentre height was 2·24 feet. Camb. Schol. 1895.

86. Describe an experiment to verify Boyle's law for pressures less than the atmospheric pressure.

Some air is in the space above the mercury in a barometer of which the tube is uniform. When the mercury stands at 29 inches in the tube the space above the mercury is 4 inches long. The tube is then pushed down into the cistern so that the space above the mercury is only 2 inches long, and now the mercury stands at 28 inches. At what height would it stand in a perfect barometer? Int. Sc. 1894.

87. *Show how to calculate the height to which a liquid can rise in a capillary tube. Prove that the excess

of pressure inside a soap-bubble of radius R, is given by $p = 2g\rho hr/R$; where h is the height to which the soap solution, of density ρ, could rise in a tube of radius r.

Calculate the excess of pressure numerically, for a liquid whose density is that of water, and in a bubble whose diameter is equal to the height the liquid ascends in a tube one-hundredth of an inch (or a quarter of a millimetre, whichever you prefer) in bore. Be careful to state the pressure completely. B.Sc. 1891.

88. *A drop of water is placed between two plane pieces of glass which are pushed close together until the film is everywhere x cm. thick and A sq. cm. in area. Show that the plates are urged together with a force equal to $2TA/x$, where T is the surface-tension between water and air. The angle of contact between glass and water is to be assumed to be zero. B.Sc. 1893.

89. A stay-rod of iron, 20 feet long, is subject to fluctuations of temperature from zero to 120° F. The coefficient of expansion of iron is 0·0000123 per degree Centigrade. What is the maximum variation of length that must be allowed for? Matric. 1893.

90. The pendulum of a clock, regulated to beat seconds at 14° C., becomes lengthened by 1/200,000 per 1° C. rise in temperature. How much does the clock lose per day when the temperature is 17° C.?

Glasgow M.A. 1893.

91. Describe the Fortin Barometer and the mode in which you would take a reading if the scale was graduated in millimetres and the vernier reads to 20ths of a millimetre.

The enclosing tube being brass, and the graduation being correct at 0° C., explain how you would apply corrections to reduce the reading to 0° C., if made at $t°$, the linear coefficient of expansion of brass being α and the volume coefficient of mercury being β.

Int. Sc. (Hons.) 1893.

92. Explain what is meant by Young's modulus, and

describe an experiment by which you would seek to determine the modulus for any given wire.

A length of piano wire is just stretched between two points at temperature 20° C.; the temperature then falls to 10° C., the length being maintained constant. Find the tension per square centimetre. [Coefficient of expansion $=0{\cdot}000011$. Young's modulus $=2\times 10^9$ grammes weight per square centimetre.]

Ind. C. S. 1891.

93. How may the coefficient of pressure-increase of a gas at constant volume be determined?

A given volume of air is at 740 mm. pressure at 17° C. What is the temperature when its pressure is 1850 mm.? Int. Sc. 1894.

94. A copper vessel contains 100 grammes of water at 12°. When 56 grammes of water at 30° are added, the resulting temperature of the mixture is 18°. What is the water-equivalent of the vessel?

A calorimeter with water-equivalent 12 contains 100 grammes of water at 12°. When 100 grammes of a metal at 100° are added, the resulting temperature of the mixture is 20°. Find the specific heat of the metal.

Int. Sc. 1894.

95. The specific gravity of a certain liquid A is ·8, that of another liquid B is ·5. It is found that the capacity for heat of 3 litres of A is the same as that of 2 litres of B. Compare the specific heats of A and B.

Describe how you would determine the specific heat of sulphuric acid. Camb. Schol. 1895.

96. State what you understand by the statement that the latent heat of water is 79, and explain how you would prove it.

On 20 lbs. of solid mercury at −60° C. 1 lb. of water at 20° C. is poured. Find the state and temperature of the mixture. The specific heat of mercury is 0·03, its latent heat of fusion 2·8, and its melting-point −40° C. Camb. Schol. 1895.

97. The latent heat of steam is 536; that of water 80. What weight of steam at 100° C. would be required to melt a kilogramme of ice? Matric. 1894.

98. How much ice at 0° C. would a kilogramme of steam at 100° C. melt if the resulting water was at 0°? S. K. 1894.

99. A lb. of liquid fusible metal at its melting-point 100° C. is poured into 1 lb. of water at 15° C. in a copper calorimeter weighing 1 lb., and the temperature rises to 22·7° C. Find the latent heat of fusion of the metal, its specific heat being ·05 and that of copper ·1. Camb. Schol. 1894.

100. *Show how to find the work done by a gas that expands at a constant temperature.

The quantity of air which occupies a volume of 50 cubic feet under a pressure of 15 lbs. per square inch is allowed to expand at a constant temperature to 60 cubic feet: find the number of foot-pounds of work done (Hyp. log. 1·2 = 0·1823216).

S. K. 1892.

101. A train whose mass is 60 tons, travelling at the rate of 30 miles per hour, arrives at the top of an incline 1 mile in length. Steam is shut off and the brakes applied with just sufficient force to keep the velocity of the train constant until it arrives at the bottom. The inclination of the plane is $\sin^{-1}\frac{1}{100}$. Find (neglecting the resistance of the air) the heat generated during the descent (J = 1400). Camb. Schol. 1895.

102. A object is placed at a distance of 9 cm. from a convex lens of 6 cm. focal length. At a distance of 58 cm. from the lens on the other side is placed a concave mirror of 16 cm. radius. Find the position and nature of the final image, showing the paths of the rays on a diagram. Camb. Schol. 1895.

103. Two thin lenses have each a focal length of 1 inch. Draw, as carefully as you can to scale, the path of a beam of light from a distant object (1) when the

lenses are in contact, (2) when they are separated by $1\frac{1}{2}$ inch, (3) when they are separated by 3 inches.

Int. Sc. (Hons.) 1894.

104. *A convex lens and a concave mirror have a common axis. The rays from a small object on the axis, after traversing the lens, are reflected at the mirror and again pass through the lens, forming a real image of the object. Give a careful diagram showing the path of the rays.

If f be the focal length of the lens, r the radius of the mirror, a the distance between the lens and the mirror, and u the distance of the object from the lens when the image and object coincide, show that

$$r = uf/(u-f) - a.$$ Camb. Schol. 1892.

105. *Fresnel's fringes are produced with homogeneous light of wave length 6×10^{-5} cm. A thin film of glass (refractive under $1 \cdot 5$) is introduced into the path of one of the interfering rays, upon which the central bright band shifts to the position previously occupied by the fifth bright band from the centre (not counting the central band itself). The ray traverses the glass perpendicularly. What is the thickness of the glass film?

S. K. (Hons.) 1894.

106. An iron rod 1 metre long is clamped in the middle, and caused to execute longitudinal vibrations. The frequency is observed to be 2500 per second. Determine the velocity of sound in the rod, and assuming its density to be 7·75 grammes per c.c., show that the Young's modulus is approximately $1 \cdot 94 \times 10^{12}$ dynes per square centimetre. S. K. 1895.

107. The top of an organ pipe is suddenly closed. What effect is produced on the pitch and character of the note?

If the vibration frequencies of the partials next above the fundamental notes in the two cases respectively differ by 256, what was the pitch of the open pipe?

S. K. 1895.

108. Having given that the cube root of 2 is 1·26, find how many beats per second there would be between the two notes which represent the major third above the C of 256 vibrations per second (1) on the natural, (2) on the equally tempered scale. S. K. 1895.

109. Define the compressibility of a liquid. The compressibilities of two liquids are in the ratio of 1 : 2, and their densities are in the ratio of 1 : 3. Compare the velocities of sound in the two liquids.

Edinb. M.A. 1893.

110. A string is attached to a weight which, when the length of the string is altered, is adjusted so as to be always equal to 400 times the weight of the string: show that the number of vibrations per second is inversely as the square root of the length. Find the number of vibrations for a length of 2 feet of string.

S. K. 1894.

111. An open organ pipe produces 8 beats per second when sounded with a tuning-fork of 256 vibrations per second, the fork giving the lower note. How much must the length of the pipe be altered to bring it into accord with the fork?

Take the velocity of sound as 1100 feet per second.

S. K. 1894.

112. If the middle C corresponds to 256 vibrations per second, how many beats would be heard per second if the note F sharp were simultaneously sounded in the natural scale and in the scale of equal temperament?

S. K. 1892.

113. What is the value in C.G.S. units of Young's modulus of elasticity for a metal of which the density is 7·5, and in which sound travels with a velocity of 5600 metres per second? S. K. 1893.

114. Explain the effect of changes of temperature on the note of a metallic organ pipe. Prove that if a solid could be found such that the note given by a pipe formed of it was the same at all temperatures, the coefficient of

cubical expansion of the solid would be about one and a half times greater than that of a gas. S. K. 1893.

115. Describe some accurate method of determining the frequency of the note emitted from a given source.

Two forks, when sounded together, give 4 beats per second. One is in unison with a length of 96 cm. of a monochord string under constant tension, and the other with 97 cm. of the same string. Find the frequencies of the two forks. Int. Sc. (Hons.) 1893.

116. State the laws connecting the time of transverse vibration, the density, the length, and stretching weight of a stretched flexible string. With what weight must a string one metre long weighing one decigramme be stretched to make it execute 256 complete vibrations per second? Int. Sc. (Hons.) 1894.

117. *A whistle produces sound waves half an inch long. Explain how by cutting concentric rings out of a large sheet of cardboard a circular grid can be formed, which, if properly placed, concentrates the sound at a focus. Calculate the dimensions of the rings if the whistle is placed at a great distance on a line perpendicular to the grid and passing through its centre, and if the focus is formed at 3 feet from the grid.

S. K. (Hons.) 1893.

118. *Define the coefficient of rigidity n, the modulus of bulk-elasticity k, and Young's modulus q. Find the relation between n, k and q, for a non-crystalline solid.

A wire 10 metres long, and ·01 sq. cm. in cross-section, is stretched ·1 cm. by a load of 2·5 kilogrammes. Find the value of Young's modulus in C.G.S. units.

B.Sc. 1893.

119. *Show how to calculate Young's modulus for a given sample of wood from the observations that a yard long of it floats vertically in water with two inches protruding, and emits, when stroked longitudinally, a note three octaves above the middle C. [Frequency of middle C is 256.] B.Sc. 1892.

120. *Discuss the modes of transverse vibration of a stretched string.

A cycle wheel has 44 spokes, each 40 cm. long, and 0·2 cm. diameter, and their density is 7·8. If they sound, when plucked, a note of frequency 256, calculate the tension of each, and find an approximate value for the thrust in the rim of the wheel. B.Sc. (Hons.) 1893.

121. Explain the meaning of "unit pole."

Calculate the magnetic force at a point 5 cm. from one end of a magnet, and situated in a continuation of the length of the magnet, whose poles are of unit strength and 3 cm. apart. Vict. Int. Sc. 1894.

122. *Assuming that the earth's magnetism is due to a very small magnet at the centre of the earth, show that $\tan D = 2 \tan \lambda$, where D is the magnetic dip at a place whose latitude is λ. N. S. Tripos. 1893.

123. Two Ledyen jars are exactly alike, except that in one the tinfoil coatings are separated by glass and in the other by ebonite. A charge of electricity is given to the glass jar, and the potential of its inner coating is measured. The charge is then shared between the two jars, and the potential falls to 0·6 of its former value. If the specific inductive capacity of ebonite be 2, what is that of the glass? S. K. 1893.

124. A short ebonite rod, with a small electrified knob at one end, is mounted so as to turn freely about its centre in a horizontal plane. In a horizontal line with this centre, and at distances from it of a quarter and half a metre respectively, are placed insulated balls that are also charged. The rod makes ten vibrations in a given time, but makes thirty vibrations in the same time if the balls are interchanged. Compare the charges on the two balls. S. K. 1893.

125. Explain the principle and construction of the attracted disc or guard ring electrometer.

If the area of the surface of the attracted disc be 88 square centimetres, the attraction between the discs

2016 dynes, when the distance between them is 3 centimetres, then the difference of the potentials of the discs equals 72 units ($\pi = 22/7$). Camb. Schol. 1895.

126. *Prove that the energy of a charge Q on an insulated conductor at potential V is $\frac{1}{2}QV$.

A plate, 1 square decimetre in area, is charged when held 1 cm. from an earth-connected plate. The electrical pull on it is 1000 dynes. What is its potential? What would be the pull on it at half the distance from the earth-connected plate when at the same potential?

Int. Sc. (Hons.) 1894.

127. A battery sends a current in succession through a tangent galvanometer which it deflects through 1°, a voltameter in which it decomposes 20 c.c. of hydrogen in a certain time, and a coil in a calorimeter in which the thermometer rises $0{\cdot}2^\circ$ in that time. The battery is then increased so that the current is 5 times as great. Describe the effect of the increase on the three instruments. Matric. 1894.

128. A battery is connected up by thick wires to a galvanometer, and the current is observed. On shunting the galvanometer with 1/21 of its own resistance the current is halved. Show that the resistance of the battery is twenty times that of the galvanometer.

Camb. Schol. 1895.

129. State and explain the laws by which the subdivision of current in a network of conductors may be determined.

A battery of 5 cells in series and a battery of 4 cells also in series have their positive terminals joined together and to one end of a wire of 10 ohms resistance, the other end being joined to the negative terminals of both batteries. If the E.M.F. of each cell = 1 volt and the resistance of each = 1 ohm, determine the current in each portion of the circuit. Vict. B.Sc. 1894.

130. *A wire in the form of a circle 10 cm. in diameter is spinning about a diameter as axis (which is

placed vertically) at the rate of 10 revolutions per second in the earth's magnetic field [$H = 0{\cdot}18$]. Calculate the maximum E.M.F. generated during a revolution.

Vict. B.Sc. 1894.

131. A Leyden jar consists of two concentric spherical surfaces of 5 and 6 cm. diameter respectively, the intervening space being filled with air. The outer sphere is uninsulated, the inner is charged with 20 units of electricity. How much work is done when the inner sphere is put to earth? S. K. 1895.

132. A fine wire is placed in 50 grammes of water in a light copper calorimeter, and 5 ampères are passed through it, an E.M.F. of 14 volts being observed between the ends of the wire. Calculate the rise of temperature in 10 minutes. [A water-gramme-degree centigrade $= 4{\cdot}2 \times 10^7$ ergs; 1 ampère $= 10^{-1}$ C.G.S. units; 1 volt $= 10^8$ C.G.S. units.] Camb. Schol. 1895.

133. A uniform wire, 4 metres long, whose resistance per metre is 6 ohms, is bent into the form of a square ABCD. The adjacent points A and B are fixed to the poles of a battery of E.M.F. 3 volts and internal resistance 4 ohms. Find the current along AB and CD, also the current between the plates of the battery.

Camb. Schol. 1895.

134. State Joule's laws relating to the quantity of heat produced by a current.

A conductor carrying a current divides into two branches whose resistances are in the ratio of 4 to 5. Compare the amounts of heat generated in the branches.

Camb. Schol. 1894.

135. Explain how it is possible to calculate approximately the E.M.F. of a cell if we know the chemical actions occurring in it, and their heat equivalents.

The electrochemical equivalent of zinc is 0·00034 gramme/ampère-second. Find the cost of the zinc used in a primary battery for each horse-power per hour, if

zinc is p pence per kilogramme, and if the cell in which the zinc is used gives V volts. [746 watts = 1 H.-P.]

B.Sc. 1895.

136. How would you show by experiment that the E.M.F. induced in a conductor is accurately proportional to the rate at which it cuts lines of magnetic induction?

A solid fly-wheel, 1 metre in diameter, spins on an axis which is directed towards the north, at a place where the horizontal intensity is 0·18. It makes 250 revolutions per minute. Calculate the difference of potential between the centre and the circumference of the wheel.

B.Sc. 1895.

ANSWERS

AND HINTS FOR SOLUTION

CHAPTER I—DYNAMICS

4. ACCELERATION = 2·5; space described = 1125 cm. **5.** 20 cm. per sec. **6.** 6666·6 dynes. **7.** 10^5 gm. **8.** 10 min. **9.** 1 hr. 23 min. 20 sec. **10.** 37·5 dynes. **13.** 2·5 ft. per sec. **14.** $v/240$. **16.** 38,400. **17.** (1) 5 oz.; (2) 2 oz. **18.** 32. **19.** 981. **20.** 9,810,000; 1/981. **21.** $4 \cdot 45 \times 10^5$ dynes. **22.** 785 cm. per sec. per sec. **23.** 12·8; 96,000. **24.** 10·25 seconds after it was thrown up. **25.** 150 ft. per sec.; 900. **26.** 4 ft. per sec. per sec. **27.** $4 \cdot 524 \times 10^7$. **28.** 1/120 of a poundal. **29.** The force is to the weight of a gramme as 25,000 to 327. **30.** Acceleration = 1/70; velocity = 1/7. **31.** Equal. **32.** 17,920 units. **34.** 8·075 oz. **35.** The force is equal to the weight of 5·6 lbs.; acceleration produced = 0·08 ft./sec.^{2} **36.** 1,774,080. **37.** As 80 : 7. **38.** 82·5 sec. **39.** Momentum = 1,471,500. **40.** $\sqrt{30}$ sec. **41.** The force is equal to 3·2 poundals, or is one-tenth of the weight of 1 lb. **42.** As 224 : 675. **43.** $32\sqrt{165}$ ft. **45.** Acceleration = 4; space described = 50 ft. **46.** Acceleration = 81; tension = 145,800 dynes. **47.** 16 ft. **48.** 3 gm. **49.** 72 ft. **50.** Acceleration = $g/4$; distance = $2g$. **51.** Tension = 16·8 lbs.; weight = 537·6 poundals. **52.** 709·1 cm. per sec. **53.** 5 : 3. **54.** 242·4 ft. **55.** 1 sec.; 32 ft. per sec. **56.** $\sqrt{3gl}$. **57.** 11,200 lbs. weight. **58.** $\text{Sin}^{-1}(1/48)$.

60. Acceleration = 1 ft./sec.^{2}; space = 6 in. **61.** Acceleration = 10 cm./sec.^{2}; 160 cm. **62.** $g = 950$. **63.** 18 gm. **65.** $g = 980$. **66.** 2 sec.

68. 26·32 poundals. **69.** Tension = weight of 4024·3 gm. **70.** 28°. **71.** 0·1113 ft./sec.^{2} **74.** $g = 981·05$. **75.** $g = 32·227$. **76.** It would have to be shortened to 3/19ths of its original length. **78.** $g = 32·191$; 39·69 in. **79.** The pendulum is 0·2717 in. too long and therefore 8·15 turns are required to correct it. **80.** 56 sec. per hour. **81.** $4\pi^2 = 39·48$.

91. $9·6 \times 10^6$ gramme-centimetres; $9·4176 \times 10^9$ ergs. **92.** 288,000 ft.-lbs. **93.** 10 : 27. **94.** $1820\sqrt{3}$ ft.-lbs. **95.** 9600 ft.-lbs. **96.** $1·44 \times 10^9$ ergs. **97.** (1) Equal; (2) inversely proportional to the masses. **98.** 20·16 ft.-lbs. **99.** 69·12 ft.-lbs. **100.** $6788\frac{4}{7}$ ft.-tons. **101.** In the ratio of 400 to 1. **102.** 10,348,800 ft.-lbs. **103.** 16,000 ft.; 400 secs. **104.** 0·48 H.-P. per ton. **105.** (1) 24 ft. per sec.; (2) 12 ft. per sec. **106.** 33 secs. **107.** A force equal to the weight of 6 lbs. **108.** 14·06 lbs. per ton. **109.** (1) 6 minutes; (2) as $513·\dot{3} : 1$. **110.** 968 ft.; total distance = 10 miles 1628 ft.; time = 21 minutes. **111.** 3,660,800 ft.-lbs. **112.** 213,367,500 ergs. **113.** 706·9 ft.-lbs. **114.** Total work = 11,000 ft.-lbs. **115.** 98,175,000 ft.-lbs.

123. (1) 336,000 ft.-lbs.; (2) 10,752,000 foot-poundals. **124.** 33 ft. **125.** 2500 ft.-lbs. **126.** $3·767 \times 10^{10}$ ergs. **127.** $9·81 \times 10^7$ ergs. **128.** $6·25 \times 10^{10}$ ergs. **129.** $4·5 \times 10^{12}$ ergs. **130.** 28,672 ft.-lbs. **131.** 4 ft.-lbs. **132.** Momentum = $8960\sqrt{2}$; K.E. = 143,360 foot-poundals, or in ft.-lbs. = 143,360/32 = 4480. **133.** 1536 foot-poundals or 48 ft.-lbs. **134.** The initial velocity must have been 96 ft. per sec., and the final energy must = K.E. at starting, $= 5 \times (96)^2/2g$ = 720 ft.-lbs. **135.** Force = 300 poundals. K.E. =

225,000 ft.-poundals. **136.** 8×10^{10} ergs. **138.** 2 tons weight. **139.** 58·8 cm. per sec.; energy wasted $= 269{,}000$ ergs. **140.** $2 \cdot 5 \times 10^{10}$ ergs. **141.** 16,940 ft.-lbs. **142.** $8 \cdot 5 \times 10^{6}$ ergs. **143.** As 1 : 4. **145.** 18 cm. **146.** K.E. = 1210 ft.-tons; a force equal to the weight of $1\frac{5}{6}$ ton. **147.** $5461\frac{1}{3}$ foot-poundals. **148.** As 1406·3 : 1. **149.** $\mu = 1/64$. **150.** $6 \cdot 25 \times 10^{10}$ dynes. **151.** Equal. **152.** Mass = 15; velocity = 8. **153.** 651 lbs. weight. **154.** As 112 : 625. **155.** $9 \cdot 375 \times 10^{8}$ dynes. **156,** 13,090 ft.-lbs. **158.** 32,000 ft.-lbs.; 320 ft. per sec. **159.** K.E. before impact = 15,000; after impact = 0. **160.** Force = 716·8 poundals; work = 114,688 foot-poundals. **161.** Mass = 45; velocity = 2/3. **162.** As $(\sqrt{3} - 1) : 1$. **165.** 13·5 poundals.

168. (1) 4561 kgm.-metres per min.; (2) $7 \cdot 456 \times 10^{9}$ ergs per sec. **169.** 38·061 cub. ft. **170.** $9\frac{1}{11}$ H.-P. **171.** $52\frac{1}{12}$ miles per hour. **172.** 16·8 H.-P. **173.** 1·973 H.-P. **174.** 151·2 H.-P. **175.** 192 H.-P. **176.** 480 H.-P. **177.** At the rate of 3·367 H.-P. **178.** The unit of work would be increased tenfold; the numerical value of the horse-power would not be changed. **179.** 12 H.-P.

EXAMINATION QUESTIONS

181. 30° North of West; $10\sqrt{3}$ miles per hour. **182.** An upward pressure equal to the weight of 105 lbs. **183.** 50 poundals or $1\frac{9}{16}$ lbs. weight. **185.** $9\frac{1}{6}$ tons weight; $4\frac{7}{12}$ tons weight. **186.** At 1.4 P.M. (4 minutes past 1 o'clock); 12 miles. **187.** As 1 : 327. **189.** $9\frac{1}{3}$ yard/min.^{2} **190.** A force equal to the weight of 200 tons. **191.** 196 ft.; 112 ft. per sec.; 4 ft. **192.** 215·2 seconds a day. **194.** 1 ton, 0·806 inch, and $0{\cdot}0458\dot{3}$ sec. **195.** $(\pi/30)^2$ lb. weight. **196.** 888·82 ft. per sec. **197.** As 6 : 1; $1\frac{1}{8}$ ft. **199.** $13\frac{1}{3}$ lbs. **200.** A force equal to the weight of (*a*) 2053 lbs.; (*b*) 342·2 lbs. **201.** 0·00347. **202.** 800 ft.-lbs.

203. $281\frac{1}{4}$ ft.-lbs.; $1\frac{9}{16}$ ft. **204.** 2182·4 yards; 43·25 ft. per sec. **205.** 0·6028 ft. **206.** 7,710,000 poundals; 5200 pound-degrees (C.). **207.** $1\frac{1}{8}$ ft. **208.** As 0·224:1. **210.** (1) 10 tons weight; (2) $13\frac{1}{8}$ tons weight. **211.** $149\frac{1}{3}$ lbs. per ton weight; 25·6 H.-P. **212.** 41·6 poundals. **213.** 18,000 poundals. **215.** $Mk^2 = 32/\pi$. **216.** 28·125 ft.-lbs. **217.** 0·268. **218.** 0·75.

Chapter II—Statics

1. Magnitude = 20; direction = 30° W. of N. **2.** Its direction is the same as that of the force of 5 lbs. and its magnitude is 6 lbs. **5.** 2 ft. 3 in. from the 4-lb. end. **6.** 96 lbs. and 40 lbs. respectively. **7.** 1 ft. further on (*i.e.*, 1 ft. from the middle). **8.** 3 ft. **9.** 5 ft. **10.** $\sqrt{3}$ lbs. **11.** $10\sqrt{3}$ gm. **12.** $\sqrt{3}$ lbs. in the horizontal string and $2\sqrt{3}$ lbs. in the vertical string. **13.** 4·375 lbs. weight. **14.** The tension will = 3·712 lbs. weight. **15.** 6·21, 16·97, and 12 lbs. weight. **16.** Equal to the weight of 125 gm. **17.** $30\sqrt{6}/(\sqrt{3}+1)$lbs.; $60/(\sqrt{3}+1)$ lbs. **18.** 0·8 lb.; 0·6 lb. **19.** 42·15 lbs.; 0·25 in. **20.** $2w/\sqrt{3}$, where w = weight of sphere. **22.** At the third division. **23.** 6 in. from the wheel. **24.** On the axis at a point $\frac{1}{8}$ of the length of the longer pillar from the surface of contact. **25.** On the diagonal and 0·4242 in. from the centre of the original square. **26.** On the median line at $\frac{1}{27}$ of its length (reckoning from the base). **27.** 3 ft. **28.** 144·9 lbs. **29.** 25 lbs. **30.** $1{\cdot}962 \times 10^6$ dynes. **31.** 119 lbs. **32.** Greater by an amount W tan a. **33.** (1) 1 ton 4 cwt.; (2) 1 ton 7 cwt. **35.** Tension of string = 12 lbs. Thrust on hinge = $6\sqrt{3}$ lbs. **36.** $\text{Tan}^{-1} 11\sqrt{3}/7$; $21\sqrt{309}/40$ lbs. **38.** A force equal to the weight of 28 lbs. with the left hand, and 11 lbs. with the right. **44.** A force equal to the weight of 3·87 kgm. **45.** Equal to the moment of a force of 5192 kgm. weight acting at A perpendicular to AB: equal to a weight of 8438 kgms.

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47. To a height of 1·25 ft. **49.** (*a*) 36·4 lbs. and 54·6 lbs.; (*b*) 78·4 lbs. and 12·6 lbs.; (*c*) there can be no equilibrium: the beam will tip round the other prop. **53.** -11 along DE; 14 along EF; and -9 along FA. **54.** The pressures are 0 and 380 lbs. **55.** $9\frac{3}{11}$ ft. **60.** Deflecting force = 34·64 lbs. Force on peg = 40 lbs. **62.** 0·834 of weight of slab.

CHAPTER III—HYDROSTATICS

3. The dimensions of pressure are the same as those of force, viz. MLT^{-2}; the dimensions of intensity of pressure (force per unit area) are $ML^{-1}T^{-2}$. **6.** 124·4 lbs. per cub. ft. **7.** 0·5787 oz. per cub. in. **8.** 1·736 oz. per cub. in. **9.** 3000 c.c. **11.** 13·824. **12.** 10·98 lbs. **13.** 0·7055 gm. per c.c. **14.** 300·8 lbs. **15.** 3·2 cub. ft. **18.** Cross-section = 0·3676 sq. cm.; diameter = 0·683 cm. **19.** 111·97 gm. **20.** 1·4. **21.** 4·5 cm. **22.** As 27 : 10. **23.** 19·712 gm. **24.** $6{\cdot}08 \times 10^{27}$ gm. **26.** 4·516. **27.** 4·57. **28.** $15s/8$. **29.** Its density is 0·925 that of air. **30.** 0·823. **31.** As 3:4:5. **32.** 1·5.

35. (1) 100 grammes weight per sq. cm; (2) 98,100 dynes per sq. cm. **36.** $4{\cdot}273 \times 10^4$. **37.** 73·53 cm. **38.** 4100 gm. weight. **39.** 206,991 dynes per sq. cm. **40.** 10·193 metres. **41.** 98,100 dynes per sq. cm. **42.** 12,750 gm. weight. **43.** 1/384 lb. per cub. in. **44.** 18,750 lbs. per sq. ft. **45.** 17·36 lbs. per sq. in

46. 138·2 ft. 47. 337,920 lbs. per sq. ft. 48. 28·02 lbs. per sq. in. 49. 10·193 metres.

50. 0·9. 51. 49·08 ft. 52. 21·3 lbs. per sq. in. 53. Sufficient to occupy 5 in. of the tube. 54. l' cos $30°(1-s)$. 55. 150 gm. 56. 68,360 gm. 57. 99,000 gm. 58. 18,150 gm. 59. 10,000 gm. on upper surface, 11,000 gm. on lower surface, 10,500 gm. on each of the vertical sides. 61. As 3 : 5 : 8. 62. A force = weight of 937·5 lbs.

64. Volume = 20 c.c.; sp. gr. = 3·1. 65. 22·04 gm. 66. 21·08 gm. 67. 34 c.c. 68. 1·538. 69. 10·5 c.c. 70. 425·9 oz. 71. 20·8 gm. 72. 27·5 gm. 74. $s_2 = s_1 m_2/(s_2 m_2 - s_1 m_1 + m_1)$. 75. 20·97 gm. 76. 21·57 gm. 77. A weight equal to that of the copper. 79. Acceleration = $20\frac{4}{7}$ ft.-sec. units; time = 1·247 sec. 80. 6·04 lbs. 81. 0·32 gm.

83. 1000 c.c. 84. 300 c.c. 85. 6437·5 cub. yds. 86. 50 c.c. 87. Sp. gr. of solid = 0·5; of liquid = 2. 88. The sphere will rest in equilibrium with $\frac{1}{7}$ of its volume immersed in the mercury. 89. 6·4 c.c. 90. As 0·108 : 0·917 or as 0·1175 : 1. 92. $m : m' = 6·7 : 7·7$. 93. $3·\dot{3}$. 94. 1·625. 95. 1·4. 96. 1·204. 97. 0·8271. 98. 1·434. 99. 0·9127. 101. 11·31. 102. Density = 21·6; volume = 1 c.c. 103. 1·948. 104. 0·7351. 105. 0·8. 106. 4·75. 107. 1·059. 108. 0·7944. 109. 3·219. 110. 0·7648. 111. 21 gm. 116. Sp. gr. of pebble = 2·74; of spirit = 0·825.

117. 240 lbs. 118. 10,080 lbs. 119. Ratio of arms = 8 : 5. 120. Pressure = 8064 lbs.; distance = 0·1042 in. 121. Mechanical advantage = 1440.

124. Apparent height = $20\sqrt{3}$ in. = 34·64 in. 125. 34 ft. 126. 13·6 in. 127. 25·197 ft. 128. (1) 0·0938; (2) 1·276. 129. 11·86 metres. 130. 10,546 kgm. per sq. metre.

132. 1,039,890 dynes per sq. cm. **133.** 14·01 lbs. per sq. in. **134.** 30,000 oz. per sq. ft. **135.** 1006·1 gm. per sq. cm., or 10,061 kgm. per sq. metre. **136.** $4{\cdot}045 \times 10^7$ dynes. **137.** A change of about half a pound (0·4917 lb.) per sq. in.

141. $p : p' = 9 : 1$. **142.** $p : p' = r'^3 : r^3$. **143.** 71·93 cm. **144.** 360 lbs. per sq. in.; 3·6 in. **145.** As 1 : 1·306. **146.** 1·395 gm. **147.** Pressure = 2 atmospheres. **148.** 96·6 cm. **149.** 306 ft. **150.** 162·2 atmospheres. **151.** It will descend 1·2 in. **152.** $p : p' = r' : r$. **154.** 54·1 em. **155.** The tube should have been raised until a length of 80 cm. stood out of the mercury. **156.** 32·49 cm. **158.** 68·85 cm. **159.** About 18 cm. **160.** $\frac{2}{3}$ cub. in. **161.** $8{\cdot}\dot{3}$ c.c. **163.** (1) 34·2 c.c.; (2) 87 c.c. **165.** 4·34 ft. **166.** 4·49 ft. **167.** It must be lowered until its top is 66 ft. below the surface. **169.** 164·3 cub. ft.; 80·3 in. **170.** 0·876 in. **171.** The jar must be sunk until its mouth is 35·64 ft. below the surface. **173.** To 77·41 cm. **175.** $D_n = (3/4)^{10}D = 0{\cdot}05631D$. **177.** 0·4363 gm. **178.** 0·741 gm. **179.** 15·1 cm. **180.** As 1 : 4. **183.** 260 kgm. **184.** 50 kgm. **185.** 1204 kgm.

EXAMINATION QUESTIONS

186. 9·6 gm. **187.** (1) 643·8 lbs.; (2) 51.44 lbs. **188.** Sp. gr. = 4; volume = 250 c.c. **189.** 0·6. **190.** On the median vertical line and at a distance of $7\frac{23}{51}$ ft. from the bottom. **191.** 8·9 oz. **192.** $2w/3$ and $w/3$ (where w is the weight of the stick), both being vertical and the latter being the tension of the string: 30°. **193.** Lead 11·42; oak 0·8507; alcohol 0·7971. **194.** As 674 : 684. **195.** $11\frac{1}{9}$ gm. **196.** 2 in. **197.** $0{\cdot}0011\dot{8}$. **198.** 63 cm. **199.** 31,250 poundals per sq. in.; 800,496 dynes per sq. cm. **200.** 60·3 cm. **201.** 30 inches of mercury. **202.** Mass of 1 cub. ft. = 0·078 lb. **203.** 10 in. **204.** 10·37 in.

Chapter IV—Heat—Expansion

1. **Expansion of Solids.**—**5.** 40·048 ft. **6.** 0·0432 in. **7.** 1·224 ft. **8.** 76·429 cm. **9.** 22·128 cm. **10.** 2·0068 metres; 150°. **11.** 153·86 cm. **12.** 263·16 cm. **13.** 87·2464 cm. **14.** 1·00095 yard. **15.** Coefficient of expansion = 0·00008; temperature = 260°. **16.** 100 cm. **17.** 0·02864 in. **18.** 0·01764 ft. **20.** 1·00649 metre. **21.** 36 cm. **23.** It will gain 20·52 secs. per day. **24.** 0·0000192. **26.** The required temperature is 196°·4, and the common length at this temperature is 251·424 cm. **27.** 12°. See § 13. for this and the next Examples. **28.** 0·0144 sq. ft. **29.** 300·912 sq. cm. **30.** Increase of volume = 2·592 cub. in. **32.** 10·22. **33.** 48·373 c.c.

2. **Expansion of Liquids.**—**34.** 0·00002797. **39.** 0·0003018. **40.** 0·0003014. **41.** $d_t : d_0 = 1 + e : 1 + e'$. **44.** 0·0001817. **46.** 103·8. **53.** 0·0001561. **54.** The coefficient of apparent expansion of the mercury is 0·0001546, and this gives 0·0000274 as the coefficient of cubical expansion of the glass. **55.** 1·574 c.c. **56.** 131°·7. **57.** 109°·65. **58.** 9·517 gm. **59.** 1·55 c.c. **60.** 100·24 c.c. **64.** The volume of the solid is 12·9752 c.c. at 10°, and 12·9914 c.c. at 95°; therefore its coefficient of cubical expansion is 0·00001468.

3. **Expansion of Gases.**—**74.** The volume becomes 28 litres. **75.** 320 c.c. **76.** 273°. **77.** 91°. **78.** (1) 12·39 litres; (2) 13·2 litres. **79.** 0·0036. **80.** 546°. **81.** 0.9487 gm. **82.** 75 c.c.; 0° C. **83.** 333°. **84.** 3·187 litres; 54°·6. **85.** (1) 11·16 litres; (2) 12·38 litres. **86.** 221·978 c.c. **87.** 0·10231 gm. **88.** 69·63 cm. at 0°; 95·12 cm. at 100°. **89.** 2190 c.c. **90.** 0·00367. **91.** 77°·6. **94.** 3·73 grammes. **95.** 18°·74. **96.** The temperature must rise from 10° to 57°·16. **98.** 322·6 c.c.

106. The final volume is two-thirds of the original volume. **107.** 383·2 c.c. **108.** The volume remains unaltered. **109.** As 1 : 0·9269. **110.** 1010 c.c. **111.** 10·47 atmospheres. **112.** As 1 : 0·7808. **113.** As 1 : 2·293. **114.** The temperature must fall to $-2°\frac{1}{3}$. **116.** 0·599 gm. **117.** 924·9 kilogrammes. **118.** 11·66 litres. **119.** 11·97 gm. **120.** 0·3743 gm. **122.** 1986 c.c. **123.** 37·98 in.; at 459°·5. **124.** As 1 : 1·02. **125.** The temperature must rise to 7°·18. **126.** (1) : (2) = 1 : 0·891. **127.** 299°. **128.** 91°. **131.** 13°·65.

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132. 85·54 in. **133.** 6·264 cm. **134.** 285°. **135.** 246·5 litres. **136.** It would lose 13·39 secs. per day. **137.** 0·0000475. **138.** 0·00006106. **139.** The capacity of the bulb must be 57·28 times that of the stem. **140.** Fraction = 5/36. **141.** 0·0000158. **143.** 0·000177. **144.** a = 0·001162 ; b = 0·000002188. **145.** An increase of 0·00088 gm. **146.** 115·49 cub. in. **147.** The volume remains unaltered. **148.** Final temperature = − 123°. **149.** 296·1 atmospheres. **153.** The weight at the sea-level is to the weight at the top of the mountain as 1 : 0·574. **154.** 496·8 kgm.; 261·8 kgm. **155.** Pressure = 0·4224 atmosphere.

CHAPTER V—SPECIFIC AND LATENT HEAT

1. **Specific Heat.**—**5.** 76,800 units. **7.** 1995 units. **8.** 4950 units. **9.** 30°. **10.** 4,804,800 units. **11.** 11·875. **13.** 0·208. **14.** Temperature = 17°·14. **15.** 88°. **16.** As 1 : 0·45$\dot{3}$. **17.** 22,666$\frac{2}{3}$. **19.** 0·031. **20.** 0·0332. **21.** 0·0962. **22.** 0·615. **23.** 0·112. **24.** 5$\frac{5}{9}$ gallons of boiling water, and 14$\frac{4}{9}$ gallons of tap-water. **25.** In the proportion of 0·03 to 1. **26.** 9°·49. **27.** 0·0556. **28.** 173°·1. **30.** As 8 : 9. **31.**

$67^\circ\cdot 9$. **32.** 306·4 units. **33.** 3263 litres. **34.** $135^\circ\cdot 3$. **35.** $853^\circ\cdot\dot{3}$. **37.** 30 gm. **38.** In the proportion of 1 to 3. **39.** $t=(s_1t_1+s_2t_2+s_3t_3)/(s_1+s_2+s_3)$. **40.** 0·424. **41.** 0·6426. **42.** $54^\circ\cdot 37$. **43.** 21·6 gm. **44.** 0·0315. **45.** Water-equivalent = 8. **46.** According to Exp. I. the specific heat is 0·03097, and according to Exp. II. 0·03185; mean value = 0·03141. **50.** Express Q_t and $Q_{t'}$ in terms of a, b, and c: the difference between these $(Q_{t'}-Q_t)$ is equal to the mean specific heat $(S_t^{t'})$ multiplied by the difference of temperature $(t'-t)$. Again, let this interval become indefinitely small; in the limit t' coincides with t, and the mean specific heat between t° and t'° becomes the true specific heat at t° (S_t).

2. **Latent Heat.**—**58.** 625 gm. **59.** $10\frac{2}{3}$ lbs. **60.** 80°. **61.** 1 lb. **62.** 176·5 gm. **63.** 80. **64.** The specific heat is the same; the latent heat is 9·612. **65.** 0·112. **66.** 0·09. **67.** 79·5. **68.** 0·0944. **69.** 0·1148. **70.** 0·0337. **72.** 15,546·6 pound-degree units. **73.** 11,312 of the same units; 5966·2 lbs. **76.** 1·5 lbs. **77.** 0·5625 lb. of ice will be melted, and the result will be a mixture of ice (0·4375 lb.) and water (1·5625 lb.) at 0°. **78.** The temperature will be lowered by $17^\circ\cdot 5$ (*i.e.* to $7^\circ\cdot 5$). **79.** 10 lbs. of water at 18°. **80.** All the snow will be melted and raised to 10°. **82.** 3·26 gm. **83.** 3/16 of the water will be frozen. **84.** 0·5. **85.** $1\cdot 6\times 10^6$ units. **86.** 16 cm. **89.** Contraction = 0·17 c.c. **90.** $888\cdot\dot{8}$ units. **91.** 0·09. **92.** 0·0336. **93.** Contraction = 0·0214 c.c. **94.** Sp. heat = 0·07724. **95.** Sp. gr. of ice = $\frac{11}{12}$ = $0\cdot 91\dot{6}$. **96.** Ice melted = 0·0873 gm.; specific heat of substance = 0·0814.

104. Vapour-pressure = 6·55 mm.; relative humidity = 0·55 (or 55 per cent). **110.** 31,200. **111.** 21,480 units. **112.** 6·29 lbs. **113.** 536·3. **114.** 37·31 gm.

116. 535. **118.** 8040 pound-degrees of heat. **119.** As 1 : 5. **121.** 541. **122.** 25·05. **123.** 0·0947.

EXAMINATION QUESTIONS

124. 0·127. **125.** 222·7 gm. **126.** There would have been a rise of 2°·4 C. **127.** (*a*) 0·625 lb. of ice will be melted; (*b*) all the ice will be melted and the resulting temperature will be 26°·6. **128.** 0·6389. **129.** 0·091. **130.** $7\frac{7}{80}$ units (pound-degree-Fahr.); 12·53 H.-P. **131.** 1·061. **132.** 43°·03. **133.** 0·1. **134.** 0·10345. **135.** 0·08 lb. **136.** The final temperature is 20°. **138.** A fall of 18°·3 C.; to raise the whole to 60° C. would require an expenditure of 585,051 ft.-lbs. of work. **139.** 0·095. **141.** 0·1108. See p. 159. **142.** Total pressure at 25° = 846·26 mm. **143.** 7·95 kgm. **144.** 540. **145.** 64°·43. **146.** 48°·92. **147.** 71°·45. **148.** 3,866,400 units. **149.** 537·7. **150.** 4859 gm. **152.** 557 units. **153.** 14,735 gm.

CHAPTER VI—CONDUCTIVITY AND THERMODYNAMICS

1. **Conductivity.**—**3.** 5,760,000. **4.** 230,400 units. **5.** 0·0003. **6.** 0·16. **7.** $4\cdot2 \times 10^6$ units; 5 kilogrammes per hour. **8.** $1\cdot476 \times 10^9$ units. **9.** 126 kilogrammes. **11.** $0\cdot1\dot{7}$. **12.** 0·0384. **13.** 3,869 kilogrammes. **14.** The multiplier for reducing to the C.G.S. system is 0·01.

2. **Thermodynamics.**—**16.** 7·512 gramme-degrees. **17.** 7000 feet. **18.** $1\cdot905 \times 10^{10}$ ergs. **19.** 683·2 metres. **20.** 0°·71 warmer. **21.** 1°·89 C. **22.** 4480 gramme-degrees; 194 metres per second. **23.** 2010 watts. **24.** 85,714 grammes. **25.** Work done = 3,710,000 ft.-lbs.; rate of working = 24,733 ft.-lbs. per minute. **27.** $6\cdot770 \times 10^9$ ergs. **28.** 362·5 metres per second. **31.** The engine is of 67 H.-P., and its efficiency is 0·0842 (or 8·42 per cent). **32.** 5·97 per cent. **33.**

3·554 per cent. **34.** $1\frac{1}{3}$ mile nearly. **35.** 3·73 lbs. **39.** 24,367 units. **43.** $1{\cdot}21 \times 10^7$. **44.** The work done is 2124 ft.-lbs., and the heat-equivalent of this would suffice to raise 1·528 lb. of water through 1° C. **46.** $J = 4{\cdot}2 \times 10^7$.

EXAMINATION QUESTIONS

49. 1800 kgm. **50.** $8{\cdot}18 \times 10^7$ heat-units. **51.** 590,400 units. **52.** 0°·88. **53.** $3{\cdot}6 \times 10^7$ units. **54.** 6100 gramme-degrees; 2586·4 kilogramme-metres. **55.** 0°·00005952. **56.** One-eighth. **57.** $\pi^2/50$ units ($g = 32$). **58.** 828. **59.** 9°·6. **60.** 1817·3 gm. **62.** 19,545 ft.-lbs.; 14·06 lb.-degree units. **63.** One-eighth. **64.** 496·3 heat-units per gramme (taking the atmospheric pressure as a megadyne, and the volume of 1 gm. of water at 100° as 1 c.c.) **66.** $1{\cdot}293 \times 10^8$ ergs. **67.** 0·0949 or 9·49 per cent. **68.** 34,543 cm. per sec. **69.** One-eighth. **70.** One-sixteenth. **71.** The 10-H.-P. engine is more nearly perfect than the other. **72.** 107°·16 C. **74.** The melting-point is raised 0°·02484 per megadyne.

CHAPTER VII—LIGHT

2. 3 ft. **5.** As 9 is to 4. **6.** As 4 is to 9. **7.** 10·24 candle-power. **8.** As 5 is to 2. **9.** 4·41 candle-power. **10.** (1) 1 yard from the candle, between this and the lamp; (2) 2 yards on the other side of the candle. **11.** (1) Between the two gas-flames and 60 cm. from the 9 C.-P. one; (2) 420 cm. beyond the 9 C.-P. flame. **15.** 5 images. **22.** 45°. (Notice that the successive angles of incidence diminish by 15°.)

32. Concave; $f = +9$ in. **33.** To a point 20 cm. in front of the mirror. **34.** Concave; $f = +15$ cm. **35.** 7·5 cm. *behind* the mirror. **36.** $p' = 90$ in.; image is 5 in. long. **37.** Real, inverted, 20 cm. in front of the

mirror; 4 cm. long. **38.** Converges to a point (1) 40 cm., (2) 60 cm. in front; (3) parallel; (4) diverges from a point 60 cm. behind. **39.** $f = +8$ in. **40.** 30 in. behind the mirror; magnification $= 6$. **41.** Virtual. **42.** $f = +9$ in. **44.** $f = \frac{12}{13}$ ft.; the mirror must be placed 1 ft. from the object. **45.** $p = 18$ in., $p' = 36$ in.; the image would be one-half the size of the object. **47.** The image is real and twice the size of the object; the magnification is the same when $p = f/2$, but in this case the image is virtual. **48.** Distance $= 3f$. **49.** (1) Image is virtual and erect, 4 in. behind the mirror, 2 in. long; (2) image 8 in. behind the mirror, 1 inch long. **50.** The image is 7·5 cm. behind the mirror and is 3 cm. long. **51.** Half that of the object.

57. 8/9; 3/4. **59.** $\sqrt{2}$. **60.** $\sqrt{2}$. **61.** Radius of circle $= 4$ inches. **63.** $1\dot{3}$. **65.** 1·528. **66.** $\sqrt{2}$. **67.** Angle of incidence $= 45°$; deviation $= 30°$. **68.** Expand the expression for μ given in Ex. 64: it will be found that $\cot \frac{A}{2} = 3·566$, and $\therefore A = 31° \ 20'$. **70.** $48' \ 36''$; $1° \ 29'$. **72.** $3°$. **78.** 60 cm. **85.** $f = -3$ in. **86.** 3 feet behind the lens. **87.** They converge to a point 5 cm. behind the lens. **89.** $f = -4\frac{1}{2}$ in. **90.** $p' = -20$ cm.; as 1 : 3. **91.** 10 cm. **92.** 2 ft. 6 in.; equal. **93.** $p' = -2$ ft.; diameter of image $= 1$ in. **95.** 1 ft. from the lens, and on the same side as the object. **97.** $f = -37·5$ cm.; as 3 : 1. **98.** $p = 1$ ft.; $p' = -3$ ft. **99.** 10 in. from the lens; $f = -8\frac{4}{7}$ in. **100.** $p = 3f$. **101.** (1) $p = 2$ ft.; (2) $p = 1$ ft. **102.** A convex lens of -5 in. focal length, held 6 in. from the wall. **103.** The three values of f are 24·9, 25·7, and 25·4 cm.: the mean value is 25·3 cm. (negative). **104.** 8 in. from the lens. **107.** As if they diverged from a point 2 in. in front of the lens. **108.** Concave: $f = 15$ cm. **109.** $p = 125$ cm. **110.** 16 cm. **112.** $F = -10$ cm. **113.** $F = -24$ cm. **114.** $f = +24$ cm. **115.** $f_1 = -3$ in.; $F = -6$ in.

and $\therefore f_2 = +6$ in. **118.** 4 cm.; magnifying power = 20/4 = 5. **119.** He will be able to see distinctly objects at a distance of 19·2 cm. (*i.e.* his range of distinct vision will be increased by 28·8 cm.). **120.** At a distance of 3 in. **121.** 120 cm. **122.** $f = +20$ cm. **125.** 58·42 cm. **126.** The three values of f are 16·84, 16·60, and 16·75 cm.: the mean value is 16·73 cm. (negative). **127.** (1) $\lambda = (\sin 3° \ 23')/001 = 0{\cdot}0005902$ mm.; (2) of $\lambda = 0{\cdot}0005904$ mm. **128.** Mean value of $\lambda = 0{\cdot}0005884$ mm.

EXAMINATION QUESTIONS

131. I : O = 2 : 1. **132.** $p = 10$ in. $I = 2\frac{2}{3}$ in. long. **133.** 2 ft.; real. **134.** The image moves through 3·7 in., *i.e.* from 24 in. behind the mirror to 20·3 in. behind it. **135.** Image is real and 0·26 cm. high. **136.** Real, inverted, 2·3 mm. broad and 7·7 mm. long. **140.** 75°. **142.** 4/9ths of the thickness of the glass. **143.** 1·2 in. **145.** The object will appear to be 5·67 cm. from the side from which it is viewed. **146.** At a distance of $14\frac{4}{7}$ ft. **147.** Magnifying power = 10/3. **148.** O : I = 1 : 2. **149.** Lens is convex; $p = 2{\cdot}25$ in. **150.** $f = -2{\cdot}4$ in. **151.** Concave: focal length = $6\frac{2}{3}$ in. **152.** A concave lens of $6\frac{2}{3}$ in. focal length. **153.** 6 in. (concave). **154.** Concave spectacles of $5\frac{1}{7}$ in. focal length. **155.** $p' = 4\frac{1}{2}$ in. **156.** A concave lens of 3 ft. focal length or a convex lens of 18 in. focal length. **157.** Concave spectacles of 6 in. focal length. **158.** At a distance of 2·28 in. **159.** $\frac{8}{9}$ in. from the lens. **160.** $\mu = 1{\cdot}5603$. **161.** 9·75 in. beyond the convex lens. **162.** $\frac{46}{83}$ in. in front of the objective. **164.** $f = +66{\cdot}\dot{6}$ cm. (concave). **167.** $f = -1{\cdot}5r$. **168.** $f_1 = -3\frac{1}{3}$ ft. (convex) and $f_2 = +5$ ft. (concave). **169.** Magnifying power = 8/3; distance = 2·5 in. **170.** 2·42 in.

CHAPTER VIII—SOUND

1. 341·0 metres per sec. **2.** 30°·4. **3.** 4·39 sec. **5.** 128·1, or almost exactly the note C. **6.** 1261·2 metres per sec. **7.** 200. **8.** 32·4 cm. **10.** 2.059×10^{10}. **12.** 2.109×10^{12}. **13.** 4.2×10^5 cm. per sec. **15.** A fifth (*c* to *g*). **16.** *g''*, *i.e.* a twelfth above the octave of *c*. **17.** 25 lbs.; an additional 11 lbs. (*i.e.* total weight = 36 lbs.). **18.** 1.024×10^8 dynes. **19.** 46·8 vibrations per sec. **20.** Stretching force = 6.759×10^6 dynes = weight of 6·89 kgm. **21.** 580. **22.** A major third (1 and 2); a fifth (2 and 3); and a seventh (1 and 3). **23.** (1) 18 grammes; (2) 14·06 grammes. **25.** 440 vibrations per sec. **28.** 460.

EXAMINATION QUESTIONS

29. 33,129 cm. per sec. **30.** 1.005×10^5 cm. per sec. **31.** 1.459×10^5 cm. per sec. **32.** 3.091×10^4 cm. per sec. **33.** The frequency will be increased in the ratio of 17 : 20. **34.** 420 metres per sec. **35.** 1·406 metres; 4·218 metres. **37.** Frequency = 1808·3. **38.** 8 vibrations per sec. **39.** 400 lbs. **40.** 120 vibrations per sec. **41.** Period = 0·004518 sec. **42.** As 2 : 1 : 1. **43.** 0·007,905 sec. **44.** 247·6 vibrations per sec., *i.e.* a little below C. **45.** 2.352×10^{12}. **46.** 65·3 vibrations per sec. **47.** 1980 vibrations per sec., *i.e.* a note a little above *b''*. **48.** C (256 vibrations per sec.). **51.** 1·593. **52.** 318·4 metres per sec.; 445·7 metres per sec. **53.** Apparent frequency = 2157.

CHAPTER IX—MAGNETISM

3. 8 dynes. **4.** 12·8 dynes. **5.** 5. **6.** 24. **7.** 2·7. **8.** 50. **10.** 18. **11.** As $\sqrt{2} : \sqrt{3}$. **16.** The force acts parallel to the base of the triangle and its magnitude is m/l^2, when m is the strength of pole and

l the side of the triangle. **19.** 37·71 oscillations per min. **20.** As 0·417 : 1. **21.** As 0·352 : 1. **22.** 18·93. **23.** 1·5 gm.

EXAMINATION QUESTIONS

26. It must be twisted 150° further. **27.** As 17 : 24. **28.** 14 times per minute. **29.** 1·8 ft. from the centre of the stronger magnet. **30.** Strength of pole = 128. **32.** Moment = 156·25 (supposing the pole to be on the perpendicular to the magnet through its centre). **33.** Change of zero-position, 36′; scale-divisions, 10·47 (mm.)

CHAPTER X—ELECTROSTATICS

2. −27. **3.** 12 cm. **4.** 8 dynes. **5.** Charge $= \pm 10{\cdot}9$. **8.** Attractive force = 2 dynes; repulsive force = 0·25 dyne. **9.** The original charge was 12 units: 12 cm. apart. **10.** The resultant force at the third corner is $\sqrt{3}/200$, and acts along the bisector of the angle. **12.** $\sigma = 10/\pi = 3{\cdot}8$. **13.** 198 units. **15.** B will be in equilibrium between A and C at a distance $\overline{AC}/3$ from A; it will also be equally repelled by both when it is placed on CA, produced so that BA = AC. **16.** The charge on A is to that on B as 9 : 4.

19. $2\sqrt{2} = 2{\cdot}828$. **20.** Potential is 40; charges are 400 and 600. **21.** Potential 15; charges 1125 and 375. **22.** $5/3 = 1{\cdot}\dot{6}$. **24.** L; 50. **26.** Distance $= 4\sqrt{5}$ cm.; force = 0·16 dyne. **28.** 8. **30.** As 8 : 1; 52·5 units. **31.** As 2 · 1. **33.** 100. **34.** As 9 : 8. **36.** 500. **38.** The potential is $(rv + r'v')/r'$. The charge $(rv + r'v')$ is wholly on the outer sphere.

43. 4284. **44.** 341·8 and 158·2. **45.** Charge on

large sphere $= 80$, on small sphere $= 440$; common potential $= 88$. **46.** 34·34 metres.

48. 75,000 ergs. **49.** 300 units; 4500 ergs. **50.** 13,440 ergs. **51.** The potentials are as 3 : 8; the energies of the charges are as 9 : 16. **52.** As 3 : 8. **53.** As 1 : 6. **55.** Potential $= 346·4$. **57.** As 1 : 3. **58.** As 1 : 5. **59.** $\frac{1}{16}$, $\frac{1}{4}$, and $\frac{1}{16}$ respectively of the energy of the original charge. **63.** The energies of the several discharges (taken in the same order as in Ex. 47) are as 9 : 3 : 4 : 2. **64.** As 4 : 1. **67.** 18·91 electrostatic units.

EXAMINATION QUESTIONS

68. 500 ergs. **69.** Potential of first jar $= 25/84$, potential of second jar $= 25/120$; hence electricity will pass from the first to the second. **70.** $2\sqrt{\pi}$; $500/\sqrt{\pi}$. **71.** As 9 : 5. **72.** 121·3 units. **74.** The charges are 108 and 36. **75.** (*a*) 100; (*b*) 266·6. **76.** As 84 : 325. **78.** 0·267 microfarad. **79.** $12·5 \times 10^6$ dynes. **80.** 338·6 volts.

CHAPTER XI—CURRENT ELECTRICITY

4. 2·823 ohms. **5.** 60 volts. **6.** 50 ohms. **7.** 0·2 ampère. **8.** 5 cells. **9.** Current $= 8\frac{1}{3}$ ampères; potential of point B $= 80$ volts. **10.** Current $= 1·2$ ampère; potential difference $= 1·44$ volt. **11.** Original resistance $= 15$ ohms; additional resistance required $= 15$ ohms. **12.** 20 cells. **13.** 6 additional cells. **14.** 1·6 ampère. **15.** 5 ohms. **16.** 4 metres. **17.** The two E.M.F.'s are equal. **18.** Resistance of battery $= 60$ ohms; resistance of wire $= 28·8$ ohms. **19.** 1·07 volt E.M.F., and 13 ohms resistance per cell; potential difference $= 14·98$ volts. **20.** E.M.F. of accumulator $= 2·2$ volts; current after re-arrangement $= 1$ milliampère. **21.** 11 ohms. **22.** Current is increased in ratio of 7 : 10. **24.** 15

.8. 0·6 mm. **29.** 181·$\dot{3}$ metres. **30.** The re-
.e of B is 81 times that of A. **31.** 72 ohms. **32.**
ohm. **33.** 6 ohms. **34.** 20·31 ohms. **35.**
.)434 ohm. **36.** 0·2903 ohm. **37.** As 1 : 4. **38.**
0·007547 ohm. **39.** 872·8 cm. **40.** If r is expressed in C.G.S. units, the specific resistance is $\pi r/40{,}000\ mn^2$. **41.** 0·2188 ohm. **42.** As 5 : 1. **43.** 1080 yards. **44.** 3·04 microhms. **45.** 32·48 microhms. **46.** 0·0783 ohm. **47.** As 1 : 1·98.

51. 5·411 ampères. **52.** 0·0754 dyne. **54.** 0·206 ohm. **55.** 6 ohms. **56.** 15 ohms. **57.** 41 ohms. **59.** As 13 : 8.

65. 14 min. 55 sec. **66.** 1·186 gm. **67.** Nearly 2 hours. **68.** 0·05055 ampère. **69.** 0·7454 ampère. **70.** 0·0003295. **71.** 0·6755 ampère. **72.** 4017 ampères. **73.** 95·1 gm. of copper and 97·5 gm. of zinc. **74.** 2·658. **75.** 0·3355.

79. The resistance is equal to that of one of the sides. **80.** $3r/4$, where r is the resistance of one of the sides. **81.** 11 ohms. **82.** $C : C' = l's : ls'$. **83.** The current will be doubled. **84.** 89·1 legal ohms. **86.** 4·5 ampères. **87.** The resistance will be diminished by one-fourth. **88.** $6{\cdot}258r$, where r is the resistance of an inch of the base wire. **89.** As $(3\sqrt{2}+2) : 3(\sqrt{2}+1)$. **91.** 0·1 ampère. **95.** 4 ohms. **96.** 4·5 ohms; 0·5 ampère. **97.** $C : C' = 5 : 6$. **98.** $C : C' = 5 : 3$. **103.** $2(R-G)$. **107.** 2 ohms; 2·125 ampères. **108.** The constants are as 3 : 7; the resistances as 7 : 3. **110.** $C = 18E/(11R+6r)$.

113. 2 rows of 2 cells each. **114.** 2 rows of 6 cells each. **115.** 3 rows of 4 cells each. **116.** 4 rows of 12 cells each; current = 0·39 ampère; potential difference = 5·85 volts. **117.** 3 rows of 6 cells each.

121. (1) : (2) $= (r_1 + r_2)^2 : r_1 r_2$. **122.** As $r_1 r_2 : (r_1 + r_2)^2$. **123.** As 5·92 : 1. **124.** 0·8. **125.** 487·5 watts. **127.** 2·81 watts per candle; 38,572 calories. **128.** 3·28 watts per candle; 7·04 H.-P. **130.** As 3 : 1 in both cases. **131.** 16°·8. **132.** 0·9$\dot{3}$ volt. **133.** 0·9165 ampère. **134.** $k = 0{\cdot}00485$.

138. 81·2%. **140.** Horse-power = 5·986; efficiency = 85·5%. **145.** Efficiency 31·9%. **148.** 3 H.-P. Efficiency = 87·34%. **149.** 12,600, 14,400, 15,000 and 14,400 watts respectively. The available power is a maximum at 100 ampères, and 15,000 watts are then wasted as heat. **150.** 0·006786 volt.

EXAMINATION QUESTIONS

151. 1·3 ohms. $C_1 : C_2 = 199 : 208$. **152.** 1·82 ampères. **153.** As 9 : 25. **154.** $13\frac{8}{9}$ ohms. **155.** The deflection of the galvanometer will increase to nearly 5°, and in the same time 100 c.c. of hydrogen will be liberated and the thermometer will rise 5°. **156.** 1395 units. **157.** 0·75 volt. **158.** Current = 0·83 ampère; Ag = 1·66 gm.; Cu = 1·465 gm.; Pb = 1·59 gm.; H liberated = 0·01537 gm.; H_2SO_4 decomposed = 1·506 gm. **159.** $\frac{1}{7}$ ohm. **160.** As 102 : 51 : 2. **162.** 1·8$\dot{3}$, 1·275, and 2·09 ampères. **165.** 0·505 ampère; $C = 0{\cdot}505 \tan\delta$. **166.** $2{\cdot}4 \times 10^{12}$ ergs; 10 ampères; 2 volts. **167.** One cell is wrongly connected. **168.** As 10 : 1. **169.** 343·25 watts in each case. **170.** 4160 watts; 2048 watts. **171.** Total watts 1200, of which 60 are used in the accumulators and 40 in the leads, leaving 1100 for the lamps, or 44 watts per lamp. The resistance of each lamp is 275 ohms. **172.** 0·00047 volt. **174.** 0·02036 volt. **175.** 2827 C.G.S. units. **176.** 71,432 ohms.

Chapter XII—Miscellaneous Examples

1. 4 cm. per sec. per sec. 2. 196 feet; 7 seconds 3. 30° from the vertical. 4. A – uniform; B – variable and increasing. 5. 6 feet per sec. 6. 10 dynes. 7. 2·736 poundals. 8. $g=980$ cm./sec.^{2} 9. 13,305,600 ft.-lbs. 10. 3926 ft.-lbs. 11. Average pressure = 13,333 poundals or $416 \cdot \dot{6}$ lbs. weight; work done = 1250 ft.-lbs. 12. 0·833 cm. 15. 7 lbs. 16. The magnitude of the resultant is 7 and it acts at a distance of $9\frac{2}{7}$ from the first force. 17. 90 lbs. 18. At a point on the median through the first angular point, and four-fifths of the length of the median from that angular point. 19. True weight = $\sqrt{WW'}$; ratio of arms, L : R = $\sqrt{W} : \sqrt{W'}$. 20. True weight = 50·0012; L : R = 1 : 1·0000024 (see Introduction, § 13). 21. 334,353 ft.-lbs. 22. 53·17 c.c. in the mercury, and 46·83 c.c. in the water. 23. 18 grammes. 24. Specific gravity of cork = 0·24. 25. Tension = weight of 99·05 gm. 26. 45·5 c.c. 27. Seven. 30. 76·126 cm. 31. 98·48 cm. 32. 0·0007407. 33. Mean apparent coefficient of expansion = $0 \cdot 001\dot{6}$. 34. $5 \cdot 575 \times 10^5$ units of heat. 35. 536. 36. 0·972. 37. $2 \cdot 3 \times 10^7$ and 964·8. 38. $9 \cdot 094 \times 10^7$ ergs. 39. 0·05279 or 5·279 per cent. 40. 8°·571. 41. 16 candle-power. 42. $\mu = 1 \cdot 5$. 43. $f = -24$ cm. 44. He should use more convergent spectacles,—of focal length = $-17\frac{1}{7}$ cm. 45. 160 vibrations. 46. 25·94 cm. 47. 300 and 320. 48. 175°·5 further (*i.e.* so that the torsion head is turned through 535°·5). 49. The potential is zero and the force unity; the force acts in a direction parallel to the base of the triangle. 50. The charge on the larger sphere was either – 6 or $-\frac{1}{6}$. 51. 0·5 ampère. 52. 8 cells. 53. (1) 0·5 ampère; (2) $\frac{6}{7}$ ampère. 54. 95 ohms, or 9·5 ohms per cell. 55.

0·6283 dyne. **56.** 0·18. **57.** 0·2 ampère through the battery; 0·1$\dot{3}$ ampère through one side of the triangle, and 0·06 ampère through the two other sides. **58.** 0·2874 ohm; 9·774 volts. **59.** 49 cells. **60.** 0·02 ampère; 40 watts.

EXAMINATION QUESTIONS

61. Pull on rope = 100 lbs. weight; work done = 20,000 ft.-lbs. **62.** The sound reaches the ground first, viz. in 52·36 sec. The fragment takes 60 secs. **63.** 1,029,600,000 ft.-lbs.; 5·2 H.-P. **64.** 2344 times the weight of the ball. **65.** Time = 0·004 sec.; force = 15,625 poundals or 488·3 lbs. weight; activity = 244,150 ft.-lbs. per sec. **66.** Momentum = 800,000 gm. cm./sec.; energy = $1·6 \times 10^{10}$ ergs; force = 8×10^6 dynes. **67.** (*a*) 80 ft.; (*b*) 13·28 ft. per sec. ($g = 32$). **68.** (*a*) Nil; (*b*) 90,450 ft.-lbs. **70.** K.E. = $2·5 \times 10^9$ ergs. **71.** From a height of 1811 cm. (taking $g = 981$). **74.** 5·03 lbs. **76.** 10·5 ft. **78.** 8198·5 ft. per sec. **80.** $1·408 \times 10^{12}$. **82.** Specific gravity of cylinder = 0·8. **84.** Density of cylinder = 2·71; of liquid = 0·795. **86.** 30 in. **87.** For a quarter of a millimetre the pressure is 49 dynes per sq. cm. **89.** 0·197 in. **90.** The clock loses 0·648 sec. per day. **92.** $2·2 \times 10^5$ dynes per sq. cm. **93.** $t = 452°$ C. **94.** Water-equivalent = 12 gm.; specific heat = 0·112. **95.** As 1 : 2·4. **96.** 20 lbs. of mercury, $\frac{7}{79}$ lb. of water, and $\frac{72}{79}$ lb. of ice, all at 0°. **97.** 125·8 gm. **98.** 7.95 kgm. **99.** 4·505. **100.** 136·7 ft.-lbs. **101.** 1640·38 units. **102.** The image is formed between the mirror and the lens, 40 cm. from the former, and is real. **105.** 0·0006 cm. **106.** 5000 metres per sec. **107.** $n = 512$. **108.** 2·56 beats per sec. **109.** As $\sqrt{6} : 1$. **110.** 40. **111.** $\frac{25}{32}$ in. **112.** 1·1 beats per sec. **113.** $2·352 \times 10^{12}$. **115.** 384 and 388. **116.** A weight of 267·2 kilo-

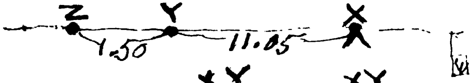
grammes. **117.** The radii are 4·24 in., 7·38 in., etc. **118.** Modulus $= 2{\cdot}45 \times 10^{12}$. **120.** Torsion $= 13{\cdot}19$ kgm. weight; thrust $= 184{\cdot}8$ kgm. weight. **121.** 0·02438 dyne. **123.** $1{\cdot}\dot{3}$. **124.** As $1 : -7$ (one charge is negative). **126.** Potential $= 80\pi$; pull at half distance $= 4000$ dynes. **127.** The galvanometer deflection becomes very nearly 5°, and in the same time 100 c.c. of hydrogen is evolved and the thermometer rises 5°. **129.** $\frac{8}{11}$, $\frac{1}{11}$, and $\frac{4}{11}$ ampère respectively. **130.** 565·5 C.G.S. units or $5{\cdot}655 \times 10^{-6}$ volt. **131.** $13\frac{1}{3}$ ergs. **133.** $\frac{24}{41}$ ampère through the battery; $\frac{18}{41}$ ampère through AB; and $\frac{6}{41}$ ampère through CD. **134.** As 5 : 4. **135.** $0{\cdot}913p/V$ pence. **136.** 1875π C.G.S. units or 0·00005891 volt.

TABLE OF NATURAL SINES AND TANGENTS.

Degrees.	Sines.	Tangents.	Degrees.	Sines.	Tangents.	Degrees.	Sines.	Tangents.
1°	·017	·017	31°	·515	·601	61°	·875	1·804
2°	·035	·035	32°	·530	·625	62°	·883	1·881
3°	·052	·052	33°	·545	·649	63°	·891	1·963
4°	·070	·070	34°	·559	·675	64°	·899	2·050
5°	·087	·087	35°	·574	·700	65°	·906	2·145
6°	·105	·105	36°	·588	·727	66°	·914	2·246
7°	·122	·123	37°	·602	·754	67°	·921	2·356
8°	·139	·141	38°	·616	·781	68°	·927	2·475
9°	·156	·158	39°	·629	·810	69°	·934	2·605
10°	·174	·176	40°	·643	·839	70°	·940	2·747
11°	·191	·194	41°	·656	·869	71°	·946	2·904
12°	·208	·213	42°	·669	·900	72°	·951	3·078
13°	·225	·231	43°	·682	·933	73°	·956	3·271
14°	·242	·249	44°	·695	·966	74°	·961	3·487
15°	·259	·268	45°	·707	1·000	75°	·966	3·732
16°	·276	·287	46°	·719	1·036	76°	·970	4·011
17°	·292	·306	47°	·731	1·072	77°	·974	4·331
18°	·309	·325	48°	·743	1·111	78°	·978	4·705
19°	·326	·344	49°	·755	1·150	79°	·982	5·145
20°	·342	·364	50°	·766	1·192	80°	·985	5·671
21°	·358	·384	51°	·777	1·235	81°	·988	6·314
22°	·375	·404	52°	·788	1·280	82°	·990	7·115
23°	·391	·424	53°	·799	1·327	83°	·993	8·144
24°	·407	·445	54°	·809	1·376	84°	·995	9·514
25°	·423	·466	55°	·819	1·428	85°	·996	11·43
26°	·438	·488	56°	·829	1·483	86°	·998	14·30
27°	·454	·510	57°	·839	1·540	87°	·999	19·08
28°	·469	·532	58°	·848	1·600	88°	·999	28·64
29°	·485	·554	59°	·857	1·664	89°	1·000	57·29
30°	·500	·577	60°	·866	1·732	90°	1·000	∞

Printed by R. & R. Clark, Limited, *Edinburgh*

	0	1		5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	5	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
11	0414	0453	3	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
12	0792	0828	9	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
13	1139	1173	4	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
14	1461	1492	8	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
15	1761	179	0	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
16	2041	2068	2	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
17	2304	233	2	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
18	2553	257	1	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
19	2788	281	9	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
20	3010	3032	6	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
21	3222	3243	2	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
22	3424	3444	7	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
23	3617	363	1	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
24	3802	382	4	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
25	3979	399	6	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
26	4150	416	7	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
27	4314	433	7	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
28	4472	448	7	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
29	4624	463	6	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
30	4771	478	4	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
31	4914	492	1	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
32	5051	506	7	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
33	5185	519	3	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
34	5315	532	8	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
35	5441	545	3	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
36	5563	557	6	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
37	5682	569	9	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
38	5798	580	2	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
39	5911	592	3	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
40	6021	603	5	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
41	6128	613	5	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
42	6232	624	5	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
43	6335	634	5	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
44	6435	644	3	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
45	6532	654	2	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
46	6628	663	9	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
47	6721	673	7	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
48	6812	682	3	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
49	6902	691	0	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
50	6990	699	5	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
51	7076	708	1	9845	9850	9854	9859	9863	0	1	1	2	2	3	3		
52	7160	716	6	9890	9894	9899	9903	9908	0	1	1	2	2	3			
53	7243	725	0	9934	9939	9943	9948	9952	0	1	1	2	2	3			
54	7324	733	4	9978	9983	9987	9991	9996	0	1	1	2	2	3			



$$S.G. = \frac{XY}{YZ} = \frac{XY}{XZ - XY}$$

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